Focusing on dark energy with weak gravitational lensing

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This work is devoted to the study of phaenomenology within the quest for Dark Energy (DE) nature. We study the impact of tomographic WL measurements on constraining dynamical and/or coupled DE models. Within this context, it will be outlined how massive neutrinos, added to the total cosmological energy balance as hot dark matter (DM), allow the consistency with present data of a higher DM–DE coupling.

1 Introduction

Cosmic acceleration, originally detected through high-redshift supernovae [1], is now supported by several cosmological observations, such as CMB [2] and large scale structure [3] data. Since it has been measured, its nature needed to be disvealed.

It is known that models with a cosmological constant $\Lambda$ (ΛCDM) apparently accommodate all available data systems. Still the physical origin of $\Lambda$ (false vacuum) causes well known fine tuning and coincidence problems. The former problem is partially eased in dynamical DE (dDE) scenarios, when self interaction is due to a tracking potential $V(\phi)$ [4]. If $V(\phi)$ is SUGRA [5] , the fit with data is at least as good as for ΛCDM [6]. Moreover, in the attempt to ease the coincidence problem, DM–DE interaction (e.g., [7,8]) was also considered, yielding an energy transfer between the dark components, so allowing a (quasi)–parallel scaling of DM and DE from a fairly high redshift until the present.

Recent works placed constraints on possible couplings, parametrized by $\beta$ (see below), by using SNIa data [9] or the redshift evolution of the Hubble parameter, $H$ [10]. Accordingly, $\beta \gtrsim 0.12–0.15$ [11, 12] is hardly consistent with observations. Unfortunately, such a low coupling level no longer eases the coincidence problem [13], but, once the genie has come out from the lamp, it is hard to put it back inside.

The aim of this work is to add a brick to the construction of this wide building, trying to study the impact of tomographic WL measurements on constraining dynamical and/or coupled DE models. Within this context, it will be outlined how massive neutrinos, added to the total cosmological energy balance, allow the consistency with present data of a higher DM–DE coupling.

The outline of the paper is as follow. In Sec. 2 we review the basic properties and definitions of coupled models. Sec. 3 describes the statistical methods used. In Sec. 4 we discuss about the possibility of slightly easing the coincidence problem in the presence of massive neutrinos. In Sec. 5 we show the results of the FM analysis with coupled models using CMB and WL. Finally, in Sec 6 we draw our conclusions.

2 Interacting dark energy

We consider a cosmological model where the DE field $\phi$ interacts with the cold DM component. The model requires the specification of the potential $V(\phi)$ and the function $f(\phi)$ characterizing the coupling. The equation of motion for $\phi$ then reads

$$\ddot{\phi} + 3H\dot{\phi} = -V_{\phi}^{\text{eff}} \quad \text{with} \quad V_{\phi}^{\text{eff}} = V + \rho_c . \quad (1)$$
Here dots denote ordinary time differentiation, $a$ is the scale factor, $H(a) = \dot{a}/a$ and $\rho_c$ is DM energy density. In turn, its evolution is governed by the following conservation equation:

$$\dot{\rho}_c + (3H + C\dot{\phi})\rho_c = 0, \quad \text{with} \quad C(\phi) = \frac{d\log(f)}{d\phi} = 4\sqrt{\frac{\pi}{3}} \frac{\beta(\phi)}{m_p}. \tag{2}$$

Here coupling function $C(\phi)$ has been parametrized by an adimensional parameter $\beta$ and $m_p$ is the Planck mass. Equation 2 can be integrated and gives $\rho_c(a) = \rho_{c,0} a^{-3} f(\phi)$. For $f = 1$, the last equation and Eq. (2) return ordinary dDE equations. The equations for the other components remain unchanged: $\rho/\rho_0 = (a_0/a)^\alpha$ with $\alpha = 3(w+1)$ ($a_0$ is a reference scale factor, not necessarily its today’s value; $\rho_0$ is the energy density when the scale factor is $a_0$). In particular, $w = 0$ for baryons, $w = 1/3$ for radiation.

In this work we consider the following expressions for the potential and the coupling function:

$$V(\phi) = \frac{\Lambda^{4+\alpha}}{\phi^\alpha} \exp\left(4\pi \frac{\phi^2}{m_{PL}^3}\right), \quad f(\phi) = \exp\left(\beta \sqrt{\frac{8\pi}{3}} \frac{\phi_0 - \phi}{m_{PL}}\right). \tag{3}$$

The potential $V(\phi)$ is called SUGRA [5] and is an example of tracking potential. It naturally arises in the context of Supergravity Theories. It depends on the slope $\alpha$ and the energy scale $\Lambda$. It is characterized by a rapid time variation of the equation of state, when DE becomes dominant. Therefore, assuming a constant $w$ for this class of potential may lead to misleading results. Fixing DE density today and $\Lambda (\alpha)$, however, determines a unique value of $\alpha$ (Lambda). CMB, SNIa and deep sample data yield $\Lambda \lesssim 10^3$GeV [6], in the absence of coupling.

