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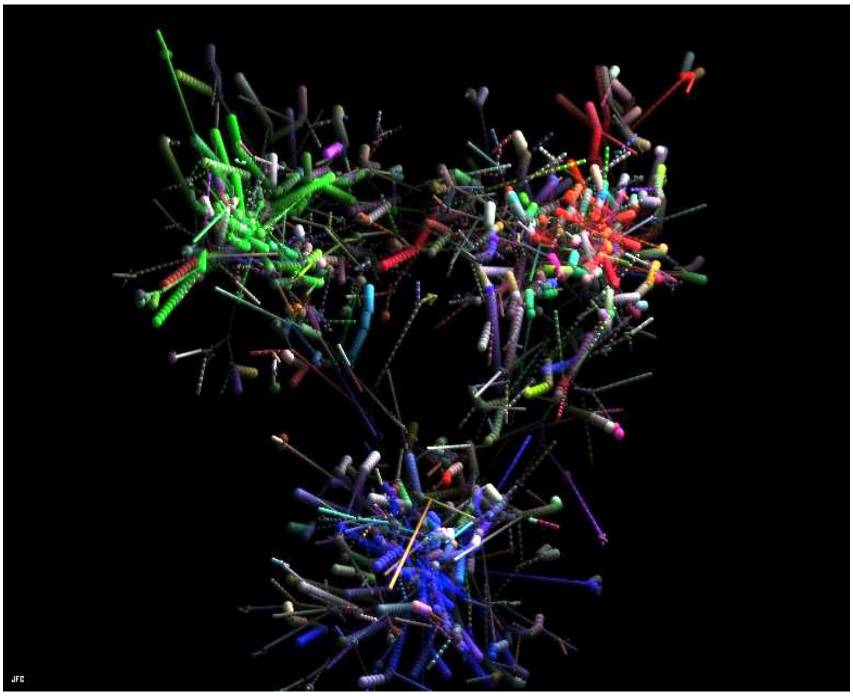
Marina Dorati



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DOTTORATO DI RICERCA IN FISICA – XXI CICLO

THE STRUCTURE OF THE NUCLEON BY ELECTROMAGNETIC PROBES IN CHIRAL EFFECTIVE FIELD THEORY

dissertation submitted by

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Introduction

In the last decades much effort has been made among physicists in order to improve our knowledge about the structure of the nucleon. The magnetic moment of the proton, first measured in 1933 by Frisch and Stern, was the earliest experimental evidence for the internal structure of the nucleon. Beginning from that discovery, a series of successful experiments had followed and thanks to the surprising progress made in experimental technology, many of the structure parameters of proton and neutron are now known to incredible precision. Although the many goals achieved by nuclear physics on both theoretical and experimental sides, a lot of opened questions around the structure of the nucleon still need to be solved, a lot of interesting issues still have to be discussed.

Quantum Chromo Dynamics (QCD) is the quantum field theory of strong interactions within the Standard Model. The nucleon is the lightest baryon in the hadron spectrum, it is a complex system of interacting quarks and gluons governed by strong interactions. In this work we are going to explore the low energy properties of this system. Because of the impossibility of QCD to allow for the description of low energy properties and processes, we will make use of an effective field theory (EFT) called Chiral Perturbation Theory (ChPT), which represents the low energy limit of QCD.

The first aim of the present work is the analysis of the so-called generalized form factors of the nucleon within the framework of Chiral Perturbation Theory. These objects are defined as the first moments of the so-called Generalized Parton Distributions (GPDs), a quite new class of functions which can e.g. be accessed in Deeply Virtual Compton Scattering (DVCS) off the nucleon. By performing the calculation up to the leading one loop order in covariant baryon ChPT (second order in the chiral expansion), we will derive the momentum transfer and quark-mass dependence of the generalized form factors. In particular, we will focus on the forward limit of these expressions, where the GPDs reduce to the well known Parton Distribution functions (PDFs) and a connection to phenomenology becomes possible. The ChPT results will be also compared to available lattice QCD data, paving the way for chiral predictions based on chiral extrapolation studies. Furthermore, an interesting aspect related to the generalized form factors is the possibility to gain from their analysis information about the contribution of the quarks to the spin of the nucleon.

The investigation of the spin structure of the nucleon is another central topic of present nuclear physics activities. To this purpose a very special role is played by the process of forward Doubly Virtual Compton Scattering (VVCS) off nucleons, a particular case of Compton scattering which involves two virtual photons. The matrix element of this process can be related to certain sum rules which have validity at all energy scales and are written in terms of the nucleon spin structure functions g_1 and g_2. Among these sum rules a really interesting task is represented by the so-called spin polarizabilities, linear combinations of moments of g_1 and g_2. Despite of the several theoretical and experimental investigations of this issue carried out in the past years, further improvement in the results is still demanded. In particular one has to better understand and estimate the contributions of the nucleon resonances to the values of the VVCS matrix element. The Δ resonance plays an important role in the nucleon spin sector and thus its inclusion in the calculation in a systematic manner is called for. A second aim of the present work is to extend the chiral analysis of the process of Doubly Virtual Compton Scattering presented in reference [BHM03b] including the Δ resonance as explicit degree of freedom in the calculation. We will present a leading-one-loop order calculation of the spin amplitude of the process of Doubly Virtual Compton Scattering in the small scale expansion scheme of covariant Chiral Perturbation Theory.

This work is organized as follows: chapter 1 will deal with essentials of Chiral Perturbation Theory. The formalism needed to perform calculations is provided, with special regards to the regularization schemes in covariant baryon ChPT and to the general properties of a chiral EFT for spin-3/2 fields. In particular, technical details about the recently introduced $\overline{\text{IR}}$ regularization scheme are given, with emphasis on the advantages that the adoption of such a scheme can bring about.
Chapter 2 will be dedicated to the discussion of the first moments of the parity-even Generalized Parton Distributions (GPDs) in a nucleon, corresponding to six (generalized) vector form factors. We explore the chiral analysis of the three (generalized) isovector and (generalized) isoscalar form factors, which are currently under investigation in lattice QCD. We study the limit of vanishing four-momentum transfer and we first concentrate on the isovector moment $\langle x\rangle_{u-d}$; our BChPT calculation predicts a new mechanism for chiral curvature, connecting the high values for this moment typically found in lattice QCD studies for large quark masses with the smaller value known from phenomenology. Likewise, we analyse the quark-mass dependence of the isoscalar moments in the forward limit and extract the contribution of quarks to the total spin of the nucleon. We also proceed with a first glance at the momentum dependence of the isoscalar C-form factor of the nucleon. We close with the analysis of the generalized axial form factors of the nucleon, presenting promising results from a combined fit of different GPDs-moments.
In chapter 3 we will first discuss the generalization of the Gerasimov-Hearn sum rule, the Burkhardt-Cottingham sum rule, the nucleon spin polarizabilities and other moments of the nucleon spin structure functions. Afterwards

we will describe in detail the calculation of the matrix element of the process of Doubly Virtual Compton Scattering (VVCS) up to leading one loop order (third order in the chiral expansion) in covariant baryon Chiral Perturbation Theory, providing the required technicalities for the inclusion of the Δ resonance as explicit degree of freedom in the calculation. The results for the spin amplitudes corresponding to the various diagrams contributing to the process are finally summarized, while for a complete and detailed discussion of the results presented in this chapter we postpone to a later communication.

Chapter 1

Notions of Chiral Perturbation Theory

Quantum Chromo Dynamics (QCD) is the fundamental quantum field theory of strong interactions. Thanks to asymptotic freedom high energy regions are probed by means of perturbartive techniques. At low momenta and energies ($Q < 1\,\mathrm{GeV}$) the runnning coupling $\alpha_s(Q)$ is of order one so that an expansion in powers of α_s is no longer practical. Moreover, QCD is written in terms of the wrong degress of freedom (quarks and gluons), while low energy experiments are performed with hadrons.
QCD can be mapped on a (chiral) effective field theory (EFT) at low energies. This effective field theory is called Chiral Perturbation Theory (ChPT) and it allows the description of low energy properties and processes within a perturbative approach.
In this chapter we are going to explore the basic concepts underlying the theory of Chiral Perturbation Theory; we provide the standard ingredients for the formulation of the theory, referring to the specific chapters for pertinent technical details.
For further information about effective field theories and ChPT we refer to reference [Wei76] and to the recent reviews [Ber07, BM06, Sch03].

1.1 Effective Lagrangian and chiral symmetry

The large value of the strong coupling constant α_s forbids a perturbative development of QCD in the low energy domain, making the testing of QCD in this region a very investigated task for theoretical physics.
In the seventies the concept of effective field theory (EFT) has emerged [Wei76], leading to the development of Chiral Perturbation Theory (ChPT) as a model-independent way of solving QCD [GL84]. The main characteristic of EFTs is the presence of a specific energy scale Λ. The value of this scale represents a threshold, it selects the relevant degrees of freedom when compared to the particular energy of working. In the case of ChPT, the scale Λ is determined by the spontaneous breaking of chiral symmetry, with consequent identification

of the meson pions (in the particular case of two light flavors) as responsible for the dynamics of a strongly interacting system. The main point of this approach is that the effective Lagrangian has to preserve all the symmetries of the original QCD Lagrangian: the effective Lagrangian has to contain all possible terms compatible with the assumed symmetry principles of the fundamental theory, in this particular case all symmetry properties of QCD.

The strong interaction as described by the Standard Model involves $N_f = 6$ different flavors of quarks described by the fermion spinor q and the gluons, vector gauge bosons that mediate strong color charge interactions and are described by the non-abelian field strength tensor $G_{\mu\nu}$. The ordinary QCD Lagrangian reads:

$$\mathcal{L}_{QCD} = \sum_{N_f} \overline{q}(i\gamma^\mu D_\mu - m_q)q - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu\, a}, \tag{1.1}$$

where m_q denotes the mass of the specific quark q, γ_μ are Dirac matrices and D_μ is the gauge covariant derivative.
According to a typical hadron scale of 1 GeV, quarks are separeted into light (u,d,s) and heavy (c,t,b). Since we are interested in processes at low energy, we choose to restrict ourselves to the light quark sector and in particular to the lightest u and d quarks. The QCD Lagrangian has the property to be invariant under parity P, charge conjugation C and time reversal T transformations. In the limit $m_q \to 0$, the corresponding Lagrangian $\mathcal{L}^0_{QCD}$ is invariant under independent left and right handed rotations of the quark fields in flavor space. Therefore the Lagrangian $\mathcal{L}^0_{QCD}$ is symmetric under the flavor group $SU(2)_L \times SU(2)_R$[1], which takes the name of chiral symmetry group. The real world reveals that chiral symmetry is a symmetry of $\mathcal{L}^0_{QCD}$ but not of the ground state: the chiral symmetry group $SU(2)_L \times SU(2)_R$ is spontaneously broken down to its vectorial subgroup $SU(2)_V$. Low energy hadron phenomenology shows evidence of the spontaneous symmetry breaking via the non-existence of degenerate parity doublets in the hadron spectrum, i.e states of equal spin, baryon number and strangeness but opposite parity. In terms of charge operators, this evidence means that the vacuum (the ground state) is not annihilated by both vector and axial charge operators, i.e. the condition $Q^V_a|0\rangle = Q^A_a|0\rangle = 0$ is not fulfilled. Assuming $Q^V_a|0\rangle = 0$, one has to conclude that $Q^A_a|0\rangle \neq 0$, i.e. the ground state of QCD is not invariant under axial transformations. According to the Goldstone's theorem [Gol61], chiral symmetry and its spontaneous symmetry breaking thus imply the existence of $N_f^2 - 1 = 3$ massless pseudoscalar (Q^A_a is an axial charge!) bosons. Because of the non-zero quark mass terms, responsible for the so-called explicit breaking of chiral symmetry, such 0^- massless bosons do not exist. The best candidates to be identified with them are are the pseudoscalar mesons π, which are

[1] The Lagrangian $\mathcal{L}^0_{QCD}$ is invariant under the global transformations of the group $SU(2)_L \times SU(2)_R \times U(1)_V \times U(1)_A$. In this work we are not concerned with the vectorial $U(1)_V$ and the axial $U(1)_A$ groups. The vector one is an exact symmetry, it results in baryonic number conservation and leads to a classification of hadrons into mesons (B=0) and baryons (B=1). The axial one is exact in the classical theory but violated in the full quantum field theory; this phenomenon is called the axial U(1) anomaly.

the lightest particles in the hadron spectrum according to the Particle Physics Booklet.

Because of the small value of the quark mass, the explicit breaking of chiral symmetry represents a small effect when compared to the mechanism of spontaneous chiral symmetry breaking and may be therefore treated as a perturbation. Given that the interaction between the pions is weak and that the quark mass is very small, an expansion in the external momentum p and in the mass m_q is possible, as long as one stays below the scale $\Lambda_\chi \sim$ 1 GeV, with $p, m_q \ll \Lambda_\chi$. Chiral Perturbation Theory takes its origin as the effective field theory of QCD with pions as the explicit degrees of freedom and with validity up to the scale Λ_χ. Concluding, in order to be the low energy limit of $\mathcal{L}_{QCD}$ the effective Lagrangian $\mathcal{L}_{ChPT}$ has to be the most general Lagrangian invariant under the discrete parity (P), charge conjugation (C) and time reversal (T) transformations, in the chiral limit $m_q \to 0$ has to be invariant under $SU(2)_L \times SU(2)_R$, while the ground state has to be invariant only under the subgroup $SU(2)_V$.

1.1.1 The mesonic sector

We now proceed with the construction of the effective Lagrangian in the mesonic sector. We denote the dynamical variables representing the pion field by the $SU(2)$ matrix field $U = exp\left(\frac{i\tau^i\pi^i}{F_\pi}\right)$, where τ^i with $i = 1, 2, 3$ are the Pauli matrices in the isospin space, π^i are the Klein-Gordon pion field operators and F_π is the pion decay constant. ChPT is characterized by counting rules: since the field U counts as a quantity $\mathcal{O}(1)$, $\partial_\mu U$ counts as $\mathcal{O}(p)$. At low energy, an expansion in powers of meson momenta is equivalent to an expansion of $\mathcal{L}_{eff}$ in powers of the derivative $\partial_\mu U$. Lorentz invariance restricts the form of the Lagrangian to only terms with an even number of derivatives, thus the general expression $\mathcal{L}_{eff} = \mathcal{L}^{(2)}_{eff} + \mathcal{L}^{(4)}_{eff} + \ldots$, where the superscripts $(n = 2, 4, \ldots)$ mark the low-energy dimension.
Let us concentrate on the leading part of the Lagrangian, given for $n = 2$, only purely meson term used in the present work. One has:

$$\mathcal{L}^{(2)}_{\pi\pi} = \frac{F_\pi^2}{4}\,\mathrm{Tr}\left[\nabla_\mu U^\dagger \nabla^\mu U + \chi^\dagger U + \chi U^\dagger\right]. \tag{1.2}$$

The covariant derivative is defined as

$$\nabla_\mu U = \partial_\mu U - i\left(v_\mu + a_\mu\right)U + iU\left(v_\mu - a_\mu\right), \tag{1.3}$$

and contains the couplings to arbitrary vector v_μ and axial a_μ fields, like for example photons and W bosons, in the order. As the partial derivative ∂_μ, the fields v_μ, a_μ count as $\mathcal{O}(p)$.
The field $\chi = 2B(s + ip)$ contains the free parameter B and is a linear combination of scalar s and pseudoscalar p fields. The constant B is related to the spontaneous symmetry breaking of the theory, measuring the strength of the

chiral condensate via $\langle 0|\overline{u}u|0\rangle = -F_\pi^2 B + \mathcal{O}(\mathcal{M})$, where $\mathcal{M} = \mathrm{diag}(m_u, m_d)$, with m_u and m_d the mass of the up and down quark.
It is straightforward to see that the symmetry breaking part of the effective Lagrangian $\mathcal{L}_{SB}^{(2)} = \frac{F_\pi^2}{4}\mathrm{Tr}\left[\chi^\dagger U + \chi U^\dagger\right]$ is proportional to the sum of the quark masses. Let us set all external sources to zero except for $s = \mathcal{M}$, , so that $\chi = 2B\,\mathrm{diag}(m_u, m_d)$. Expanding in powers of the pion fields one finds:

$$\begin{aligned} \mathcal{L}_{\pi\pi}^{(2)} &= -F_\pi^2 B(m_u + m_d) \\ &+\frac{1}{2}\left[\partial_\mu\pi^i\partial^\mu\pi^i - B(m_u+m_d)\pi^i\pi^i\right] + \mathcal{O}(\pi^4). \end{aligned} \tag{1.4}$$

The first term is thus related to the vacuum expectation value of the mass term $-(m_u\langle 0|\overline{u}u|0\rangle + m_d\langle 0|\overline{u}u|0\rangle)$, while the contribution at order π^2 corresponds to the Klein-Gordon equation for a free pion of mass $m_\pi^2 = B(m_u + m_d)$. Thanks to this relation one can always translate the quark-mass dependence of physical observables into a pion-mass dependence up to a certain order. According to the standard ChPT scenario, we assume that B counts as $\mathcal{O}(1)$ and m_q is $\mathcal{O}(p^2)$, so that m_π has to be counted as a quantity of order p. Beside these chiral counting rules a systematic method to estimate the importance of loop diagrams is also demanded. We make use of Weinberg power-counting scheme [Wei79], according to which a one to one correspondence between loop and chiral order exists, that is diagrams with N_L meson loops are suppresed by powers of $(p^2)^L$. The chiral dimension D of a loop diagram in the mesonic sector can then be calculated as $D = 2 + \sum_d N_d(d-2) + 2N_L$, where N_d is the number of vertices with chiral dimension d, i.e. generated by a Lagrangian of chiral dimension d, and N_L is the number of pion loops.