The coupling function $f(\phi)$ depends on $\beta$, and $\phi_0$ is the field value today. In this work we assume a constant $\beta \geq 0$ (see however [14] for a different approach); data place the upper limit $\beta \lesssim 0.12 - 0.15$ [11,12]. For reasonable values of the cosmological parameters and of $\Lambda$, we expect coupling effects not to be relevant for $\beta \lesssim 0.01$, so that the dynamically interesting values for the coupling lies in the range $0.01 < \beta < 0.10$.

## 3 Methods: the Fisher matrix

The Fisher matrix (FM) approach [15, 16] assumes a reference model as the most probable one, i.e. as the maximum of $\mathcal{L}(\vec{x}|\vec{\theta})$, the likelihood distribution of the data system $\vec{x}$ given the model. If we take the set $(\vec{\theta}_i)$ as representative of the reference model, one can approximate $\mathcal{L}$ by a multivariate Gaussian distribution, built using its second derivatives in respect to the parameters at $(\vec{\theta}_i)$. Nevertheless, as is known, this technique is limited by the actual non–Gaussian behavior of data.

Therefore, FM is nothing but the Hessian of the log-likelihood function:

$$F_{ij} = -\left(\frac{\partial}{\partial \theta_i} \frac{\partial}{\partial \theta_j} \log \mathcal{L}\right)_0 = \sum_{\ell \ell'} \frac{\partial x_{\ell}}{\partial \theta_i} \text{Cov}_{\ell \ell'}^{-1}(\vec{\theta}_0) \frac{\partial x_{\ell'}}{\partial \theta_j} \tag{4}$$

In this definition, the covariance matrix $\text{Cov}$ has an expression which is typical of the observable $x_\ell$ used to build the FM. Moreover, the Cramér–Rao theorem [17] directly relates FM to the precision by which each $\theta_i$ can be measured.

In the $M$–$D$ space of the parameters, the hyper–ellipsoid of constant probability density defined by $Q(\theta, \vec{\theta}) = \Delta \theta^T F_{ij} \Delta \theta = \Delta \chi^2(N = 2, \sigma)$, can be projected in the two–parameter subspace, marginalizing over the other parameters:

$$\begin{pmatrix} \Delta \theta_i \\ \Delta \theta_j \end{pmatrix} \begin{pmatrix} (F^{-1})_{ii} & (F^{-1})_{ij} \\ (F^{-1})_{ij} & (F^{-1})_{jj} \end{pmatrix}^{-1} \begin{pmatrix} \Delta \theta_i \\ \Delta \theta_j \end{pmatrix} = \Delta \chi^2(N = 2, \sigma)$$

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The ellipse obtained can be interpreted as an estimate of the confidence region within a given confidence level \( \sigma \) for the two parameters \( \beta_1 \) and \( \beta_2 \) [18].

Let us now characterize CMB FM [19]. Let \( C_{\ell}^{XY} \) be the angular spectra of the input model, to which we must add a white noise signal, to obtain

\[
C_{\ell}^{XY} = C_{\ell}^{XY} + N_{\ell}^{XY}
\]

with

\[
N_{\ell}^{XY} = \delta_{XY}\sigma_{XY}^2 \exp \left[ l(l+1)\frac{\theta_{FWHM}^2}{8 \ln 2} \right].
\]

(6)

If \( f_{sky} \) is the effective fraction of sky, the expression of the covariance matrix is

\[
[C_{\ell}]_{\alpha\beta} = \frac{1}{l(l+1/2)f_{sky}} \begin{pmatrix}
(C_{\ell}^{TT})^2 & (C_{\ell}^{TE})^2 & C_{\ell}^{TE}C_{\ell}^{TT} \\
(C_{\ell}^{TE})^2 & (C_{\ell}^{EE})^2 & C_{\ell}^{TE}C_{\ell}^{EE} \\
C_{\ell}^{TE}C_{\ell}^{TT} & C_{\ell}^{TE}C_{\ell}^{EE} & \frac{1}{2}[(C_{\ell}^{TE})^2 + C_{\ell}^{TT}C_{\ell}^{EE}] 
\end{pmatrix}.
\]