1.1.2 Chiral Perturbation Theory with baryons

Chiral Perturbation Theory can be extended to include baryons. We construct a theory able to describe the dynamics of baryons, their interaction with pions and external fields at low energy. In particular, we are interested in the pion-nucleon (πN) system.
We still makes use of the matrix field operator $U(x)$ representing the pion and we denotes by Ψ_N the isospin doublet nucleon field. We introduce the covariant derivative D_μ acting on the nucleon field

$$D_\mu\Psi_N = \left(\partial_\mu + \Gamma_\mu - iv_\mu^{(s)}\right)\Psi_N, \tag{1.5}$$

where

$$\Gamma_\mu = \frac{1}{2}\left[u^\dagger, \partial_\mu u\right] - \frac{i}{2}u^\dagger\left(v_\mu + a_\mu\right)u - \frac{i}{2}u\left(v_\mu - a_\mu\right)u^\dagger \tag{1.6}$$

is the so-called chiral connection expressed in terms of the external vector and axial background fields v_μ, a_μ, $v_\mu^{(s)}$ defines an isosinglet vector field and $u^2 = U$. Together with the covariant derivative, another building block with

one derivative is the so-called vielbein $u_\mu \equiv i\left(u^\dagger \nabla_\mu u - u \nabla_\mu u^\dagger\right)$, which under parity transforms as an axial vector.
In order to write the leading order Lagrangian we fix some counting rules assigning chiral dimension p^0 to the the elements $\{M_0,\, \Psi,\, \overline{\Psi},\, D_\mu\Psi,\, \overline{\Psi}\Psi,\, \overline{\Psi}\gamma_\mu\Psi,\, \overline{\Psi}\gamma^\mu\gamma_5\Psi,\, \overline{\Psi}\sigma^{\mu\nu}\Psi,\, \overline{\Psi}\sigma^{\mu\nu}\gamma_5\Psi\}$ and chiral dimension p^1 to the objects $\{(i\not{D} - M_0)\Psi, \overline{\Psi}\gamma_5\Psi\}$, where M_0 defines the nucleon mass in the chiral limit.
The leading order pion-nucleon Lagrangian thus takes the form [GSS88]:

$$\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi}_N \left[i\gamma^\mu D_\mu - M_0 + \frac{g_A}{2}\gamma^\mu\gamma_5 u_\mu \right] \Psi_N, \tag{1.7}$$

where g_A is the chiral limit of the axial coupling constant.

For explicit expressions of higher order up to $\mathcal{O}(p^4)$ πN Lagrangian we refer to reference [FMMS00]. Their general form can be written in terms of field monomials $\mathcal{O}_i^{(n)}$ (the superscript n denotes as usual the chiral order) as following:

$$\mathcal{L}_{\pi N}^{(2)} = \sum_{i=1}^{7} c_i \overline{\Psi}_N \mathcal{O}_i^{(2)} \Psi_N, \;\; \mathcal{L}_{\pi N}^{(3)} = \sum_{i=1}^{23} d_i \overline{\Psi}_N \mathcal{O}_i^{(3)} \Psi_N, \;\; \mathcal{L}_{\pi N}^{(4)} = \sum_{i=1}^{118} e_i \overline{\Psi}_N \mathcal{O}_i^{(4)} \Psi_N. \tag{1.8}$$

The coefficient of the monomials are low energy constants (LECs) of the theory. At higher order the Lagrangians becomes very long and the number of the LECs explodes rapidly. Their values are not constrained by symmetries and most of them show a scale dependence. Indeed, calculation involving one loop graphs yield divergent integrals thus the need for renormalization. Since we are working within the framework of an effective field theory, all possible structures conforming to the symmetries of the system are already contained in the Lagrangians. The LECs are therefore the best suited objects to act as counterterms for the renormalization of the theory. As will be discussed later on, loop diagrams in the nucleon sector start to appear at chiral dimension $D = 3$. This implies that the low energy constants appearing in lower order Lagrangians are not affected by loop effects and are therefore scale-independent. In particular, the constants c_i belonging to $\mathcal{L}_{\pi N}^{(2)}$ have been determined by some accuracy from low-energy hadron phenomenology. In order to develop a regularized theory, the dimension-three and -four LECs are decomposed into a finite and infinite part [Eck94, FMS98, MMS00]:

$$d_i \equiv d_i^r(\lambda) + \frac{\beta_i}{F_\pi^2} L, \tag{1.9}$$

where

$$L = \frac{\lambda^{d-4}}{16\pi^2} \left[\frac{1}{d-4} + \frac{1}{2}(\gamma_E - 1 - \ln(4\pi)) \right], \tag{1.10}$$

with λ the renormalization scale, d denotes the space-time dimension and γ_E the Euler-Mascheroni constant. The β_i appearing in the infinite part of the decomposition denotes the β-function associated with the corresponding couterterm and depends on coupling constants of the theory. The value of the renormalized LEC $d_i^r(\lambda)$ can be finally fixed from phenomenology. The $d_i^r(\lambda)$ at two different scales λ_1, λ_2 are connected by the following relation:

$$d_i^r(\lambda_2) = d_i^r(\lambda_1) - \beta_i \log\frac{\lambda_2}{\lambda_1}. \tag{1.11}$$

The appearance of a new energy scale, namely the nucleon mass M_0, prevents the extension of the power-counting scheme introduced in the mesonic sector to include the pion-nucleon system. Because of the zeroth component, one can not say that a derivative acting on the baryon field results in a small four-momentum. Since the nucleon mass M_0 is of about the same size as the scale $4\pi F_\pi$ appearing in the calculation of pion-loop contribution, terms coming from higher order diagrams are no more suppressed and a consistent hierarchy of loop diagrams is no more ensured. This issue represents an historical problem of ChPT, first noted by the authors of reference [GSS88] when dealing with chiral loops within the scheme of dimensional regularization ($\overline{\mathrm{MS}}$). Many physicists have faced the task during the years and several interesting alternative solutions have been suggested. The problem was first cleared up by Jenkins and Manohar [JM91a], who proposed to apply to the pion-nucleon sector the heavy quark effective field theory methods used in heavy quark physics. The idea is to consider the baryons as extremely heavy sources surrounded by a cloud of light particles (the pions) and to split up their momentum into a large piece of the order of their mass and a small residual component. One has $p^\mu = M_0 v^\mu + r^\mu$, where v^μ is a velocity vector satisfying $v^2 = 1$ and $v \cdot r \ll M_0$. At this point we introduce the velocity projector operators $P_v^\pm \equiv \frac{1 \pm \not{v}}{2}$ and we apply them to the nucleon field defining the two velocity-dependent fields $N_v \equiv e^{-iM_0 v\cdot x} P_v^+ \Psi_N$ and $h_v \equiv e^{-iM_0 v\cdot x} P_v^- \Psi_N$. The nucleon field can now be written as $\Psi_N = e^{-iM_0 v\cdot x}(N_v + h_v)$, where we note that the dependence on the nucleon mass is factorized out through the exponential term. Inserting this expression of the nucleon field into the leading order Lagrangian of eq.(1.7) one finds that the heavy component h_v is formally suppressed by the power of $1/M_0$ relative to the light component N_v. The non-relativist limit of $\mathcal{L}_{\pi N}^{(1)}$ finally reads

$$\mathcal{L}_{\pi N}^{(1)} = \overline{N}\left[iv \cdot D + g_A S \cdot u\right] N, \tag{1.12}$$

with $S_\mu = \frac{i}{2}\gamma_5 \sigma_{\mu\nu} v^\nu$ the so-called Pauli-Lubanski spin vector. A systematic formulation of this approach was developed in ref.[BKKM92].

Since it is based on the idea of treating baryons as extremely heavy, this framework is called Heavy Baryon Chiral Perturbation Theory (HBChPT). This construction of the effective Lagrangian guarantees that terms of the kind $\frac{M_0}{4\pi F_\pi} \sim 1$ will not appear in the calculation, allowing for the formulation of a power-counting scheme analogous to the mesonic sector. In the non-relativistic approach of HBChPT the one-to-one correspondence between the loop and the small momentum expansion is newly restored. The chiral dimension D of a Feynman diagram can be then derived via

$$D = 2N_L + 1 + \sum_d (d-2) N_d^M + \sum_d (d-1) N_d^{MB}, \tag{1.13}$$

	power counting	analyticity	non-relativistic limit	ultraviolet regularization
$\overline{\text{IR}}$	✓	✓	✓	✓
standard IR	✓	-	✓	(✓)
$\overline{\text{MS}}$	-	✓	-	✓
EOMS [FGJS03]	✓	✓	-	✓

Table 1.1: Overview over several renormalization schemes of BChPT frequently discussed in literature and how they behave with respect to the four conditions listed in the text [Gai07]. The $\overline{\text{IR}}$ scheme is designed in such a way to fulfill all of the four conditions.

where N_L again denotes the number of loops in the diagram, N_d^M is the number of vertices generated by a meson Lagrangian of order d and N_d^{MB} the number of vertices from a meson-baryon Lagrangian of order d. The power-counting is very similar to the mesonic one. We note that $D \geq 1$ and the loop starts contributing at $D = 3$. This implies that the second order LECs from $\mathcal{L}_{\pi N}^{(2)}$ are scale-independent, they do not contribute to the renormalization of the infinities from one-loop calculations[2].
However, the HBChPT prescription still shows some deficiencies: under certain circumstances the heavy baryon approach may generate Green functions which do not satisfy the analytic properties which characterize a fully relativistic field theory [BKM96]. The search for a method which encloses the advantages of the heavy baryon approach (presence of a consistent power-counting scheme) as well as of a relativistic theory (absence of non-analyticities) has led to the development of the so-called Infrared regularization scheme (IR) , which will be the topic of the next section. As we will see, even this technique has to be modified in order to develop a consistent renormalization scheme for covariant baryon ChPT (BChPT). Essentials of this recently introduced innovative scheme will also be given.

1.2 Regularization schemes in covariant baryon ChPT

In this section basic aspects of renormalization in covariant BChPT will be discussed, with particular emphasis on the infrared (IR) regularization scheme and a new scheme called $\overline{\text{IR}}$ recently introduced by the authors of reference [GH06] and described in detail in reference [Gai07]. See these references for further information about this modified version of the infrared regularization prescription.
The infrared regularization scheme (IR) was first introduced by Becher and

[2] We underline that these conditions are fulfilled for strongly interacting systems in the presence of vector (v), axial (a), scalar (s) and pseudoscalar (p) fields. As we will see in chapter 2, the presence of additional fields nullifies those conditions.

Leutwyler [BL99] as solution to the known deficiency of a rigorous power-counting in the dimensional $\overline{\text{MS}}$ scheme of covariant BChPT and in order to avoid the production of non-analytic structures as can happen in the heavy baryon formulation of ChPT. The basic idea underlying infrared regularization consists of splitting the relativistic one-loop integrals into "soft" and "hard" parts. The "soft" part is regulated by the power-counting scheme as in HBChPT, while the terms contributing to the "hard" part can be absorbed in the LECs of the theory. Strictly speaking this means that by a particular choice of Feynman parametrization any dimensionally regularized one-loop integral H can be separated into an infrared singular part I and a regular part R. In the low energy region H and I have the same analytic properties while the contribution of R, which corresponds to an infinite sum of terms analytic in the quark mass, can be taken into account just by a proper modification of the low energy constants appearing in the general effective Lagrangian. A calculation which adopts the IR regularization scheme is obtained by replacing the general integral H by its infrared singular part I and by dropping the regular part R. The method of infrared regularization produces a consistent power-counting, to be specific it is characterized by a power-counting scheme which follows the same formula of eq.(1.13) as the heavy baryon approach; the chiral dimension D of a Feynman diagram in the IR regularization scheme is then obtained via

$$D \;=\; 2N_L + 1 + \sum_d (d-2)N_d^M + \sum_d (d-1)N_d^{MB}. \tag{1.14}$$

Besides the existence of a meaningful power-counting prescription, the IR scheme also owns the property that an expansion in $1/M_0$ of the covariant result exactly reproduces the corresponding non-relativistic heavy baryon expression. This implies that all coupling constants are defined in both schemes (IR and HBChPT) at the same way and their finite parts do have the same numerical values.
However, in addition to power-counting and non-relativistic limit, a consistent renormalization scheme in BChPT has to fulfill two further conditions: it has to provide a satisfying ultraviolet regularization and to guarantee the absence of unphysical non-analyticities. Unfortunately, the IR prescription misses both requirements. Indeed, a loop integral calculated in IR can contain divergences and scale dependent logarithms beyond the order of the calculation. Since at a given order only counterterms from the effective Lagrangian corresponding to the order of the calculation can contribute to the result, the higher order divergences and scale-dependent logarithm can not be properly absorbed into the LECs. The authors of reference [BL99] have argued that the matching between the HBCHPT approach and the relativistic theory requires $\lambda = M_0$ and have suggested to remove by hand the divergences which can not be absorbed by the LECs since they represent higher order corrections. This procedure can in principle bring to acceptable numerical results up to the physical pion mass but is off course constrained by the choice of the scale λ and definitely not completely satisfactory.
Furthermore, one finds out that the infrared regular parts of the loop integrals

How to calculate loop diagrams in $\overline{\text{IR}}$ *- a practitioner's guide -*
1.   Calculate all diagrams in $\overline{\text{MS}}$ as usual.
2.   The infrared regular part of this result is found if all integrals H_{11} appearing in the $\overline{\text{MS}}$ result are replaced by R_{11}.
3.   The $\overline{\text{IR}}$ result is found, if the regular part expanded in the pion-mass up to the power at which a counter-term is available is added to the $\overline{\text{MS}}$ result.

Table 1.2: The three steps which have to be performed in an $\overline{\text{IR}}$ calculation in practice [Gai07].

can be non-analytic, with bad consequences on analysis of chiral extrapolation. An example for an analyticity problem in IR can be seen in the investigation of the quark-mass dependence of the nucleon mass. Indeed, when looking at the $\mathcal{O}(p^3)$ covariant expression for the nucleon mass in the IR scheme given in [PHW04], one can easily observe that for $m_\pi \to 2M_0$ the arccos structure diverges and becomes complex afterwards. This singularity becomes numerically significant for values of the pion mass already outside the range of applicability of the theory (above 700 MeV), but the quark-mass dependence of some quantities - e.g the isovector magnetic moment κ_v [GH07, Gai07]- can be anyway strongly influenced for values of the pion mass not far away from the physical one.
As alternative to the procedure dictated by infrared regularization, the authors of [GH06] has developed a new scheme which fulfills all four conditions discussed above. The new scheme ensures that only the terms of the regular part which can really be absorbed by the proper counterterms (that is by the counterterms corresponding to the order of working) will appear in the calculation. This is achieved by introducing a new -modified- regular part obtained by expanding the standard regular part of the loop integral up to the power of the pion mass at which a corresponding counterterm is available. The final result in the $\overline{\text{IR}}$ regularization scheme can be then calculated as the sum of the $\overline{\text{MS}}$ result plus the new regular part as defined above. Table 1.2 summarizes the steps one has to follow in order to reproduce a calculation in the $\overline{\text{IR}}$ prescription, while table 1.1 shows in a schematic way how the regularization schemes known from the literature fulfill the four basic conditions which set up a consistent regularization scheme.
An example of calculation and relative results within the $\overline{\text{IR}}$ scheme will be

provided in chapter 2.
We conclude listing the main properties and advantages of this new renormalization scheme:

- ◇ within this framework all ultraviolet divergences cancel and the result is independent of the renormalization scale;
- ◇ a rigorous power-counting scheme exists, allowing for a well defined hierarchy of the Feynman diagrams. As happens for the IR scheme, even in this case the power-counting formula which provides the chiral dimension of a Feynman diagram is exactly the same as in HBChPT (see eq.(1.13));
- ◇ the $1/M_0$ expansion of the $\overline{\text{IR}}$ result reproduces exactly the non-relativistic limit of HBChPT ;
- ◇ the LECs are defined in the same way in IR and $\overline{\text{IR}}$ as well as in HBChPT, so that new information about the couplings has validity in both schemes;
- ◇ in contrast to the standard infrared scheme, $\overline{\text{IR}}$ results show no unphysical cuts, singularities or imaginary parts.

1.3 Including the $\Delta(1232)$ as an explicit degree of freedom

In section 1.1.2 we have seen how to include spin-$\frac{1}{2}$ baryons in the framework of a chiral effective field theory. Pions and nucleons are the dynamical degrees of freedom in Baryon Chiral Perturbation Theory. Resonant baryons are assumed to be very heavy compared to the nucleon, so that they can be integrated out and their contribution appears in the form of counterterms. This approach can be considered reasonable for heavier resonances such as the Roper and higher states but looses its validity when applied to the case of the $\Delta(1232)$ resonance. Indeed, the spin-3/2 isospin-3/2 $\Delta(1232)$ resonance lies only about 300 MeV above the nucleon mass and couples very strongly to the $\pi - N$ sector. Phenomenology indicates that the inclusion of the Δ effects via a mere modification of the low energy constants of the theory is not always sufficient in order to estimate the real contribution provided by the $\Delta(1232)$ in many low- and medium-energy processes. Thus the need to include the $\Delta(1232)$ as an explicit degree of freedom in the chiral EFT, as was first suggested by Jenkins and Manohar in reference [JM91b].

In the second half of the nineties the authors of [HHK98] have introduced the so-called small scale expansion scheme (SSE), a proper power-counting scheme based on the essentials of HBChPT but developed in such a way that nucleon and Δ degrees of freedom appear simultaneously in the effective Lagrangian. The main idea underlying the SSE scheme is to introduce an additional small parameter, that is the Δ-nucleon mass splitting $\Delta \equiv m_\Delta - M_0 = 294\,\text{MeV}$ and to set up a low energy expansion in power of the parameter ϵ with

$$\epsilon \in \left\{ \frac{p}{\Lambda_\chi}, \frac{m_\pi}{\Lambda_\chi}, \frac{\Delta}{\Lambda_\chi} \right\}, \tag{1.15}$$

where $\Lambda_\chi \simeq 1\,\mathrm{GeV}$ denotes as usual the scale of chiral symmetry breaking. The mass splitting Δ is therefore treated as a small parameter together with external momenta and pion mass. We also observe that it is a dimensionful parameter of the theory which stays finite in the chiral limit. Because it is based on the main concepts of the heavy baryon approach and it consists in a *phenomenological* expansion in the small parameter ϵ, the SSE scheme can be regarded as a phenomenological extension of HBChPT.
Alternative approaches to the inclusion of spin-3/2 particles in chiral EFT have been presented in references [DJM94, PP03].

Following reference [HHK98], we now proceed to the development of a chiral EFT for spin-3/2 fields. We adopt the Rarita Schwinger formalism [RS41] and we represent the Δ field as a vector-spinor field $\Psi_\mu(x)$. The standard general form for the relativistic Lagrangian for a spin-3/2 field of mass m_Δ is:

$$\mathcal{L}_{3/2} = \overline{\Psi}^{\alpha}\, {}^{\frac{3}{2}}\Lambda(A)_{\alpha\beta}\Psi^{\beta}, \tag{1.16}$$

where α,β are Lorentz indeces and ${}^{\frac{3}{2}}\Lambda(A)$ denotes a matrix depending on a free unphysical parameter A ($A \neq -1/2$). This dependence is introduced in order to ensure that the Lagrangian is invariant under the so-called point transformation

$$\begin{aligned} \Psi_\mu(x) &\rightarrow \Psi_\mu(x) + a\gamma_\mu\gamma_\nu\Psi^\nu(x), \\ A &\rightarrow \frac{A-2a}{1+4a}. \end{aligned} \tag{1.17}$$

This transformation means that the presence of a quantity a ($a \neq -1/4$) of "off-shell" spin-1/2 components $\gamma_\nu\Psi^\nu(x)$, which always appear in the relativistic spin-3/2 field $\Psi_\mu(x)$, can be compensated by a corresponding change in the parameter A, so that the Lagrangian remains invariant. One can think of transferring the A-dependence of the matrix ${}^{\frac{3}{2}}\Lambda(A)$ to another matrix $O^A_{\mu\nu}$ defined as

$$O^A_{\alpha\mu} = g_{\alpha\mu} + \frac{1}{2}A\gamma_\alpha\gamma_\beta, \tag{1.18}$$

so that the Lagrangian takes the form:

$$\mathcal{L}_{3/2} = \overline{\Psi}^{\alpha} O^A_{\alpha\mu}\, {}^{\frac{3}{2}}\Lambda^{\mu\nu} O^A_{\nu\beta}\Psi^{\beta}. \tag{1.19}$$

If we now redefine the spin-3/2 field as

$$\psi_\mu(x) = O^A_{\mu\nu}\Psi^\nu(x), \tag{1.20}$$

we can finally write

$$\mathcal{L}_{3/2} = \overline{\psi}^{\frac{3}{2}}_{\mu} \Lambda^{\mu\nu} \psi_{\nu}, \tag{1.21}$$

and we can thus work with an A-independent Lagrangian written in terms of the ψ_μ fields.

The next step is to find a way to match spin-3/2 dynamics and the isospin properties of the Δ resonance. The $\Delta(1232)$ carries isospin $I = 3/2$. The four physical states of the Δ can be reproduced if we treat the spin-3/2 field $\psi_\mu(x)$ as an isospin-doublet and we attach to it an additional isovector index $i = 1, 2, 3$. Because of this double index, the Δ field is a vector and spinor in both spin and isospin spaces. This construction has to be constrained by the condition

$$\tau^i \psi^i_\mu(x) = 0, \quad i = 1, 2, 3 \tag{1.22}$$

with τ^i a 2-component Pauli matrix in the isospin space, in order to eliminate the two extra degrees of freedom generated by such a notation. A representation for the three isospin doublets is for example:

$$\begin{aligned}
\psi^1_\mu &= \frac{1}{\sqrt{2}} \begin{bmatrix} \Delta^{++} - \frac{1}{\sqrt{3}}\Delta^0 \\ \frac{1}{\sqrt{3}}\Delta^+ - \Delta^- \end{bmatrix}_\mu \\
\psi^2_\mu &= \frac{i}{\sqrt{2}} \begin{bmatrix} \Delta^{++} + \frac{1}{\sqrt{3}}\Delta^0 \\ \frac{1}{\sqrt{3}}\Delta^+ - \Delta^- \end{bmatrix}_\mu \\
\psi^3_\mu &= -\sqrt{\frac{2}{3}} \begin{bmatrix} \Delta^+ \\ \Delta^0 \end{bmatrix}_\mu
\end{aligned} \tag{1.23}$$

Finally, the general form of the relativistic Lagrangian for a spin-3/2, isospin-3/2 field reads:

$$\mathcal{L}_{S=3/2, I=3/2} = \overline{\psi}^{\mu\,\frac{3}{2}}_{i} \Lambda^{(0)ij}_{\mu\nu} \psi^{\nu}_{j}. \tag{1.24}$$

In addition to this Lagrangian which will be used to describe the Δ system, the general effective Lagrangian relevant for the study of a general interacting system made up of pions, nucleons and Δ resonances will off course include the following structures:

$$\mathcal{L}^{\text{eff}} = \mathcal{L}_{\pi N} + \mathcal{L}_{\pi\pi} + \mathcal{L}_{\pi N \Delta} + \mathcal{L}_{\pi\Delta} + \mathcal{L}_{\Delta}. \tag{1.25}$$

Explicit expressions of some terms of the effective Lagrangian will be given in chapter 3, where the formalism introduced in this section will be applied to the description of a particular low energy process.

Adopting the choice $A = -1$, the propagator for a spin-3/2 isospin-3/2 particle within the Rarita-Schwinger formalism in d space-time dimensions has the general form:

$$G_{\mu\nu}(p) = G^{ij}_{\mu\nu}(p)\xi^{ij}_{3/2}, \tag{1.26}$$

where

$$G^{ij}_{\mu\nu}(p) = -i\frac{\not p + m_\Delta}{p^2 - m_\Delta^2 + i\epsilon}\{g_{\mu\nu} - \frac{1}{d-1}\gamma\mu\gamma\nu - \frac{(d-2)}{(d-1)}\frac{p_\mu p_\nu}{m_\Delta^2} + \frac{p_\mu\gamma_\nu - p_\nu\gamma_\mu}{(d-1)m_\Delta}\}, \tag{1.27}$$

and $\xi^{ij}_{3/2}$ is an isospin projection operator, which together with the correspondent isospin-1/2 term has the following properties:

$$\begin{aligned} \xi^{3/2}_{ij} + \xi^{1/2}_{ij} &= \delta_{ij}, && (1.28)\\ \xi^I_{ij}\xi^J_{jk} &= \delta^{IJ}\xi^J_{ik}, && (1.29) \end{aligned}$$

with

$$\begin{aligned} \xi^{3/2}_{ij} &= \delta^{ij} - \frac{1}{3}\tau^i\tau^j = \frac{2}{3}\delta^{ij} - \frac{i}{3}\epsilon_{ijk}\tau^k, && (1.30)\\ \xi^{1/2}_{ij} &= \frac{1}{3}\tau^i\tau^j = \frac{1}{3}\delta^{ij} + \frac{i}{3}\epsilon_{ijk}\tau^k. && (1.31) \end{aligned}$$

The Δ propagator can be rewritten via the projection operators $P^{3/2}_{\mu\nu}$ and $P^{1/2}_{\mu\nu}$ into its spin-3/2 and 1/2 components [BHM03a]:

$$\begin{aligned} G_{\mu\nu}(p) &= -\frac{\not p + m_\Delta}{p^2 - m_\Delta^2}P^{3/2}_{\mu\nu} - \frac{1}{\sqrt{d-1}m_\Delta}\left(\left(P^{1/2}_{12}\right)_{\mu\nu} + \left(P^{1/2}_{21}\right)_{\mu\nu}\right) && (1.32)\\ &\quad + \frac{d-2}{(d-1)m_\Delta^2}(\not p + m_\Delta)\left(P^{1/2}_{22}\right)_{\mu\nu}. \end{aligned}$$

We observe that the spin-1/2 pieces do not propagate and thus in principle can be absorbed in purely polynomial terms with consequent redefinition of the LECs of the theory. This has been shown for example in reference [BHM03a, TE96] for the nucleon mass. That calculation has demonstrated how the spin-1/2 contribution to the nucleon self-energy can be completely absorbed into polynomial terms appearing in the chiral expansion of the nucleon mass beyond leading-one-loop order. This reference represents one of the first attempts to develop a consistent extension of the infrared regularization method in the presence of spin-3/2 fields, that is a covariant formulation of the small scale expansion, up to that time mainly applied in its heavy baryon approach. We remark that the fact that spin-1/2 terms simply redefine counterterms is not a characteristic of the covariant formalism but the same happens also in the non-relativistic SSE scheme [HHK98]. The calculation of the nucleon mass in covariant SSE has been later performed at the next-to-leading one loop order in reference [BHM05], where results for the Δ mass are also provided.

Another example of calculation in the covariant SSE scheme will be presented in chapter 3, where further technical details about chiral EFT with inclusion of the Δ resonance will be given.

An effective field theory based on the chiral symmetry properties of QCD in the zero-mass limit has been introduced. Working in a relativistic framework we have written the main Lagrangians describing the nucleon-system and its interaction with pions. The advantages/disadvantages of different renormalization schemes have been discussed and a systematic manner to include spin-3/2 fields as explicit degree of freedom in the calculation has been explored. We are now ready to start our chiral analysis of the structure of the nucleon by electromagnetic probes; further pertinent technical details necessary for the understanding of the discussed calculations can be found in the specific chapters.

Chapter 2

The Generalized Form Factors of the Nucleon

2.1 Introduction

About a decade ago the concept of Generalized Parton Distributions (GPDs) has emerged among theorists, constituting a universal framework bringing a host of seemingly disparate nucleon structure observables like form factors, moments of parton distribution functions, etc. under one theoretical roof. For reviews of this very active field of research we refer to references [Ji98, Die03, BR05, BP07].
Working in twist-two approximation, the parity-even part of the structure of the nucleon is encoded in two Generalized Parton Distribution functions $H^q(x,\xi,t)$ and $E^q(x,\xi,t)$. For a process where the incoming (outgoing) nucleon carries the four-momentum p_1^μ (p_2^μ) we define two new momentum variables

$$q^\mu = p_2^\mu - p_1^\mu; \quad \bar{p} = (p_1^\mu + p_2^\mu)/2. \tag{2.1}$$

The GPDs can e.g. be accessed in Deeply Virtual Compton Scattering (DVCS) off the nucleon where – in the parton model of the nucleon – the GPD variable x can be interpreted as the fraction of the total momentum of the nucleon carried by the probed quark q. $t = q^2$ denotes the total four-momentum transfer squared to the nucleon, whereas the "skewdness" variable $\xi = -n \cdot q/2$ with $n \cdot \bar{p} = 1$ interpolates between the t- and the x dependence of the GPDs, for details see the reviews [Ji98, Die03, BR05, BP07].
The three-dimensional parameter space of GPDs is vast and rich in information about nucleon structure. The experimental program for their determination is only at the beginning at laboratories like CERN, Desy, JLAB, etc. [Ji98, Die03, BR05, BP07]. However, moments of GPDs can be interpreted much easier and are connected to well established hadron structure observables. E.g. the zeroth order Mellin moments in the variable x correspond to the contribution of quark q to the well known Dirac- and Pauli form factors $F_1(t)$ and $F_2(t)$ of

the nucleon:

$$\int_{-1}^{1} dx\, H^q(x,\xi,t) = F_1^q(t), \tag{2.2}$$

$$\int_{-1}^{1} dx\, E^q(x,\xi,t) = F_2^q(t), \tag{2.3}$$

We recall that the Dirac- and Pauli form factors appear when writing the most general form of the nucleon matrix element of the vector current $V_\mu = \overline{q}\gamma_\mu q$:

$$\langle p_2|V_\mu|p_1\rangle = \overline{u}(p_2)\left[F_1^q(t)\gamma_\mu + F_2^q(t)\frac{i\sigma_{\mu\alpha}q^\alpha}{2M_N}\right]u(p_1) \tag{2.4}$$

where $u\,(\overline{u})$ is a Dirac spinor of the incoming (outgoing) nucleon of mass M_N for which the quark matrix element is evaluated.
The form factors play a central role in the understanding the structure of the nucleon. For the case of 2 light flavors the isoscalar and isovector Dirac and Pauli form factors of the nucleon have been studied at low values of t at the one-loop level in Chiral Effective Field Theory, both in non-relativistic [BFHM98] and in covariant [GSS88, KM01, SGS05] schemes. The chiral extrapolation of these form factors for lattice QCD data with the help of ChEFT has been discussed in refs.[HW02, G$^+$05, AKNT06, PPV06], whereas the status of the experimental situation is reviewed in ref.[dJ06].