(7)

When dealing with matter power spectra, we use the following definition [20, 21]:

\[
[C_{\ell}]_{ij} \approx \delta_{ij} \frac{V_{\ell}}{V_{ij}(k_{\text{sky}})} 2P^2(k_{\text{sky}}),
\]

(8)

where \( V_{ij}(k_{\text{sky}}) = \frac{4\pi k_{\text{sky}}^2 \delta k}{\delta_{ij}} \) and \( V_{\ell} = (2\pi)^3/V \) is the volume of the fundamental cell in \( k \) space, \( V \) the volume of the survey and \( V_{ij}(k_{\text{sky}}) = 4\pi k_{\text{sky}}^2 \delta k \) the volume of the shell of width \( \delta k \) centered on \( k_{\text{sky}} \).

Finally, in the tomographic WL case [22–24], the cosmic shear power spectrum for the \( i \)-th and \( j \)-th bin, \( P_{(ij)}(\ell) \), will receive a shot-noise contribution [25]:

\[
P_{(ij)}(\ell) = P_{k}^{(ij)}(\ell) + \delta_{ij} \frac{\sigma_\kappa^2}{n_i}
\]

(9)

where \( \sigma_\kappa \approx 0.22 \) [26] is the rms shear due to intrinsic ellipticity and measurement noise, while \( n_i \) is the average number density of galaxies per steradians in the \( i \)-th bin. The covariance matrix is

\[
\text{Cov} \left[ P_{(ij)}^{(\text{obs})}(\ell) , P_{(jn)}^{(\text{obs})}(\ell') \right] = \frac{\delta_{ij}}{2(l+1)\Delta \ell f_{sky}} \left[ P_{(im)}^{(\text{obs})}(\ell) P_{(jn)}^{(\text{obs})}(\ell') + P_{(im)}^{(\text{obs})}(\ell) P_{(jn)}^{(\text{obs})}(\ell') \right]
\]

(10)

where \( f_{sky} \) is the sky fraction covered by the survey and \( \Delta \ell \) is the bin width centred at \( \ell \). In Eq. (10) we have not included the non-Gaussian term, due to the contribution of the shear trispectrum [27, 28].

We consider a 9 parameter model: \( \omega_b = \Omega_b h^2 \), \( \omega_c = \Omega_c h^2 \) baryons and CDM density parameters, \( H_0 \) Hubble parameter, \( A_s \) scalar fluctuation amplitude, \( n_s \) spectral index, \( \tau_{\text{opt}} \) cosmic opacity to CMB photons, \( \Lambda \) GeV energy scale in SUGRA potential, \( \beta \) DM–DE coupling parameter, \( \sum m_n/eV \) total neutrino mass. We estimate the neutrino mass density parameter, \( \Omega_\nu h^2 \), via the relation \( \Omega_\nu h^2 = \frac{\sum m_n}{0.68\, eV} \). We compute the CMB anisotropies (temperature and polarisation) power spectra and the transfer functions, used to calculate linear matter power spectrum, using a modified version of CAMB [29].

Numerical derivatives were evaluated considering a 5% stepsize. The observational features for CMB and galaxy surveys mission considered in the paper are listed in Table 1.

Table 1: CMB (left) and galaxy surveys (right) specifications used in the paper. In the right table, scales and volumes are in Mpc/h and (Mpc/h)^3, respectively.

<table>
<thead>
<tr>
<th>Mission</th>
<th>( l_{\text{max}} )</th>
<th>( f_{sky} )</th>
<th>( \theta_{FWHM} )</th>
<th>( \sigma_T )</th>
<th>( \sigma_P )</th>
<th>Mission</th>
<th>( k_{\text{min}} )</th>
<th>( k_{\text{max}} )</th>
<th>Volume</th>
</tr>
</thead>
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<tr>
<td>WMAP [30]</td>
<td>1000</td>
<td>0.8</td>
<td>13'</td>
<td>260</td>
<td>500</td>
<td>2dF [32]</td>
<td>0.02</td>
<td>0.1</td>
<td>10^8</td>
</tr>
<tr>
<td>PLANCK [31]</td>
<td>2500</td>
<td>0.8</td>
<td>7.1'</td>
<td>42</td>
<td>80</td>
<td>SDSS [33]</td>
<td>0.02</td>
<td>0.15</td>
<td>0.72 \times 10^9</td>
</tr>
</tbody>
</table>

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Fig. 1: Transfer functions (CMB anisotropy spectra) for the models indicated in the frame are plotted in the left (right) panel. In the lower plot of the right panel, we show the residual differences between the models with $m_\nu = 0$, $\beta = 0$ and with $m_\nu \neq 0$, $\beta \neq 0$. Dotted lines comprise the range of the expected cosmic variance. The tiny oscillations at large $l$ would be further reduced by $l$ shifts of 1 or 2 units.