We want to focus on the *first* moments in x of these nucleon GPDs

$$\int_{-1}^{1} dx\, x\, H^q(x,\xi,t) = A_{2,0}^q(t) + (-2\xi)^2 C_{2,0}^q(t), \tag{2.5}$$

$$\int_{-1}^{1} dx\, x\, E^q(x,\xi,t) = B_{2,0}^q(t) - (-2\xi)^2 C_{2,0}^q(t), \tag{2.6}$$

where one encounters *three generalized form factors* $A_{2,0}^q(t)$, $B_{2,0}^q(t)$ and $C_{2,0}^q(t)$ of the nucleon for each quark flavor q. For the case of two light flavors the *generalized isoscalar and isovector form factors* have been analysed in a series of papers at leading one loop order in the nonrelativistic framework of HBChPT, starting with the pioneering analyses of Chen and Ji as well as Belitsky and Ji [CJ02, BJ02, ACK06, DMS06, DMS07]. Our aim is to provide the first analysis of these generalized form factors utilizing the methods of covariant Baryon Chiral Perturbation Theory (BChPT) for two light flavors pioneered in reference [GSS88]. The leading one loop order calculation presented here relies on the $\overline{\text{IR}}$ renormalization prescription introduced in section 1.2 which we consider to be inevitable for a consistent analysis of quark mass dependencies in BChPT. We note that (at $t = 0$) a covariant BChPT calculation differs from a nonrelativistic one – provided both are performed at the same chiral order D – by an infinite series of terms $\sim (m_\pi/M_0)^i$ where M_0 denotes the mass of the nucleon in the chiral limit, estimated to be around 890 MeV [PHW04, AK$^+$04, BHM05, PMW$^+$06]. Such terms quickly become relevant once the pion mass m_π takes on values larger than 140 MeV, as it typically occurs in present day lattice QCD simulations of generalized form factors. Aside

from this resummation property in $(1/M_0)^i$, we emphasize again, that the power-counting analysis determining possible operators and allowed topologies for loop diagrams at a particular chiral order (see section 2.3.3) is *identical* between covariant and nonrelativistic frameworks. Both schemes organize a perturbative calculation as a power series in $(1/(4\pi F_\pi))^D$. Finally we note that the first moments of nucleon GPDs have also been studied in constituent quark models (e.g. see ref.[BPT03]) and chiral quark soliton models (e.g. see ref.[G$^+$07]) of the nucleon which – in contrast to ChEFT – can also provide dynamical insights into the short-distance structure present in the generalized form factors.
This chapter is organized as follows: In the next section we specify the operators with which we are going to obtain information on the three generalized isoscalar and the three generalized isovector form factors of the nucleon. In section 2.3 we portray the effective chiral Lagrangian required for the calculation, immediately followed by the sections containing our leading one loop order results of the generalized form factors in the isovector (section 2.4) and in the isoscalar (section 2.5) matrix elements. Finally, a last section (section 2.6) extends the discussion to the generalized axial form factors. A summary of the main results concludes this chapter, while a few technical details regarding the calculation of the amplitudes in covariant BChPT are relegated to the appendix section A. We have published the main results of this chapter in reference [DGH08]. After this publication has appeared the results of this analysis have been applied successfully to new sets of lattice data in refs.[H$^+$08, B$^+$07b]

2.2 Extracting the First Moments of GPDs

2.2.1 The generalized form factors of the nucleon

In eqs.(2.5,2.6) of the introduction of this chapter it was shown that the first moments of nucleon GPDs are connected to three generalized form factors. In lattice QCD one can directly access the contribution of quark flavor q to these generalized form factors of the nucleon by evaluating the matrix element [Ji98, Die03, BR05]

$$i\langle p_2|\overline{q}\gamma_{\{\mu}\overleftrightarrow{D}_{\nu\}}\,q|p_1\rangle = \overline{u}(p_2)\left[A^q_{2,0}(q^2)\gamma_{\{\mu}\overline{p}_{\nu\}} - \frac{B^q_{2,0}(q^2)}{2M_N}\,q^\alpha i\sigma_{\alpha\{\mu}\overline{p}_{\nu\}} + \frac{C^q_{2,0}(q^2)}{M_N}\,q_{\{\mu}q_{\nu\}}\right]u(p_1). \tag{2.7}$$

The brackets $\{\ldots\}$ denote the completely symmetrized and traceless combination of all indices: $a_{\{\mu}b_{\nu\}} = a_\mu b_\nu + a_\nu b_\mu - \frac{2}{d}g_{\mu\nu}a\cdot b$. In eq.(2.7) the generalized form factors $A(q^2)$, $B(q^2)$ and $C(q^2)$ are real functions of the momentum transfer squared. In ChEFT we employ the same philosophy and also extract information about the first moments of nucleon GPDs of eq.(2.5,2.6) via a calculation of the generalized form factors according to eq.(2.7).
Studying a strongly interacting system with two light flavors in the nonpertur-

bative regime of QCD with the methods of ChEFT one works in the basis of singlet (s) and triplet (v) contributions of the quarks to the three form factors:

$$i\langle p_2|\overline{q}\gamma_{\{\mu}\overleftrightarrow{D}_{\nu\}}\,q|p_1\rangle_{u+d} = \\ \overline{u}(p_2)\left[A^s_{2,0}(q^2)\gamma_{\{\mu}\overline{p}_{\nu\}} - \frac{B^s_{2,0}(q^2)}{2M_N}\,q^\alpha i\sigma_{\alpha\{\mu}\overline{p}_{\nu\}} + \frac{C^s_{2,0}(q^2)}{M_N}\,q_{\{\mu}q_{\nu\}}\right]\frac{\mathbb{1}}{2}\,u(p_1), \tag{2.8}$$

$$i\langle p_2|\overline{q}\gamma_{\{\mu}\overleftrightarrow{D}_{\nu\}}\,q|p_1\rangle_{u-d} = \\ \overline{u}(p_2)\left[A^v_{2,0}(q^2)\gamma_{\{\mu}\overline{p}_{\nu\}} - \frac{B^v_{2,0}(q^2)}{2M_N}\,q^\alpha i\sigma_{\alpha\{\mu}\overline{p}_{\nu\}} + \frac{C^v_{2,0}(q^2)}{M_N}\,q_{\{\mu}q_{\nu\}}\right]\frac{\tau^a}{2}\,u(p_1). \tag{2.9}$$

Note that the 2×2 unit matrix $\mathbb{1}$ and the Pauli matrices τ^a with $a = 1, 2, 3$ on the right hand sides of eqs.(2.8,2.9) operate in the space of a (proton,neutron) doublet field.

At present not much is known yet experimentally about the momentum dependence of these 6 form factors. The main source of information at the moment is provided by lattice QCD studies of these objects (e.g. see refs.[G$^+$04, H$^+$03, E$^+$06a, H$^+$08, B$^+$07b]). Given that present day lattice simulations work with quark masses much larger than those realized for u and d quarks in the standard model, one also needs to know the quark mass dependence of all 6 form factors in order to extrapolate the lattice QCD results down to the real world of light u and d quarks. This information is also encoded in the ChEFT results, typically expressed in form of a pion mass dependence of the observables under study. (The connection between the explicit breaking of chiral symmetry due to non-zero quark-masses and the resulting effective pion-mass is addressed in section 2.3.2). The analysis of the quark-mass dependence of the generalized form factors proceeds as follow: we first extract numerical values for presently unknown ChPT coupling constants by fitting the pion mass dependent BChPT results to lattice data at different quark masses. Thanks to the extrapolation function provided by the EFT framework we are then able to bridge the gap between the domain of large quark-masses used in present day lattice simulations and the physical world of small quark masses. The information revealed by this crosstalk between lattice QCD and ChPT allows us to give predictions for the physical observables under study.

The need for a chiral extrapolation of lattice QCD results for the generalized form factors of the nucleon leads to a further complication in the analysis: One needs to be aware that it is *common practice in current lattice QCD analyses* that the mass parameter M_N in Eqs.(2.8,2.9) does *not* correspond to the physical mass of a nucleon, instead, it represents a (larger) nucleon mass consistent with the values of the quark-masses employed in the simulation. The quark-mass dependence of M_N has been studied in detail in ChEFT, both in non-relativistic [BHM04] and in covariant frameworks [PHW04, BHM05, PMW$^+$06]. The next-to-leading one loop chiral formulae of ref.[PHW04] provide a stable extrapolation function up to effective pion-masses ~ 600 MeV. We therefore utilize the $\mathcal{O}(p^4)$ $\overline{\text{IR}}$ renormalized BChPT

g_A	1.2
F_π [GeV]	0.0924
M_0 [GeV]	0.889
c_1 [GeV^{-1}]	-0.817
c_2 [GeV^{-1}]	3.2
c_3[GeV^{-1}]	-3.4
e_1^r(1GeV) [GeV^{-3}]	1.44

Table 2.1: Input values used in this chapter for the numerical analysis of the chiral extrapolation functions [Gai07].

result [Gai07]

$$\begin{aligned} M_N(m_\pi) &= M_0 - 4c_1 m_\pi^2 \\ &+ \frac{3g_A^2 m_\pi^3}{8\pi^2 F_\pi^2 \sqrt{4 - \frac{m_\pi^2}{M_0^2}}} \left(-1 + \frac{m_\pi^2}{4M_0^2} + c_1 \frac{m_\pi^4}{M_0^3} \right) \arccos\left(\frac{m_\pi}{2M_0} \right) \\ &+ 4e_1^r(\lambda) m_\pi^4 - \frac{3m_\pi^4}{128\pi^2 F_\pi^2} \left[\left(\frac{6g_A^2}{M_0} - c_2 \right) \right. \\ &\left. + 4\left(\frac{g_A^2}{M_0} - 8c_1 + c_2 + 4c_3 \right) \log\left(\frac{m_\pi}{\lambda} \right) \right] \\ &- \frac{3c_1 g_A^2 m_\pi^6}{8\pi^2 F_\pi^2 M_0^2} \log\left(\frac{m_\pi}{M_0} \right) + \mathcal{O}(p^5), \end{aligned} \tag{2.10}$$

in order to correct for the mass effects of eqs.(2.8,2.9). The coupling constants occurring in this formula are described in detail in reference [PHW04, AK+04, BHM05, PMW+06]. Possible effects of higher orders can be estimated as $\mathcal{O}(p^5) \sim \delta_M \frac{m_\pi^5}{(4\pi F_\pi)^4}$ where δ_M could be varied within natural size estimates $-3 < \delta_M < +3$ [Gai07].
We note that the trivial, purely kinematical effect of $M_N = M_N(m_\pi)$ in eqs.(2.8,2.9) could induce quite a strong quark mass dependence into the form factors $B_{2,0}^{s,v}(t)$ and $C_{2,0}^{s,v}(t)$ and might even be able to mask any “intrinsic” quark mass dependence in these form factors. We are reminded of the analysis of the Pauli form factors $F_2^{s,v}(t)$ in ref.[G+05] where the absorption of the analogous effect into a “normalized” magneton even led to a different slope (!) for the isovector anomalous magnetic moment $\kappa_v = F_2^v(t = 0)$ when compared to the quark mass dependence of the “unnormalized” lattice data. We therefore urge the readers that this effect should be taken into account in any quantitative (future) analysis of the quark mass dependence of the generalized form factors $B_{2,0}^{s,v}(t)$ and $C_{2,0}^{s,v}(t)$ as well. For convenience we give the numerical values for the appearing coupling constants in table 2.1 and emphasize that we use the same values for all parameters throughout this work.

Finally we note that in the forward limit $t \to 0$ the generalized form factors $A_{2,0}^{s,v}(t = 0)$ can be understood as moments of the ordinary Parton Distribution

Functions (PDFs) $q(x)$ and $\bar{q}(x)$ [Ji98, Die03, BR05]:

$$\langle x\rangle_{u\pm d} \;=\; A^{s,v}_{2,0}(t=0) = \int_0^1 dx\, x\,(q(x)+\bar{q}(x))_{u\pm d}\,. \qquad (2.11)$$

Experimental results exist for $\langle x\rangle$ in proton- and "neutron" targets, from which one can estimate the isoscalar and isovector quark contributions at the physical point [xpr] at a regularization scale μ. In this work we choose $\mu = 2$ GeV for our comparisons with phenomenology[1]. In section 2.4.1 we attempt to connect the physical value for $\langle x\rangle_{u-d}$ with recent lattice QCD results from the LHPC collaboration [H$^+$08], whereas in section 2.5.1 we analyse the quark mass dependence of $\langle x\rangle_{u+d}$ with (quenched) lattice QCD results from the QCDSF collaboration [G$^+$04].

2.2.2 The generalized form factors of the pion

The first moment of a generalized parton distribution function in a pion $H^q_\pi(x,\xi,t)$ can be defined analogously to the case of the nucleon discussed above. One obtains [Ji98, Die03, BR05]

$$\int_{-1}^1 dx\, x\, H^q_\pi(x,\xi,t) \;=\; A^q_\pi(t) + (-2\xi)^2\, C^q_\pi(t). \qquad (2.12)$$

The two functions $A^q_\pi(t)$ and $C^q_\pi(t)$ are the generalized form factors of the pion, generated by contributions of quark flavor q. In the forward limit one recovers the first moment of the ordinary parton distribution functions in a pion:

$$\langle x\rangle_\pi \;=\; A^q_\pi(t=0) = \int_0^1 dx\, x\,(q(x)+\bar{q}(x))\,. \qquad (2.13)$$

In the analysis of the isoscalar GPD moments of a nucleon we encounter tensor fields directly interacting with the pion-cloud of the nucleon. One therefore needs to understand the relevant pion-tensor couplings in terms of the two generalized form factors $A_\pi(t)$ and $C_\pi(t)$. We note that the two generalized form factors of the pion have been analysed at one loop level already in ref.[DL91] for the total sum of quark and gluon contributions, whereas the quark contribution to the form factors as defined in eq.(2.12) has been the focus of the more recent works [KP02, DMS05].

2.3 Formalism

2.3.1 Leading order nucleon Lagrangian

In this section we apply the basic concepts of ChEFT introduced in chapter 1 to the present discussion, adapting them to the particular case of interest.

[1] Note that this μ dependence is not part of the ChEFT framework, as it clearly involves short-distance physics. However, all chiral tensor couplings specified in section 2.3 carry an implicit μ dependence (which we do not indicate) as soon as they are fitted to lattice QCD data or phenomenological values which do depend on this scale.

We briefly introduce the relevant parts of the chiral Lagrangian which are necessary for an evaluation of the tensor currents eqs.(2.8,2.9) at leading one loop level.
We first extend the Lagrangian of eq.(1.7) to the interaction between *external tensor fields* and a strongly interacting system at low energies. In this work we focus on *symmetric, traceless tensor fields* with positive parity in order to calculate the generalized form factors of the nucleon. In particular, we utilize the chiral tensor structures

$$\begin{aligned} V^{\pm}_{\mu\nu} &= \frac{1}{2}\left(g_{\mu\alpha}g_{\nu\beta}+g_{\mu\beta}g_{\nu\alpha}-\frac{2}{d}g_{\mu\nu}g_{\alpha\beta}\right)\times\left(u^\dagger V_R^{\alpha\beta}u\pm uV_L^{\alpha\beta}u^\dagger\right), \\ V^{0}_{\mu\nu} &= \frac{1}{2}\left(g_{\mu\alpha}g_{\nu\beta}+g_{\mu\beta}g_{\nu\alpha}-\frac{2}{d}g_{\mu\nu}g_{\alpha\beta}\right)v^{\alpha\beta}_{(s)}\,\frac{\mathbb{1}}{2}\,. \end{aligned} \tag{2.14}$$

The right- and left handed fields $V^{(R,L)}_{\alpha\beta}$ are related to the symmetric *isovector* tensor fields of definite parity $v^i_{\alpha\beta}$ and $a^i_{\alpha\beta}$ with $i=1,2,3$ via

$$\begin{aligned} V^R_{\alpha\beta} &= \left(v^i_{\alpha\beta}+a^i_{\alpha\beta}\right)\times\frac{\tau^i}{2}\,, \\ V^L_{\alpha\beta} &= \left(v^i_{\alpha\beta}-a^i_{\alpha\beta}\right)\times\frac{\tau^i}{2}\,, \end{aligned} \tag{2.15}$$

while $v^{(s)}_{\alpha\beta}$ denotes the symmetric *isoscalar* tensor field of positive parity. In order to study possible interactions with external tensor fields originating from the leading order Lagrangian eq.(1.7), we rewrite it into the equivalent form

$$\mathcal{L}_{t\pi N} = \bar{\Psi}_N\left[\frac{1}{2}\left(i\gamma^\mu\widetilde{g_{\mu\nu}}\overrightarrow{D}^\nu-i\overleftarrow{D}^\nu\widetilde{g_{\mu\nu}}\gamma^\mu\right)-M_0+\ldots\right]\Psi_N, \tag{2.16}$$

where we have introduced

$$\widetilde{g_{\mu\nu}} = g_{\mu\nu}+a^s_{2,0}V^0_{\mu\nu}+\frac{a^v_{2,0}}{2}V^+_{\mu\nu}. \tag{2.17}$$

The coupling $a^s_{2,0}$ ($a^v_{2,0}$) has been defined such that it corresponds to the chiral limit value of ($\langle x\rangle_{u+d}$) ($\langle x\rangle_{u-d}$) defined in eq.(2.11). We note that the coupling $a^s_{2,0}$ is allowed to be different from unity, as we only sum over the $u+d$ quark contributions in the isoscalar moments but neglect the contributions from gluons. While this separation between quark- and gluon contributions does not occur in nature where one obtains the sum rule for the total angular momentum $A^{q+g}_{2,0}(0)=1$, it can be implemented in lattice QCD analyses at a fixed renormalization scale (e.g. see refs.[G+04, H+03, E+06a, H+08]).
Although the construction of the parity-even tensor interactions with a strongly interacting system started from the $\mathcal{O}(p^1)$ BChPT Lagrangian, an inspection of the resulting Lagrangian eq.(2.16) reveals that the leading interactions actually start out at $\mathcal{O}(p^0)$. This appears as a consequence of the fact that we do not assign a nonzero chiral dimension p^n, $n\geq 1$ to any of the tensor fields. Furthermore, symmetries allow the addition of the parity-odd tensor interaction

$\sim V^-_{\mu\nu}$. We finally obtain [DGH08]

$$\mathcal{L}^{(0)}_{t\pi N} = \frac{1}{2}\bar{\Psi}_N \left[i\gamma^\mu \left(a^s_{2,0} V^0_{\mu\nu} + \frac{a^v_{2,0}}{2} V^+_{\mu\nu} + \frac{\Delta a^v_{2,0}}{2} V^-_{\mu\nu}\gamma_5 \right) \overrightarrow{D}^\nu - i\overleftarrow{D}^\nu \gamma^\mu \left(a^s_{2,0} V^0_{\mu\nu} + \frac{a^v_{2,0}}{2} V^+_{\mu\nu} + \frac{\Delta a^v_{2,0}}{2} V^-_{\mu\nu}\gamma_5 \right) \right] \Psi_N, \quad (2.18)$$

with the coupling $\Delta a^v_{2,0}$ corresponding to the chiral limit value of the axial quantity $\langle \Delta x \rangle_{u-d}$ (see section 2.6). The $\mathcal{O}(p^1)$ part of the leading order pion-nucleon Lagrangian in the presence of external symmetric, traceless tensor fields with positive parity then reads [DGH08]

$$\mathcal{L}^{(1)}_{t\pi N} = \bar{\Psi}_N \left\{ i\gamma^\mu D_\mu - M_0 + \frac{g_A}{2}\gamma^\mu\gamma_5 u_\mu + \frac{b^v_{2,0}}{8M_0}\left(i\sigma_{\alpha\mu} \left[\overrightarrow{D}^\alpha, V^{\mu\nu}_+ \right] \overrightarrow{D}_\nu + h.c. \right) + \frac{b^s_{2,0}}{4M_0}\left(i\sigma_{\alpha\mu} \left[\overrightarrow{\nabla}^\alpha, V^{\mu\nu}_0 \right] \overrightarrow{D}_\nu + h.c. \right) + \ldots \right\} \Psi_N, \quad (2.19)$$

where we have introduced $\nabla^\alpha = \partial^\alpha - i v^\alpha_{(s)}$. The two new couplings $b^v_{2,0}$ and $b^s_{2,0}$ can be interpreted as the chiral limit values of isovector- and isoscalar anomalous gravitomagnetic moments $B^v_{2,0}(0)$ and $B^s_{2,0}(0)$. No further structures enter our calculation at this order[2]. Finally we note that the coupling $b^s_{2,0}$ is only allowed to exist because we do not sum over the quark- *and gluon* contributions in the isoscalar moments, otherwise the anomalous gravitomagnetic moment is bound to vanish in the forward limit $B^{q+g}_{2,0}(t=0) = 0$ [Ter99].