4 Softening limits on neutrino mass through DM–DE coupling

There seem to be little doubt left: at least one neutrino mass eigenstate or, possibly, two of them exceed $\approx 0.055$ eV (direct or inverse hierarchy). This follows solar [34] and reactor [35] neutrino experiments, yielding $\Delta m_{12}^2 \approx 8 \times 10^{-5}$eV$^2$ and, namely, atmospheric [36] and accelerator beam [37] experiments yielding $\Delta m_{23}^2 \approx 3 \times 10^{-3}$eV$^2$.

Cosmology is also sensitive to neutrino mass. Valdarnini & Bonometto (1985) [38] made a detailed analysis of transfer functions in mixed DM models, where a part of DM is due to massive neutrinos. In the Nineties, large deal of work on this subject took place, widely testing both the linear and the non–linear theory.

Using advanced astrophysical data, such as CMB and deep sample, increasingly stringent limits on neutrino masses could be computed. Standard limits on neutrino masses were recently summarized by Komatsu et al (2008), within the WMAP5 release [39], setting $\sum m_\nu < 0.61$ eV ( 95% C.L.) within $\Lambda$CDM models (see also [40]). These limits, however, rely on implicit assumptions concerning the dark cosmic sector, requiring two non interacting components characterized by state parameters $w \approx 0$ and $\approx -1$. As we know this hypothesis is only partially satisfactory.

Let us then consider first in a SUGRA quintessence model the impact of coupling and neutrino mass on the spectra of density fluctuations $P(k)$ and CMB anisotropies $C_l$. All through this paper, only cases with 2 equally massive and 1 massless $\nu$’s will be shown. Results for different mass distributions exhibit minor numerical differences.

In Fig. 1 the power of $k$ and $l$ is selected so to magnify the spectral differences in the plot. In the spatially flat models considered it is $\Omega_{\Lambda, c} = 0.23$, $\Omega_{\Lambda, b} = 0.04$, $h = 0.71$. It is evident that massive $\nu$’s and coupling act on the spectra in opposite directions so that the model with massive $\nu$’s and coupled DE can approach quite well the uncoupled model with no massive $\nu$’s.

Using a FM technique, we try to estimate how far we can go, simultaneously increasing $\beta$ and $\Omega_\nu$, without conflicting with data. We consider then two different experimental contexts. The first one assumes that CMB spectra are measured at WMAP sensitivity and $P(k)$ is measured with the sensitivity of the 2dF experiment. The second assumes Planck sensitivity for CMB spectra and SDSS sensitivity for $P(k)$.

In the left panel of Fig. 2 1– and 2–$\sigma$ likelihood contours are plotted assuming a SUGRA cosmological model with $\Lambda/\text{GeV} = 1.1$, with neither $\nu$’s nor coupling. We plot the two data systems considered: (A)
WMAP and 2dF; (B) PLANCK and SDSS. This figure describes very well the degeneracy between the neutrino density parameter $\Omega_\nu$ and the coupling $\beta$. It is worth noting that the A case allows for couples $(\Omega_\nu, \beta)$ which are more than a factor $\sim 2$ far from the known observational limits.

At the level of sensitivity allowed by currently available data (WMAP & 2dF), and fully trusting in the confidence ellipses as obtained by a FM technique, we can locate in the $\Omega_\nu$ vs. $\beta$ plane some points which are consistent with the reference model. For example, (0.005, 0.049), (0.011, 0.1) lay within 1–σ ellipse, while (0.02, 0.12) appears consistent with data within $\sim 2$–σ’s. In the right panel of Fig. 2, we show the scale dependence of the density parameters for these models. Such high levels of coupling, allowed due to the presence of massive $\nu$’s, are able to slightly ease the coincidence problem.

5 Gravitational lensing constraints on coupled dark energy

As already discussed in Sec. 2, in coupled models, also for quite low $\beta$’s, the non–standard time evolution of the dark components can leave an imprint on both the expansion history of the Universe, and the growth of (matter) fluctuations, at the linear and non–linear levels (e.g. [41]). However, any detected evolution of $H$ can be reproduced through a suitable redshift dependence of DE density $\Omega_{de}$ and state parameter $w$, when $\Lambda$ approaches $m_P$. A risk is that, if matter and DE are coupled, fitting observations leads to an estimate of a phantom equation of state ($w < -1$), even if $w > -1$ at all redshifts [42].