2.3.2 Consequences for the meson Lagrangian

The choice of assigning the chiral power p^0 to the symmetric tensor fields $V^{L,R,0}_{\mu\nu}$ also has the consequence that the well known leading order chiral Lagrangian for two light flavors in the meson sector [GL84](see eq.(1.2)) is modified [DGH08]:

$$\mathcal{L}^{(2)}_{t\pi\pi} = \frac{F_\pi^2}{4}\,\mathrm{Tr}\left[\nabla_\mu U^\dagger \left(g^{\mu\nu} + 4x^0_\pi V^{\mu\nu}_0 \right) \nabla_\nu U + \chi^\dagger U + \chi U^\dagger \right]. \quad (2.20)$$

We note that the new coupling x^0_π has been defined such that it corresponds to the chiral limit value of $\langle x \rangle_\pi$ of eq.(2.13). It is allowed to differ from unity because we only sum over the quark-distribution functions in the isoscalar channel and neglect the contributions from gluons.

The explicit breaking of chiral symmetry via the finite quark masses is encoded via

$$\chi = 2\,B_0\,(s + ip)\,, \quad (2.21)$$

if one switches off the external pseudoscalar background field p and identifies the two flavor quark mass matrix $\mathcal{M} = \mathrm{diag}(m_u, m_d)$ with the scalar background field s. To the order we are working here we obtain the resulting pion

[2] We only show those terms where the tensor fields couple at tree level without simultaneous emission of pions, photons, etc., as these are the relevant terms for our $\mathcal{O}(p^2)$ calculation of the form factors according to the power-counting analysis of subsection 2.3.3.

mass m_π via

$$m_\pi^2 = 2\, B_0\, \hat{m} + \mathcal{O}({m_q}^2), \tag{2.22}$$

where $\hat{m} = (m_u + m_d)/2$ and B_0 is connected to the value of the chiral condensate. The other free parameter at this order F_π can be identified with the value of the pion-decay constant (in the chiral limit).

2.3.3 Power-counting in BChPT with tensor fields

We start from the general power-counting formula of Baryon ChPT:

$$D \;=\; 2N_L + 1 + \sum_d (d-2)N_d^M + \sum_d (d-1)N_d^{MB}. \tag{2.23}$$

Here D denotes the chiral dimension p^D of a particular Feynman diagram, N_L counts the number of loops in the diagram, whereas the variables $N_d^{M,MB}$ count the number of vertices of chiral dimension d from the pion (M) and pion-nucleon (MB) Lagrangians. To leading order[3] $D = 0$ we only have the tree level contributions from the order p^0 Lagrangian of eq.(2.18) with $N_L = 0$, $N_2^M = 0$ and $N_0^{MB} = 1$. At next-to-leading order $D = 1$ we find additional tree level contributions from the order p^1 Lagrangian of eq.(2.19) with $N_L = 0$, $N_2^M = 0$ and $N_1^{MB} = 1$. The first loop contributions enter at $D = 2$ with $N_L = 1$, $N_2^M = 0$ and $N_0^{MB} = 1$ plus possible contributions from N_1^{MB}. The corresponding diagrams are shown in fig.2.1. Diagram (e) in that figure represents loop corrections from the nucleon Z-factor (given in appendix A.4) which at this order only renormalizes the tree level tensor couplings of the order p^0 Lagrangian. Note that there is an additional possibility of obtaining $D = 2$ contributions via $N_L = 0$, $N_2^M = 0$ and $N_2^{MB} = 1$, corresponding to further tree level contributions discussed in the next subsection.
In this work we stop with our analysis of the generalized form factors at the $D = 2$, i.e. $\mathcal{O}(p^2)$ level, corresponding to a leading one loop order calculation. Preliminary results for the next-to-leading one loop effects of $D = 3$ are given in appendix A.7 but their analysis is postponed to a later communication. The Feynman rules pertinent to the calculation are listed in Appendix A.1. The (perhaps) surprising finding of this power-counting analysis is the observation that the tensor coupling to the pion field controlled by the coupling x_π^0 in eq.(2.20) does *not* contribute at leading one loop order! Here it only starts to enter at $D = 3$ via $N_L = 1$, $N_2^M = 1$ and $N_1^{MB} = 1$ or 2. (The corresponding diagram for $N_1^{MB} = 2$ is shown in fig.2.2 while the not-shown diagram for $N_1^{MB} = 1$ is expected to sum to zero due to isospin-symmetry.) We therefore note that the generalized form factors of the nucleon behave quite different from the standard Dirac- and Pauli form factors of the nucleon where the pion-cloud interactions with the external source à la fig.2.2 are part of the leading one loop order result and play a prominent role in the final result. We discuss the impact of this particular $D = 3$ contribution further in section 2.5.3

[3] Note that in contrast to the analyses presented in the previous chapters where the leading order contributions were of chiral dimension $D = 1$, the calculation in this chapter starts at $D = 0$ due to the presence of external fields with zero chiral dimension.

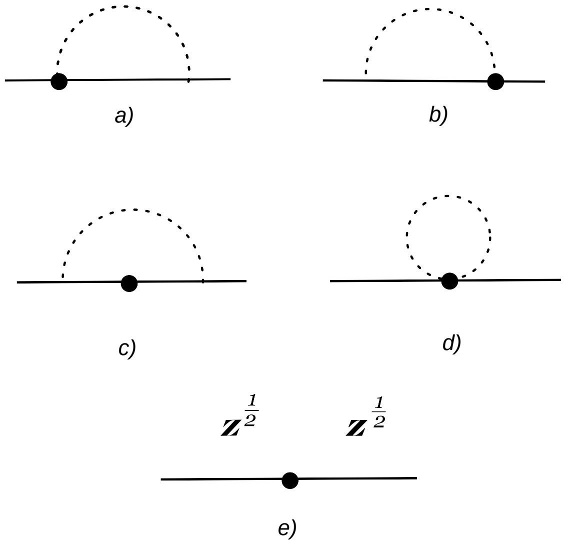

Figure 2.1: The loop diagrams contributing to the first moments of the GPDs of a nucleon at leading one loop order in BChPT. The solid- and dashed lines represent nucleon and pion propagators, respectively. The solid dot denotes a coupling to a tensor field from the $\mathcal{O}(p^0)$ Lagrangian of eq.(2.18).

when we try to estimate the possible size of higher order corrections to our $\mathcal{O}(p^2)$ analysis.
We point out that the chiral dimension we are talking about is the one of the contributing loop diagrams and thus of the currents in eqs.(2.9),(2.8), it is not necessarily the chiral dimension of the respective contributions to the generalized form factors. Indeed, because of the presence of one power of a small parameter in front of $B_{2,0}(q^2)$ and two powers of a small parameter in front of $C_{2,0}(q^2)$ in eqs.(2.9),(2.8), if the currents are evaluated at a chiral dimension D, the corresponding contributions to $B_{2,0}(q^2)$ are of chiral dimension $D-1$, the contributions to $C_{2,0}(q^2)$ are of chiral dimension $D-2$. The form factor $A_{2,0}(q^2)$ does not carry such a small prefactor and is therefore not affected by this consideration. Consequently at the $D=2$ level systematic uncertainties due to possible higher order effects are expected to be already moderate for $A_{2,0}(q^2)$ but still large for $C_{2,0}(q^2)$. We now move on to a discussion of the tensor interactions in the $\mathcal{O}(p^2)$ Lagrangian which contribute to the generalized form factors at $D=2$ according to our power-counting analysis.

2.3.4 Next-to-leading order nucleon Lagrangian

At next-to-leading order the covariant BChPT Lagrangian for two flavor QCD contains seven independent terms in the presence of general scalar, pseudoscalar, vector- and axial vector background fields, governed by the couplings

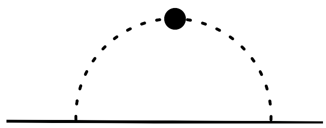

Figure 2.2: (Isoscalar) tensor field coupling to the pion-cloud of the nucleon. This process only starts to contribute at next-to-leading one loop order in BChPT.

$c_1, \ldots, c_7$ [BKM95]. Extending this scenario to *symmetric* and *traceless* tensor background fields with positive parity, symmetries allow the construction of *six* additional, independent terms which describe the coupling of a tensor field to the nucleon at next-to-leading order tree level [DGH08]:

$$\begin{aligned}
\mathcal{L}^{(2)}_{t\pi N} =& \frac{c_8}{4M_0^2}\,\bar{\Psi}_N\left\{\mathrm{Tr}(\chi_+)V^+_{\mu\nu}\gamma^\mu i\overrightarrow{D}^\nu + h.c.\right\}\Psi_N \\
&+ \frac{c_9}{2M_0^2}\,\bar{\Psi}_N\left\{\mathrm{Tr}(\chi_+)\gamma^\mu i\overrightarrow{D}^\nu + h.c.\right\}\Psi_N V^0_{\mu\nu} \\
&+ \frac{c^v_{2,0}}{2M_0}\bar{\Psi}_N\left\{\left[\overrightarrow{D}^\mu,[\overrightarrow{D}^\nu,V^+_{\mu\nu}]\right]\right\}\Psi_N \\
&+ \frac{c^s_{2,0}}{M_0}\bar{\Psi}_N\left\{\left[\overrightarrow{\nabla}^\mu,[\overrightarrow{\nabla}^\nu,V^0_{\mu\nu}]\right]\right\}\Psi_N \\
&+ \frac{c_{12}}{4M_0^2}\bar{\Psi}_N\left\{\left[\overrightarrow{D}^\alpha,[\overrightarrow{D}_\alpha,V^+_{\mu\nu}]\right]\gamma^\mu i\overrightarrow{D}^\nu + h.c.\right\}\Psi_N \\
&+ \frac{c_{13}}{2M_0^2}\bar{\Psi}_N\left\{\gamma^\mu i\overrightarrow{D}^\nu + h.c.\right\}\Psi_N\,\left[\overrightarrow{\nabla}^\alpha,[\overrightarrow{\nabla}_\alpha,V^0_{\mu\nu}]\right] \\
&+ \ldots
\end{aligned} \tag{2.24}$$

with $\chi_+ = u^\dagger\chi u^\dagger + u\chi^\dagger u$. The physics behind these couplings c_i where $i = 8, \ldots 13$ with respect to the generalized form factors of the nucleon is quite simple: c_8 and c_9 govern the leading quark mass insertion in $\langle x\rangle_{u-d}$ and $\langle x\rangle_{u+d}$, respectively, whereas the couplings c_{10} and c_{11} give the values of the generalized form factors $C^v_{2,0}(0)$ and $C^s_{2,0}(0)$ in the double limit $t \to 0$ and $m_\pi \to 0$. We can therefore denote them as $c^v_{2,0}$ and $c^s_{2,0}$. Finally the couplings c_{12} and c_{13} parametrize the contributions of short-distance physics to the slopes of the generalized form factors $A^v_{2,0}(t)$ and $A^s_{2,0}(t)$ in the chiral limit. Note that the operator controlled by the coupling c_9 is not allowed to exist when we add the gluon-contributions on the left hand side of eq.(2.8) [H$^+$].
After laying down the necessary effective Lagrangians for our calculation, we are now proceeding to the results of our calculation.

2.4 The Generalized Isovector Form Factors in $\mathcal{O}(p^2)$ BChPT

2.4.1 Moments of the isovector GPDs at $t = 0$

In this subsection we present our results for the generalized isovector form factors of the nucleon at $t = 0$. For the PDF-moment $A^v_{2,0}(t = 0)$ we obtain to $\mathcal{O}(p^2)$ in $\overline{\text{IR}}$ renormalized BChPT

$$\begin{aligned}
A^v_{2,0}(0) &= \langle x \rangle_{u-d} \\
&= a^v_{2,0} + \frac{a^v_{2,0} m_\pi^2}{(4\pi F_\pi)^2} \Bigg[-(3g_A^2+1)\log\frac{m_\pi^2}{\lambda^2} - 2g_A^2 \\
&\quad + g_A^2 \frac{m_\pi^2}{M_0^2}\left(1 + 3\log\frac{m_\pi^2}{M_0^2}\right) - \frac{1}{2} g_A^2 \frac{m_\pi^4}{M_0^4}\log\frac{m_\pi^2}{M_0^2} \\
&\quad + g_A^2 \frac{m_\pi}{\sqrt{4M_0^2 - m_\pi^2}}\left(14 - 8\frac{m_\pi^2}{M_0^2} + \frac{m_\pi^4}{M_0^4}\right)\arccos\left(\frac{m_\pi}{2M_0}\right)\Bigg] \\
&\quad + \frac{\Delta a^v_{2,0} g_A m_\pi^2}{3(4\pi F_\pi)^2}\Bigg[2\frac{m_\pi^2}{M_0^2}\left(1 + 3\log\frac{m_\pi^2}{M_0^2}\right) - \frac{m_\pi^4}{M_0^4}\log\frac{m_\pi^2}{M_0^2} \\
&\quad + \frac{2m_\pi(4M_0^2 - m_\pi^2)^{\frac{3}{2}}}{M_0^4}\arccos\left(\frac{m_\pi}{2M_0}\right)\Bigg] \\
&\quad + 4m_\pi^2 \frac{c_8^{(r)}(\lambda)}{M_0^2} + \mathcal{O}(p^3). \qquad (2.25)
\end{aligned}$$

Many of the parameters in this expression are well known from analyses of chiral extrapolation functions. Numerical estimates for their chiral limit values can be found in table 2.1. Furthermore, in a first fit to lattice data we constrain the coupling $\Delta a^v_{2,0}$ from the phenomenological value of $\langle \Delta x \rangle^{phen.}_{u-d} \approx 0.21$ via (see section 2.6)

$$\langle \Delta x \rangle_{u-d} = \Delta a^v_{2,0} + \mathcal{O}(m_\pi^2) \qquad (2.26)$$

and perform a fit with two parameters: The couplings $a^v_{2,0}$ and $c_8^{(r)}(1\text{GeV})$ at the regularization scale $\lambda = 1$ GeV. We fit to the LHPC data for this quantity as given in ref.[H$^+$08], including lattice data up to effective pion masses of $m_\pi \approx 600$ MeV. The resulting values for the fit parameters together with their statistical errors are given in table 2.2. The resulting chiral extrapolation function is shown as the solid line in figure 2.3. We note that the extrapolation curve tends towards smaller values for small quark masses but does not quite reach the phenomenological value at the physical point which was *not* included in the fit. We therefore again estimate possible corrections to the solid curve arising from higher orders. From dimensional analysis we know that the leading chiral contribution to $\langle x \rangle_{u-d}$ beyond our calculation takes the form $\mathcal{O}(p^3) \sim \delta_A \frac{m_\pi^3}{(4\pi F_\pi)^2 M_0} + \ldots$. Repeating the fit with values for δ_A between[4] $-1, \ldots, +1$ and

[4] The natural scale of all couplings in the observables considered here is below one, as all coupling estimates in this section refer to a moment of a parton distribution itself normalized

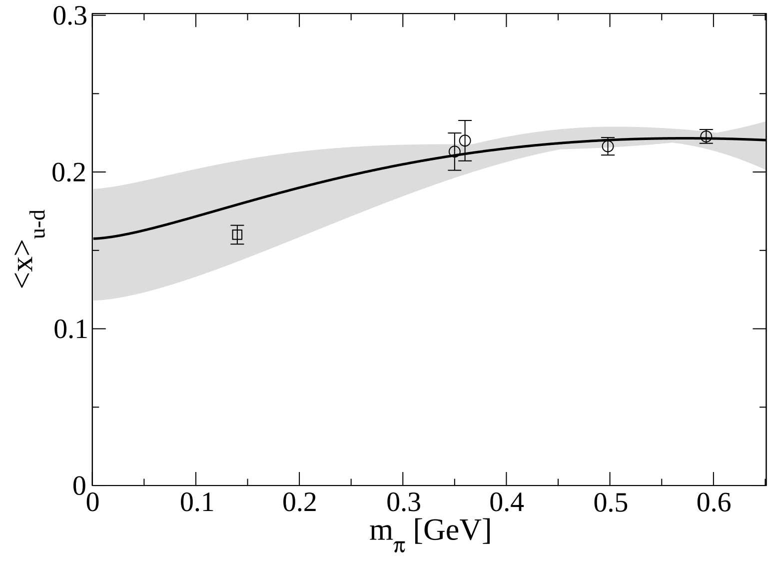

Figure 2.3: The quark mass dependence of the isovector parton distribution in a nucleon. The solid line displays the best fit curve of the $\mathcal{O}(p^2)$ result of eq.(2.25) to the LHPC lattice data of ref.[H$^+$08]. The corresponding parameters are given in Table 2.2. Note that the phenomenological value at the physical pion mass was not included in the fit. The grey band indicates the size of possible $\mathcal{O}(p^3)$ corrections as discussed in the text.

accounting for statistical errors one ends up with an array of curves covering the grey shaded area of figure 2.3. Reassuringly, the phenomenological value for $\langle x\rangle_{u-d}$ lies well within that band of uncertainties due to possible next-order corrections, giving us *no indication* that something may be inconsistent with the large values for $\langle x\rangle_{u-d}$ typically found in lattice QCD simulations at large quark masses. The resulting values for the couplings $a^v_{2,0}$ and $c_8^{(r)}(1\text{ GeV})$ of Fit I are also well within expectations. We can conclude that thanks to the combined (leading one loop order) BChPT plus lattice analysis the physical value of this observable can predicted with a precision of approximately 30%.