In principle, this risk can be excluded providing information both on $H(z)$ and the growth factor, $G(z) = \delta(a)/a$. Experiments, or combinations of experiments, probing them are then needed. We have already discussed the low constraining power of CMB and deep samples that can only put upper limits on $\beta$. At the available sensitivity level, such data systems provide just weighted integrals of $H(z)$ and $G(z)$, which remain consistent with a rather wide set of options. On the contrary, WL tomography, probing the power spectrum $P(k)$ at different redshifts, is well suited to constrain $G(z)$. In combination with CMB data, it will certainly be a powerful tool for the analysis of DE.

Leaving aside the massive $\nu$’s option, in this section we aim to deepen the case of coupling, by performing a FM analysis of future WL surveys (DUNE–like [26,43]) and CMB experiments (PLANCK–like).

Table 2 lists the estimated errors on the various parameters considered. For each data set, we compare forecasts for the target model, a coupled SUGRA model ($\beta = 0.1$), with results for an uncoupled SUGRA...
model with the same values of the relevant parameters. The table clearly shows that a PLANCK–like experiment is able to provide a measurement of a direct DE–DM interaction at 68% confidence level, even for moderate values of $\beta$. Nevertheless, allowing for a direct interaction strongly degrades the experimental sensitivity on the parameters characterizing the matter density, $\omega_m$, $\Omega_m$, and the normalization of the power spectrum of density fluctuations, $\sigma_8$.

On the contrary, WL surveys alone perform significantly better with respect to these parameters, with a factor of 2, or more, improvement over CMB experiments. In particular, WL data alone can clearly distinguish the target model from a non–coupled model, or a cosmological constant, even at the $3\sigma$ level, viceversa assuming a reference SUGRA model with $\beta = 0$, we can expect to put an upper limit $\beta \lesssim 0.03$, at the same confidence level (see Fig. 3 left panel).

A completely new situation occurs if both CMB and WL measurements are simultaneously used. In this case, the opening of the coupling option causes just a marginal increase of the errors on most parameters. Moreover, as particularly evident from the right panel of Fig. 3, the degeneracies between $\beta$ and other parameters, such as $\sigma_8$ or $\Omega_m$, disappear. This is a clear indication of the complementarity of the CMB and WL measurements, as described in [44]. Breaking degeneracies is the main aim when different observables are simultaneously considered. We see that, from this point of view, the efficiency of using both CMB and WL measures can be hardly overestimated.

Table 2: Estimated errors on model parameters for a SUGRA model, with and without coupling.

<table>
<thead>
<tr>
<th></th>
<th>CMB</th>
<th>WL</th>
<th>WL+CMB</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta = 0.$</td>
<td>$\beta = 0.1$</td>
<td>$\beta = 0.$</td>
<td>$\beta = 0.1$</td>
</tr>
<tr>
<td>$100\times\omega_b$</td>
<td>0.016</td>
<td>0.019</td>
<td>0.5</td>
</tr>
<tr>
<td>$\omega_m$</td>
<td>0.002</td>
<td>0.006</td>
<td>0.016</td>
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<tr>
<td>$\Omega_m$</td>
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<td>0.12</td>
<td>0.002</td>
</tr>
<tr>
<td>$n_s$</td>
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<td>0.005</td>
<td>0.012</td>
</tr>
<tr>
<td>$\sigma_8$</td>
<td>0.07</td>
<td>0.13</td>
<td>0.0026</td>
</tr>
<tr>
<td>$\lambda$</td>
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<td>9.5</td>
<td>0.89</td>
</tr>
<tr>
<td>$\beta$</td>
<td>–</td>
<td>0.04</td>
<td>–</td>
</tr>
<tr>
<td>$\tau$</td>
<td>0.005</td>
<td>0.006</td>
<td>–</td>
</tr>
</tbody>
</table>
6 Conclusions

One basic question, still unanswered in Cosmology, is about the nature of its “Dark Side”. In this work we debated the ideas of a dynamical form of DE coupled to DM. In particular, we showed that the option of a coupling in the Dark sector can cause a critical fallout in determining cosmological parameters, sometimes allowing for unexpected consequences, as for the coincidence problem with massive $\nu$’s. This presses for new and better data in order to break degeneracies among cosmological models. In this scenario, WL surely will contribute in a decisive way to the present research in Cosmology.

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References