At this point some remarks are needed. First of all, in our analysis we have compared the results of a SU(2) theory with the phenomenology of a six flavor world. There might therefore be slight differences between our results and phenomenology, which are anyway for sure beyond the precision of our calculation. Secondly, we have made use of lattice results calculated with $2 + 1$ flavors. The validity of our SU(2) ChPT calculation can be extended to a world with a large, fixed strange quark-mass, the effects of such inclusion being encoded in the coupling constants of the theory. However, the values we would find for those coupling constants might differ from the ones in a pure two flavor scenario.
We note that the mechanism of the downward-bending at small quark masses in $A^v_{2,0}(0)$ found in eq.(2.25), see figure 2.4, is quite different from what has been

to unity. This expectation is confirmed by the fit values of tables 2.2 and 2.4 found for the investigated coupling constants.

	Fit I (4 points - 2 parameter)	Fit II (6+1 points - 3 parameter)
$a^v_{2,0}$	0.157 ± 0.006	0.141 ± 0.0057
$\Delta a^v_{2,0}$	0.210 (fixed)	0.144 ± 0.034
$c^r_8(1\text{GeV})$	-0.283 ± 0.011	-0.213 ± 0.03

Table 2.2: The values of the coupling constants resulting from the two fits of the $\mathcal{O}(p^2)$ BChPT result given in eq.(2.25) to the LHPC lattice data for $\langle x \rangle_{u-d}$. The errors shown are of only statistical origin and do neither include uncertainties from possible higher order corrections in ChEFT nor from systematic uncertainties connected with the lattice simulation.

discussed in literature so far within the nonrelativistic HBChPT framework (e.g. see ref.[DMN$^+$01]). In order to demonstrate this we truncate eq.(2.25) at leading order in $1/M_0$ to obtain the exact $\mathcal{O}(p^2)$ HBChPT limit of our results, agreeing with the findings of references [AS02, CJ01]:

$$\begin{aligned} A^v_{2,0}(0)|^{p^2}_{HBChPT} &= a^v_{2,0}\left\{1 - \frac{m_\pi^2}{(4\pi F_\pi)^2}\left(2g_A^2 + (3g_A^2+1)\log\frac{m_\pi^2}{\lambda^2}\right)\right\} \\ &\quad +4m_\pi^2\frac{c_8^{(r)}(\lambda)}{M_0^2} + \mathcal{O}\left(\frac{m_\pi^3}{16\pi^2 F_\pi^2 M_0}\right). \end{aligned} \tag{2.27}$$

As we already stated in chapter 1 and in the Introduction, the covariant BChPT scheme used in this work is able to exactly reproduce the corresponding nonrelativistic HBChPT result at the same order by the appropriate truncation of the $1/M_0$ expansion. At leading one loop order no recoil effects are included in HBChPT loop results and our covariant results have thus to be truncated at order $\left(\frac{1}{M_0}\right)^0$ in order to obtain the according nonrelativistic limit. In the following we therefore denote this truncation by $1/M_0 \to 0$.

All differences between the HBChPT limits presented in this chapter and the findings of previous HBChPT studies [AS02, CJ01, ACK06, DMS06, DMS07] are due to the inclusion of selected terms of higher order, i.e. terms with $D = 3$ (and even larger values of D) according to the counting formula eq.(2.23) in those references.
Fit I is certainly constricted by the assumption that we use the physical value of $\langle \Delta x \rangle^{phen.}_{u-d} \approx 0.21$ for the coupling $\Delta a^v_{2,0}$ which presumably takes a value in the chiral limit which is a bit smaller than the phenomenological value at the physical point [DH]. Furthermore, in order to also numerically compare the $\mathcal{O}(p^2)$ HBChPT result of eq.(2.27) with the $\mathcal{O}(p^2)$ covariant BChPT result of eq.(2.25) we perform a second fit: We fit the covariant expression for $\langle x \rangle_{u-d}$ of eq.(2.25) again to the LHPC lattice data, this time however, we constrain the coupling $\Delta a^v_{2,0}$ in such a way that the resulting chiral extrapolation curve reproduces the phenomenological value of $\langle x \rangle^{phen.}_{u-d} = 0.160 \pm 0.006$ [xpr] exactly for physical quark masses. The parameter values for this Fit II are again given in table 2.2, whereas the resulting chiral extrapolation curve of the covariant $\mathcal{O}(p^2)$ expression of eq.(2.25) is shown as the solid line in fig.2.4. First,

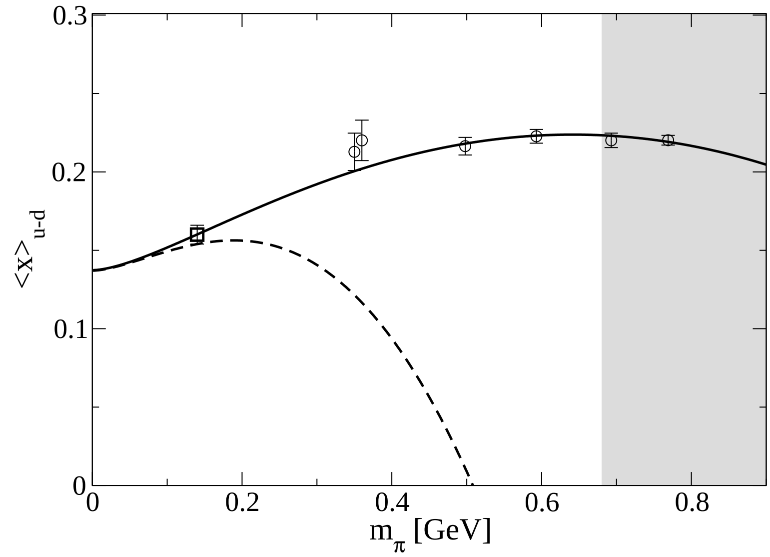

Figure 2.4: "Fit II" of the $\mathcal{O}(p^2)$ BChPT result of eq.(2.25) to the LHPC lattice data of ref.[H+08] *and* to the physical point (solid line). The corresponding fit-parameters are given in table 2.2. The dashed curve shown corresponds to the $\mathcal{O}(p^2)$ result in the HBChPT truncation (see eq.(2.27)). The shaded area indicates the region where one does not expect that ChEFT can provide a trustworthy chiral extrapolation function due to the large pion masses involved. As explained in the text, the difference between the two curves gives indication of the size of the higher order corrections provided by the BChPT result but does not allow us to make strong statements about the radius of convergence of HBChPT.

we emphasize that the curve looks very reasonable, connecting the physical point with the lattice data of the LHPC collaboration in a smooth fashion. Second, we note that the resulting values for the coupling constants $a^v_{2,0}$ and $\Delta a^v_{2,0}$ underlying this curve are very reassuring, indicating that both $\langle x\rangle_{u-d}$ and $\langle\Delta x\rangle_{u-d}$ are slightly smaller in the chiral limit than at the physical point! Likewise, the unknown quark mass insertion $c_8^{(r)}(\lambda)$ contributes in a strength just as expected from natural size estimates. For the comparison with HBChPT we now utilize the very same values[5] for $a^v_{2,0}$ and $c_8^{(r)}(\lambda)$ of Fit II as given in table 2.2. The resulting curve based on the $\mathcal{O}(p^2)$ HBChPT formula of eq.(2.27) is shown as the dashed curve in fig.2.4. One observes that this leading one loop HBChPT expression agrees with the covariant result between the chiral limit and the physical point but is not able to extrapolate on towards the lattice data. Analogous behaviour has been found for the corresponding leading one loop HBChPT expressions for the axial coupling constant of the nucleon [HPW03a] and for the anomalous magnetic moments of the nucleon [HW02]. These examples show the limited usefulness of leading HBChPT extrapolation

[5] We emphasize again that due to the definition of the $\overline{\text{IR}}$ renormalization scheme the numerical values of the ChPT coupling constants have to be the same in HBChPT and $\overline{\text{IR}}$ renormalized covariant BChPT.

		$\langle x \rangle_{u-d}$
PHENOMENOLOGY		0.16 ± 0.006[xpr]
EXTRAPOLATED values at m_π^{phys}	LHPC	0.157 ± 0.006 [H+08]
	QCDSF/UKQCD	0.198 ± 0.008 [B+07b]

Table 2.3: The phenomenological values of the observables $\langle x \rangle_{u-d}$, together with the values obtained from chiral extrapolation studies of lattice data of refs.[H+08] and [B+07b] based on our formula of eq.2.25.

formulae, which seems to successfully describe the quark mass dependence only between the chiral limit and the physical point.
However, we note that all our *numerical* comparisons with HBChPT results shown in the figures of section 2.4 and 2.5 are based on the assumption that the $D = 2$ fit values found in tables 2.2 and 2.4 are already reliable estimates of the true, correct values of these couplings in low energy QCD. Clearly, the shown HBChPT curves might have to be revised if future $D = 3$ analyses [DHH] lead to substantially different numerical values for these couplings. The true range of applicability of HBChPT versus covariant BChPT can only be determined once the stability of the employed couplings is guaranteed. A study of higher order effects is therefore essential also in this respect. Ideally we would like to reanalyse the results of Fit II by first fixing the couplings from a fit of the HBChPT formula of Eq.(2.27) and then study the resulting $\mathcal{O}(p^2)$ BChPT curve for this observable. However, a fit to the present set of lattice data shown in Fig.2.3 leads to a curve that is not compatible with the 2 lightest lattice points and lies significantly below the grey band shown in Fig.2.3—even down to the chiral limit. We will study this issue further in ref.[DHH].
At this point we conclude that the smooth extrapolation behaviour of the covariant $\mathcal{O}(p^2)$ BChPT expression for $\langle x \rangle_{u-d}$ of eq.(2.25) between the chiral limit and the region of present lattice QCD data is due to an *infinite tower* of $\left(\frac{m_\pi}{M_0}\right)^i$ terms. According to our analysis the chiral curvature resulting from the logarithm of eq.(2.27) governing the leading nonanalytic quark mass behaviour of this moment is *not* responsible for the rising behaviour of the chiral extrapolation function as has been hypothesized in ref.[DMN+01].
Subsequent to the publication of these results for the momentum fraction $\langle x \rangle_{u-d}$ in ref.[DGH08], the LHPC and QCDSF/UKQCD collaborations utilized the formula of eq.(2.25) to perform chiral extrapolation studies on new dynamical lattice data of $A_{2,0}^{u-d}(0)$. The corresponding extrapolated values are reported in table 2.3. Before we proceed to the discussion of these values, we have to mention an important point, that we will have to keep in mind also for future analysis of the other form factors: because of the serious differences in normalization between LHPC and QCDSF/UKQCD data, the comparison between the fits performed by the two collaborations should be done with due caution. Both collaborations make use of our formula to extrapolate down to the physical pion mass, but as clearly shown in the table the obtained values for $\langle x \rangle_{u-d}$ are not consistent with each other. In particular, the LHPC value

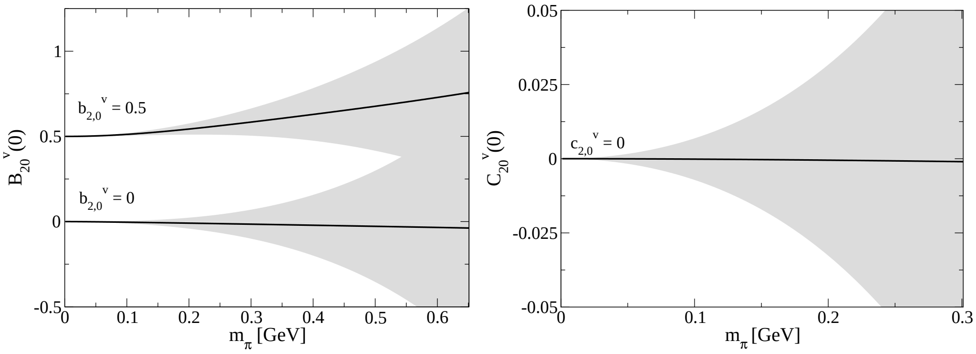

Figure 2.5: Quark mass dependence of the isovector moments $B^v_{2,0}(t=0)$ and $C^v_{2,0}(t=0)$. In $B^v_{2,0}(t=0)$ we have varied the (unknown) chiral limit value $b^v_{2,0}$ between 0 and +0.5, as lattice analyses [G+04, H+03, H+08, B+07b] suggest that this moment has a large positive value. For the chiral limit value $c^v_{2,0}$ of $C^v_{2,0}(t=0)$ we have chosen the value zero, as preliminary lattice QCD analyses suggest that this moment is consistent with zero [H+08]. The grey bands shown indicate the size of possible higher order corrections to these $\mathcal{O}(p^2)$ results.

agrees very well with both our BChPT value and phenomenology, while the QCDSF/UKQCD value lies higher than the phenomenological point. We will discuss once again the extrapolated results for this observable in section 2.6.1, where the possibility to gain new information from combined fits of different observables will be explored.

Finally we are discussing the $\mathcal{O}(p^2)$ BChPT results at $t=0$ for the remaining two generalized isovector form factors of the nucleon at twist-two level. One obtains

$$\begin{aligned} B^v_{2,0}(t=0) &= b^v_{2,0}\frac{M_N(m_\pi)}{M_0} + \frac{a^v_{2,0}\,g_A^2 m_\pi^2}{(4\pi F_\pi)^2}\left[\left(3+\log\frac{m_\pi^2}{M_0^2}\right)\right. \\ &\quad -\frac{m_\pi^2}{M_0^2}\left(2+3\log\frac{m_\pi^2}{M_0^2}\right) + \frac{m_\pi^4}{M_0^4}\log\frac{m_\pi^2}{M_0^2} \\ &\quad \left. -\frac{2m_\pi}{\sqrt{4M_0^2-m_\pi^2}}\left(5-5\frac{m_\pi^2}{M_0^2}+\frac{m_\pi^4}{M_0^4}\right)\arccos\left(\frac{m_\pi}{2M_0}\right)\right] \\ &\quad +\mathcal{O}(p^3), \end{aligned} \tag{2.28}$$

$$\begin{aligned} C^v_{2,0}(t=0) &= c^v_{2,0}\frac{M_N(m_\pi)}{M_0} + \frac{a^v_{2,0}g_A^2 m_\pi^2}{12(4\pi F_\pi)^2}\left[-1+2\frac{m_\pi^2}{M_0^2}\left(1+\log\frac{m_\pi^2}{M_0^2}\right)\right. \\ &\quad -\frac{m_\pi^4}{M_0^4}\log\frac{m_\pi^2}{M_0^2} + \frac{2m_\pi}{\sqrt{4M_0^2-m_\pi^2}}\left(2-4\frac{m_\pi^2}{M_0^2}+\frac{m_\pi^4}{M_0^4}\right) \\ &\quad \left.\times\arccos\left(\frac{m_\pi}{2M_0}\right)\right] + \mathcal{O}(p^3). \end{aligned} \tag{2.29}$$

As one can easily observe, one encounters plenty of nonanalytic terms and even chiral logarithms in these covariant $\mathcal{O}(p^2)$ BChPT results. However, we note that in the HBChPT limit to the same chiral order $\mathcal{O}(p^2)$ one would only obtain the chiral limit values $b_{2,0}^v$ and $c_{2,0}^v$, nothing else [Dor05]. E.g. the chiral logarithms calculated in ref.[BJ02] for $B_{2,0}^v(t=0)$ and $C_{2,0}^v(t=0)$ would only show up in a full $\mathcal{O}(p^3)$, respectively $\mathcal{O}(p^4)$ BChPT calculation of the generalized form factors. From the viewpoint of power-counting in BChPT they are to be considered part of higher order corrections (i.e. $D > 2$) to the full $\mathcal{O}(p^2)$ results given in eqs.(2.28,2.29).
Regrettably, at this point no information from experiments exists for these two structure quantities of the nucleon. From phenomenology one would expect that $B_{2,0}^v(t=0)$ has a "large" positive value at $m_\pi = 140$ MeV, as it corresponds to the next-higher moment of the *isovector* Pauli form factor $F_2^v(t=0) = \kappa_v = 3.71$ n.m. (Compare eq.(2.6) and eq.(2.3) at $\xi = 0$). Lattice QCD analyses seem to support this expectation [G$^+$04, H$^+$03, H$^+$08, B$^+$07b]. In contrast, the value of $C_{2,0}^v(t=0)$ cannot be estimated from information known about nucleon form factors. State-of-the-art lattice QCD analyses (e.g. see ref.[H$^+$08]) suggest that it is consistent with zero. In fig.2.5 we have indicated how the corresponding extrapolation curves based upon this information might look like. As a caveat we note that in both form factors the intrinsic quark mass dependence is small and we would see a dominant influence of the quark mass dependence stemming from the kinematical factor $M_N(m_\pi)$ in eqs.(2.28) and (2.29), if the corresponding chiral limit values $b_{2,0}^v$ and $c_{2,0}^v$ are nonzero. A further observation is that the uncertainties connected with possible higher order corrections from $\mathcal{O}(p^3)$ to $B_{2,0}^v(t=0)$ and $C_{2,0}^v(t=0)$ could already become substantial for pion masses around 300 MeV. For both quantities they can be estimated via $\mathcal{O}(p^3) \sim \delta_{B,C} \frac{m_\pi^2 M_N(m_\pi)}{(4\pi F_\pi)^2 M_0}$ where $-1 < \delta_{B,C} < 1$. In order to ultimately test the stability of the results in eqs.(2.28,2.29) it will be very useful to extend this analysis to next-to-leading one loop order. The systematic errors of our calculation of $B_{2,0}^v(t=0)$ and $C_{2,0}^v(t=0)$ already become large at the lowest pion masses at which lattice data are available: we therefore prefer to omit a study of those two form factors based on fits to lattice data. Subsequent to the publication of the results for these two form factors in [DGH08], the LHPC collaboration utilized our results to analyse the form factors $B_{2,0}^v(t)$ and $C_{2,0}^v(t)$ in ref.[H$^+$08].

2.4.2 The slopes of the generalized isovector form factors

In order to discuss the generalized isovector form factors $A_{2,0}^v(t)$, $B_{2,0}^v(t)$ and $C_{2,0}^v(t)$ at nonzero values of t, we first analyse their slopes ρ_X, defined via

$$X_{2,0}^v(t) = X_{2,0}^v(0) + \rho_X^v\, t + \mathcal{O}(t^2); \qquad X = A, B, C. \tag{2.30}$$

To $\mathcal{O}(p^2)$ in BChPT we find

$$\begin{aligned}
\rho_A^v &= \frac{c_{12}}{M_0^2} - \frac{a_{2,0}^v g_A^2}{6(4\pi F_\pi)^2}\frac{m_\pi^2}{(4M_0^2-m_\pi^2)}\Bigg\{26 + 8\log\frac{m_\pi^2}{M_0^2} - \frac{m_\pi^2}{M_0^2}\left(30 + 32\log\frac{m_\pi^2}{M_0^2}\right) \\
&\quad + \frac{m_\pi^4}{M_0^4}\left(6 + \frac{39}{2}\log\frac{m_\pi^2}{M_0^2}\right) - 3\frac{m_\pi^6}{M_0^6}\log\frac{m_\pi^2}{M_0^2} \\
&\quad - \frac{m_\pi}{\sqrt{4M_0^2-m_\pi^2}}\left(90 - 130\frac{m_\pi^2}{M_0^2} + 51\frac{m_\pi^4}{M_0^4} - 6\frac{m_\pi^6}{M_0^6}\right)\arccos\left(\frac{m_\pi}{2M_0}\right)\Bigg\} \\
&\quad + \frac{m_\pi \delta_A^t}{(4\pi F_\pi)^2 M_0},
\end{aligned} \tag{2.31}$$

$$\begin{aligned}
\rho_B^v &= \frac{a_{2,0}^v g_A^2}{18(4\pi F_\pi)^2}\frac{1}{(4M_0^2-m_\pi^2)}\Bigg\{4M_0^2 + m_\pi^2\left(83 + 24\log\frac{m_\pi^2}{M_0^2}\right) \\
&\quad - 114\frac{m_\pi^4}{M_0^2}\left(1 + \log\frac{m_\pi^2}{M_0^2}\right) + \frac{m_\pi^6}{M_0^4}\left(24 + 75\log\frac{m_\pi^2}{M_0^2}\right) - 12\frac{m_\pi^8}{M_0^6}\log\frac{m_\pi^2}{M_0^2} \\
&\quad - \frac{6m_\pi^3}{\sqrt{4M_0^2-m_\pi^2}}\left(50 - 80\frac{m_\pi^2}{M_0^2} + 33\frac{m_\pi^4}{M_0^4} - 4\frac{m_\pi^6}{M_0^6}\right)\arccos\left(\frac{m_\pi}{2M_0}\right)\Bigg\} \\
&\quad + \delta_B^t\frac{M_N(m_\pi)}{(4\pi F_\pi)^2 M_0},
\end{aligned} \tag{2.32}$$

$$\begin{aligned}
\rho_C^v &= \frac{a_{2,0}^v g_A^2}{180(4\pi F_\pi)^2}\frac{1}{(4M_0^2-m_\pi^2)}\Bigg\{4M_0^2 - 13m_\pi^2 + 12\frac{m_\pi^4}{M_0^2}\left(4 + 3\log\frac{m_\pi^2}{M_0^2}\right) \\
&\quad - 3\frac{m_\pi^6}{M_0^4}\left(4 + 11\log\frac{m_\pi^2}{M_0^2}\right) + 6\frac{m_\pi^8}{M_0^6}\log\frac{m_\pi^2}{M_0^2} \\
&\quad + \frac{6m_\pi^3}{\sqrt{4M_0^2-m_\pi^2}}\left(10 - 30\frac{m_\pi^2}{M_0^2} + 15\frac{m_\pi^4}{M_0^4} - 2\frac{m_\pi^6}{M_0^6}\right)\arccos\left(\frac{m_\pi}{2M_0}\right)\Bigg\} \\
&\quad + \delta_C^t\frac{M_N(m_\pi)}{(4\pi F_\pi)^2 M_0}.
\end{aligned} \tag{2.33}$$

The parameters δ_A^t, δ_B^t and δ_C^t are not part of the covariant $\mathcal{O}(p^2)$ result. They are only given to indicate the size of possible higher order corrections from $\mathcal{O}(p^3)$. A numerical analysis of the formulae given above suggests that the size of pion-cloud contributions to the slopes of the *generalized isovector* form factors is very small! The physics governing the size of these objects seems to be hidden in the counter-term contributions c_{12}, δ_B^t and δ_C^t which dominate numerically when assuming natural size estimates $-1 < c_{12}, \delta_B^t, \delta_C^t < +1$. We note that this situation reminds us of the *isoscalar Dirac- and Pauli* form factors of the nucleon $F_1^s(t)$ and $F_2^s(t)$, where the t dependence in SU(2) ChEFT calculations is also dominated by counter-terms (e.g. see the discussion in reference [BFHM98]).
Finally truncating the covariant results of eqs.(2.31-2.33) at leading order in

$1/M_0$ we obtain the (trivial) HBChPT expressions to $\mathcal{O}(p^2)$:

$$\rho_A^v = \frac{c_{12}}{M_0^2} + \mathcal{O}(p^3), \quad (2.34)$$

$$\rho_B^v = 0 + \mathcal{O}(p^3), \quad (2.35)$$

$$\rho_C^v = 0 + \mathcal{O}(p^3). \quad (2.36)$$

As already mentioned, the nonzero slope results found in the HBChPT calculations of refs.[ACK06, DMS06, DMS07] are of higher order from the point of view of our power-counting. Most of them can already be added systematically to our covariant $\mathcal{O}(p^2)$ results of eqs.(2.31-2.33) at $\mathcal{O}(p^3)$.

2.4.3 The generalized isovector form factors of the nucleon

In this subsection we present the full t dependence of the generalized isovector form factors of a nucleon to $\mathcal{O}(p^2)$ in BChPT. We note that for all three generalized form factors at this order only the amplitude of diagram c) of fig.2.1 depends on t. The resulting expressions at this order are therefore quite simple:

$$A_{2,0}^v(t) = A_{2,0}^v(0) + \frac{a_{2,0}^v g_A^2}{192\pi^2 F_\pi^2} F_{2,0}^v(t) + \frac{c_{12}}{M_0^2}\, t + \mathcal{O}(p^3), \quad (2.37)$$

with $A_{2,0}^v(0)$ given above and

$$\begin{aligned} F_{2,0}^v(t) = {} & t - \frac{2m_\pi^3\left(50M_0^4 - 43m_\pi^2 M_0^2 + 8m_\pi^4\right)}{M_0^4\sqrt{4M_0^2 - m_\pi^2}} \arccos\left(\frac{m_\pi}{2M_0}\right) \\ & + \int_{-\frac{1}{2}}^{\frac{1}{2}} du \Bigg\{ \frac{2m_\pi^3 M_0^2}{\tilde{M}^8\sqrt{4\tilde{M}^2 - m_\pi^2}} \Big[-10\tilde{M}^6 + \left(17m_\pi^2 + 60M_0^2\right)\tilde{M} \\ & -4m_\pi^2\left(m_\pi^2 + 15M_0^2\right)\tilde{M}^2 + 12m_\pi^4 M_0^2\Big] \arccos\left(\frac{m_\pi}{2\tilde{M}}\right) \\ & + \frac{2\left(M_0^2 - \tilde{M}^2\right)}{M_0^2\tilde{M}^6}\Big[12m_\pi^4 M_0^4 + 2m_\pi^2\left(4m_\pi^2 - 9M_0^2\right)M_0^2\tilde{M}^2 \\ & + \left(m_\pi^2 - 2M_0^2\right)\left(8m_\pi^2 + M_0^2\right)\tilde{M}^4\Big] \\ & + \frac{2M_0^2}{\tilde{M}^8}\Big[2\tilde{M}^8 + 3m_\pi^2\left(3m_\pi^2 + 4M_0^2\right)\tilde{M}^4 - 4m_\pi^4\left(m_\pi^2 + 9M_0^2\right)\tilde{M}^2 \\ & + 12m_\pi^6 M_0^2\Big] \log\left(\frac{\tilde{M}}{M_0}\right) + \frac{2m_\pi^2}{M_0^4\tilde{M}^8}\Big[m_\pi^4\left(-12M_0^8 + 4\tilde{M}^2 M_0^6 + 8\tilde{M}^8\right) \\ & + 9m_\pi^2\tilde{M}^2\left(4M_0^8. - \tilde{M}^2 M_0^6 - 3\tilde{M}^6 M_0^2\right) + 12M_0^4\tilde{M}^4\left(\tilde{M}^4 - M_0^4\right)\Big] \\ & \times \log\left(\frac{m_\pi}{M_0}\right)\Bigg\}, \end{aligned} \quad (2.38)$$

where we have introduced $\tilde{M}^2 = M_0^2 + \left(u^2 - \frac{1}{4}\right) t$. Note that $F_{2,0}^v(t=0) = 0$ by construction. A conservative estimate for the size of possible higher order corrections indicated in eq.(2.37) can be obtained via $\mathcal{O}(p^3) \sim \delta_A \frac{m_\pi^3}{(4\pi F_\pi)^2 M_0} + \delta_A^t \frac{m_\pi}{(4\pi F_\pi)^2 M_0} t$, in complete analogy to the discussion in the two previous subsections. For the remaining two generalized isovector form factors we obtain

$$\begin{aligned}
B_{2,0}^v(t) &= b_{2,0}^v \frac{M_N(m_\pi)}{M_0} + \frac{a_{2,0}^v g_A^2 M_0^2}{48\pi^2 F_\pi^2} \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{du}{\tilde{M}^8} \Bigg\{ \left(M_0^2 - \tilde{M}^2\right) \tilde{M}^6 + 9 m_\pi^2 M_0^2 \tilde{M}^4 \\
&\quad - 6 m_\pi^4 M_0^2 \tilde{M}^2 + 6 m_\pi^2 M_0^2 \left(m_\pi^4 - 3 m_\pi^2 \tilde{M}^2 + \tilde{M}^4\right) \log \frac{m_\pi}{\tilde{M}} \\
&\quad - \frac{6 m_\pi^3 M_0^2}{\sqrt{4\tilde{M}^2 - m_\pi^2}} \left[m_\pi^4 - 5 m_\pi^2 \tilde{M}^2 + 5\tilde{M}^4\right] \arccos\left(\frac{m_\pi}{2\tilde{M}}\right) \Bigg\} \\
&\quad + \delta_B \frac{m_\pi^2 M_N(m_\pi)}{(4\pi F_\pi)^2 M_0} + \delta_B^t \frac{M_N(m_\pi)}{(4\pi F_\pi)^2 M_0} t, \qquad (2.39)
\end{aligned}$$

$$\begin{aligned}
C_{2,0}^v(t) &= c_{2,0}^v \frac{M_N(m_\pi)}{M_0} + \frac{a_{2,0}^v g_A^2 M_0^2}{48\pi^2 F_\pi^2} \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{du\, u^2}{\tilde{M}^8} \Bigg\{ 2\left(M_0^2 - \tilde{M}^2\right) \tilde{M}^6 \\
&\quad - 3 m_\pi^2 M_0^2 \tilde{M}^4 + 6 m_\pi^4 M_0^2 \tilde{M}^2 - 6 m_\pi^4 M_0^2 \left(m_\pi^2 - 2\tilde{M}^2\right) \log \frac{m_\pi}{\tilde{M}} \\
&\quad + \frac{6 m_\pi^3 M_0^2}{\sqrt{4\tilde{M}^2 - m_\pi^2}} \left[m_\pi^4 - 4 m_\pi^2 \tilde{M}^2 + 2\tilde{M}^4\right] \arccos\left(\frac{m_\pi}{2\tilde{M}}\right) \Bigg\} \\
&\quad + \delta_C \frac{m_\pi^2 M_N(m_\pi)}{(4\pi F_\pi)^2 M_0} + \delta_C^t \frac{M_N(m_\pi)}{(4\pi F_\pi)^2 M_0} t. \qquad (2.40)
\end{aligned}$$

The parameters δ_B, δ_C, δ_B^t and δ_C^t have again been inserted to study the possible corrections of higher orders to our $\mathcal{O}(p^2)$ results. Varying these parameters between -1 and +1, we conclude that a full $\mathcal{O}(p^3)$ calculation is required before one wants to make any strong claims regarding the t dependence of $B_{2,0}^v(t)$ and $C_{2,0}^v(t)$ *beyond* the linear t dependence discussed in the previous subsection. On the other hand, the predicted t dependence of the form factor $A_{2,0}^v(t)$ of eq.(2.37) appears to be more reliable at this order, as possible higher order contributions only affect terms beyond the leading linear dependence on t. However, due to the unsettled situation in the t dependence of those form factors at leading one loop level, we only present fits to dipole-Q^2-extrapolated lattice data throughout this chapter and do not dare to fit to lattice data directly before the next-to-leading one loop order effects are calculated. To get a rough idea about such direct fits in the sector of generalized nucleon form factors the reader is again pointed to reference [H$^+$08].

2.5 The Generalized Isoscalar Form Factors in $\mathcal{O}(p^2)$ BChPT

At the time of the publication of these results in reference [DGH08], only quenched lattice QCD data were available for the moments of the isoscalar GPDs [G+04]. Subsequent to the publication of our results both LHPC [H+08] and QCDSF [B+07b] collaborations utilized our chiral extrapolation formulae for the analysis of their new lattice data obtained with dynamical fermion simulations.

2.5.1 Moments of the isoscalar GPDs at $t = 0$

To $\mathcal{O}(p^2)$ in two flavor covariant BChPT the only nonzero loop contributions to the isoscalar moment $A_{2,0}^s(t=0)$ (see eq.(2.11)) arise from diagrams c) and e) in fig.2.1. One obtains [DGH08]

$$\begin{aligned}
A_{2,0}^s(0) &= \langle x\rangle_{u+d} \\
&= a_{2,0}^s + 4m_\pi^2\frac{c_9}{M_0^2} - \frac{3a_{2,0}^s g_A^2 m_\pi^2}{16\pi^2F_\pi^2}\left[\frac{m_\pi^2}{M_0^2} + \frac{m_\pi^2}{M_0^2}\left(2-\frac{m_\pi^2}{M_0^2}\right)\log\left(\frac{m_\pi}{M_0}\right)\right. \\
&\quad \left. + \frac{m_\pi}{\sqrt{4M_0^2-m_\pi^2}}\left(2-4\frac{m_\pi^2}{M_0^2}+\frac{m_\pi^4}{M_0^4}\right)\arccos\left(\frac{m_\pi}{2M_0}\right)\right] \\
&\quad + \mathcal{O}(p^3). \qquad (2.41)
\end{aligned}$$

Eq.(2.41) should provide a similarly successful chiral extrapolation function for $\langle x\rangle_{u+d}$ as the covariant $\mathcal{O}(p^2)$ BChPT result of eq.(2.25) did for the LHPC lattice data for $\langle x\rangle_{u-d}$ in section 2.4.1. The uncertainty arising from higher orders can be estimated to scale as $\mathcal{O}(p^3) \sim \delta_A^0\frac{m_\pi^3}{(4\pi F_\pi)^2 M_0}$ where δ_A^0 should again be a number between $-1,\ldots,+1$, according to natural size estimates. We note that the coupling $\Delta a_{2,0}^v$ which played an essential role in the chiral extrapolation function of $\langle x\rangle_{u-d}$ is *not* present in the quark mass dependence of the isoscalar moment $\langle x\rangle_{u+d}$. The resulting chiral extrapolation function is therefore presumably quite different from the one in the isovector channel. The absence of a chiral logarithm $\sim m_\pi^2\log m_\pi$ in $\langle x\rangle_{u+d}$ (compare eq.(2.27) and eq.(2.42)) presumably only leads to a difference in the chiral extrapolation functions between the isovector- and the isoscalar moment for $m_\pi < 140$ MeV. Note that from eq.(2.41) in the limit $1/M_0 \to 0$ we reproduce the leading HBChPT result for $\langle x\rangle_{u+d}$ of ref.[AS02] which found a complete cancelation of the nonanalytic quark mass dependent terms in this channel:

$$A_{2,0}^s(0)|_{HBChPT}^{p^2} = a_{2,0}^s + 4m_\pi^2\frac{c_9}{M_0^2} + \mathcal{O}\left(\frac{m_\pi^3}{16\pi^2F_\pi^2M_0}\right). \qquad (2.42)$$

The coupling c_9 is therefore scale independent (in dimensional regularization) and constitutes the leading correction to the chiral limit value $a_{2,0}^s$ of $\langle x\rangle_{u+d}$. At $t=0$ we also find nontrivial results for the two other generalized isoscalar form factors of the nucleon. To order p^2 in the covariant calculation they read [DGH08]

$$\begin{aligned} B^s_{2,0}(0) &= b^s_{2,0}\frac{M_N(m_\pi)}{M_0} - \frac{3\,a^s_{2,0}\,g_A^2 m_\pi^2}{(4\pi F_\pi)^2}\left[\left(3+\log\frac{m_\pi^2}{M_0^2}\right) - \frac{m_\pi^2}{M_0^2}\left(2+3\log\frac{m_\pi^2}{M_0^2}\right)\right. \\ &\quad \left. +\frac{m_\pi^4}{M_0^4}\log\frac{m_\pi^2}{M_0^2} - \frac{2m_\pi}{\sqrt{4M_0^2-m_\pi^2}}\left(5-5\frac{m_\pi^2}{M_0^2}+\frac{m_\pi^4}{M_0^4}\right)\arccos\left(\frac{m_\pi}{2M_0}\right)\right] \\ &\quad +\delta_B^0\frac{m_\pi^2 M_N(m_\pi)}{(4\pi F_\pi)^2 M_0}, \qquad (2.43) \\ C^s_{2,0}(0) &= c^s_{2,0}\frac{M_N(m_\pi)}{M_0} - \frac{a^s_{2,0} g_A^2 m_\pi^2}{4(4\pi F_\pi)^2}\left[-1+2\frac{m_\pi^2}{M_0^2}\left(1+\log\frac{m_\pi^2}{M_0^2}\right)\right. \\ &\quad \left. -\frac{m_\pi^4}{M_0^4}\log\frac{m_\pi^2}{M_0^2} + \frac{2m_\pi}{\sqrt{4M_0^2-m_\pi^2}}\left(2-4\frac{m_\pi^2}{M_0^2}+\frac{m_\pi^4}{M_0^4}\right)\arccos\left(\frac{m_\pi}{2M_0}\right)\right] \\ &\quad +\delta_C^0\frac{m_\pi M_N(m_\pi)}{(4\pi F_\pi)^2}. \qquad (2.44) \end{aligned}$$

The parameters δ_B^0 and δ_C^0 have been added "by hand" to these results in order to indicate possible effects of higher order (i.e. $\mathcal{O}(p^3)$) corrections. Presumably they take on values $-1,\ldots,+1$. Note that our $\mathcal{O}(p^2)$ BChPT prediction for $C^s_{2,0}(0)$ is strongly affected by possible corrections from higher orders. This is due to the fact that one only receives a nonzero result for this form factor starting at $\mathcal{O}(p^2)$ – the order we are working at. Eq.(2.44) should therefore only be considered to provide a rough estimate for the quark mass dependence of this form factor at $t=0$. For a true, quantitative analysis of its chiral extrapolation behaviour the complete[6] $\mathcal{O}(p^3)$ corrections should first be added. In order to obtain a *rough estimate* of the chiral extrapolation functions resulting from eqs.(2.41,2.43) we utilize the quenched[7] data of the QCDSF collaboration [G$^+$04] as input.

However, lattice simulations (especially in the isoscalar channel) are characterized by several sources of systematic errors. Values for the generalized form factors of the nucleon at different lattice spacings and volumes at small pion masses are not available yet. Further simulations are needed in order to control possible finite size and discretization effects. Moreover, because of large calculational expenses, lattice simulations in the isoscalar sector often neglect disconnected diagrams, that is processes where the incoming external field is not directly coupled to a valence-quark. Consequently, present lattice results for the isoscalar form factors contain an unknown systematic error.

[6] The most prominent correction from $\mathcal{O}(p^3)$ arises from the triangle diagram of fig.2.2 and is given in appendix A.5.2 via $\Delta C^s_{h.o.}(t=0,m_\pi)$. We note, however, that this is not the only next-order correction.

[7] The quark masses employed in ref.[G$^+$04] are so large that one does not expect to find differences between quenched and dynamical simulations, see e.g. the discussion in reference [HW02]. Note that we are utilizing the lattice data of reference [G$^+$04] with the scale set by r_0, as we consider the alternative way of scale-setting (via a *linear* extrapolation to the physical mass of the nucleon) also discussed in ref.[G$^+$04] to be obsolete in the light of the detailed chiral extrapolation studies of refs.[PHW04, AK$^+$04, BHM05, PMW$^+$06].

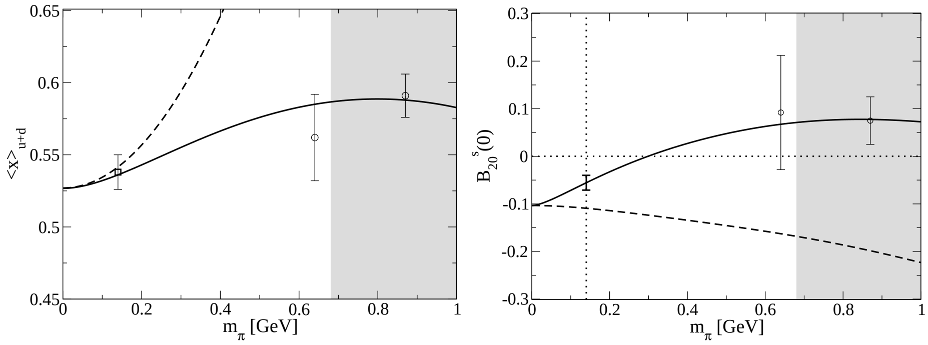

Figure 2.6: Solid lines: The quark mass dependence of the $\mathcal{O}(p^2)$ BChPT result for $\langle x\rangle_{u+d}$ of eq.(2.41) and $B^s_{2,0}(0)$ of eq.(2.43). The values for the three previously unknown parameters resulting from a *combined* fit to the shown QCDSF data of ref.[G+04] and to the phenomenological value for $\langle x\rangle_{u+d}$ [xpr] are given in table 2.4. At the physical point we obtain $B^s_{2,0} = -0.056 \pm 0.016$, with the shown error only being statistical. Due to the poor data situation with its unknown systematic errors, however, these results should only be considered as a rough estimate of the true quark mass dependence. The dashed lines correspond to the respective HBChPT results at this order which are again found to only be applicable for very low pion masses. We stress once again that the analysis of higher order terms is demanded in order to draw conclusions about the radius of convergence of HBChPT.

Performing a combined fit of eqs.(2.41,2.43) to the lattice data shown in fig.2.6 and including the phenomenological value of $\langle x\rangle_{u+d} \sim 0.54$ [xpr], we obtain the two solid curves shown in fig.2.6. The resulting parameters of the fit are given in table 2.4. Interestingly, despite the large quark masses and the huge error bars in the data of ref.[G+04] we obtain reasonable chiral extrapolation curves with natural size coupling constants. The analysis of the QCDSF data in combination with the physical value for $\langle x\rangle_{u+d}$ suggests that the chiral limit value of this isoscalar PDF-moment is smaller than the value at the physical point, leading to a monotonically rising chiral extrapolation function as shown in the left panel of fig.2.6. As a second observation we note that the value for the generalized form factor $B^s_{2,0}(t = 0)$ could take on a *small negative* value at the physical point according to the right panel of fig.2.6, albeit with a large uncertainty due to the poor data situation and large systematic errors of the BChPT calculation at this order. Because of the small (negative!) value of the isoscalar Pauli form factor $F^s_2(t = 0) = \kappa_s = -0.12$ n.m., it is somewhat expected that the next higher moment yields a value close to zero. However, fig.2.6 now opens the possibility that $B^s_{2,0}(t = 0) \approx -0.06$ might be as large as 50% of its $F^s_2(t = 0)$ analogue. It will be very interesting to observe whether this feature can be reproduced when the new data of QCDSF [LQ] at small pion masses and a next-to-leading one loop order BChPT calculation of the generalized form factors become available. Fig.2.6 also demonstrates that the corresponding $\mathcal{O}(p^2)$ HBChPT results are again not sufficient for a chiral extrapolation at this order. However, we note again that the true range of ap-

$a^s_{2,0}$	$b^s_{2,0}$	c_9	$\langle x\rangle^{phen.}_{u+d}(\mu = 2\text{GeV})$
0.527 ± 0.007	-0.103 ± 0.016	0.147 ± 0.002	0.538 ± 0.012 (fixed)

Table 2.4: The values for the three tensor coupling constants entering $A^s_{2,0}(t)$ and $B^s_{2,0}(t)$ at order p^2 as extracted from a combined fit to the lattice results for $A^s_{2,0}(0)$ and $B^s_{2,0}(0)$ [G+04] shown in fig.2.6 and to the physical point of $A^s_{2,0}(t = 0, m_\pi = 0.14\text{ GeV}) = \langle x\rangle^{phen.}_{u+d}$. Note that we have obtained a small negative value for $b^s_{2,0}$. The indicated uncertainties are the statistical errors arising in the fit of eqs.(2.41) and (2.43) to the data of references [G+04, xpr] and do not reflect the much larger systematic uncertainties from both the lattice simulations (which e.g. neglect all contributions from disconnected diagrams) and possible higher order contributions to our chiral analysis.

		$B^s_{2,0}(t = 0, m_\pi^{\text{phys}})$
Extrapolated	BChPT	-0.056 ± 0.016[DGH08]
values	LHPC	-0.094 ± 0.050 [H+08]
at m_π^{phys}	QCDSF/UKQCD	-0.120 ± 0.023 [B+07b]

Table 2.5: Extrapolated values for the isoscalar generalized form factor $B^s_{2,0}(t = 0)$ at the physical pion mass, as obtained from the combined fit showed in figure 2.6 and from chiral extrapolations to the lattice data of refs.[H+08] and [B+07b].

plicability of HBChPT versus covariant BChPT can only be determined once the stability of the employed couplings is guaranteed, see the similar discussion for $\langle x\rangle_{u-d}$ in section 2.4.1.
The ChPT results for the form factors provided in this section have been subsequently used by LHPC [H+08] and QCDSF/UKQCD [B+07b] collaborations to analyse new dynamical lattice QCD data. The combination of lattice calculations in the chiral regime (with pion mass values as low as 350 MeV) and our formulae obtained in chiral perturbation theory had allowed significant progress in the chiral extrapolation of the generalized form factors of the nucleon. In table 2.5.1 we show the values obtained by the two collaborations for the forward limit of the form factor $B^s_{2,0}$ at the physical pion mass. Although these results have to be taken with due caution because of the reasons discussed above, it seems that the results obtained from chiral extrapolations studies on more recent dynamical lattice data confirm that the form factor $B^s_{2,0}(t = 0)$ takes on a *small negative* value at the physical pion mass.

2.5.2 The contribution of u and d quarks to the spin of the nucleon

In the past few years a lot of interest in generalized isoscalar form factors of the nucleon has focused on the values of $A^s_{2,0}$ and $B^s_{2,0}$ at the point $t = 0$ since one can determine the contribution of quarks to the total spin of the nucleon

		$J_{u+d}(t=0, m_\pi = 0.14\text{GeV})$
EXTRAPOLATED	BChPT	0.24 ± 0.05[DGH08]
VALUES	LHPC	0.213 ± 0.026[H$^+$08]
AT m_π^{phys}	QCDSF/UKQCD	0.226 ± 0.013[B$^+$07b]

Table 2.6: Values for the contribution of u and d quarks to the spin of a nucleon, as obtained from chiral extrapolations in references [DGH08], [H$^+$08] and [B$^+$07b]. All three analysis are based on the same formulae given in ref.[DGH08].

via these two structures [Ji97]:

$$J_{u+d} = \frac{1}{2}\left[A^s_{2,0}(t=0) + B^s_{2,0}(t=0)\right]. \tag{2.45}$$

To $\mathcal{O}(p^2)$ in BChPT we find [DGH08]

$$\begin{aligned} J_{u+d} = \frac{1}{2}\Bigg\{ & a^s_{2,0} + b^s_{2,0}\frac{M_N(m_\pi)}{M_0} + \frac{a^s_{2,0} m_\pi^2}{(4\pi F_\pi)^2}\Bigg[\frac{3g_A^2 m_\pi}{\sqrt{4M_0^2 - m_\pi^2}}\left(8 - 6\frac{m_\pi^2}{M_0^2} + \frac{m_\pi^4}{M_0^4}\right) \\ & \times \arccos\frac{m_\pi}{2M_0} \\ & -3g_A^2\left(3 - \frac{m_\pi^2}{M_0^2} + \left(2 - 4\frac{m_\pi^2}{M_0^2} + \frac{m_\pi^4}{M_0^4}\right)\log\frac{m_\pi}{M_0}\right)\Bigg] \\ & +4m_\pi^2\frac{c_9}{M_0^2}\Bigg\} + \mathcal{O}(p^3). \end{aligned} \tag{2.46}$$

Note that despite the plethora of nonanalytic quark mass dependent terms contained in the $\mathcal{O}(p^2)$ BChPT result of eq.(2.46), the two chiral logarithms calculated in ref.[CJ02] within the HBChPT framework are *not yet* contained in our result. Both terms ($\sim a_{(q)\pi}$, $b_{(q)N}$ in the notation of ref.[CJ02]) are part of the complete $\mathcal{O}(p^3)$ result according to our power-counting and will appear in the calculation of the next order[8]. We further note that the two logarithms of ref.[CJ02] are UV-divergent and are accompanied by a counter-term, whereas the $\mathcal{O}(p^2)$ BChPT result of eq.(2.46) happens to be UV-finite to the order we are working. In ref.[CJ02] the authors also reported that the two chiral logarithms (of $\mathcal{O}(p^3)$) which they describe presumably are canceled *numerically* by pion-cloud contributions around an intermediate $\Delta(1232)$ state. We can confirm that this possibility exists, as the described Δ contributions also start at $\mathcal{O}(p^3)$, assuming a power-counting where the nucleon-Δ mass difference in the chiral limit is counted as a small parameter of chiral dimension one $\sim p^1$.

Utilizing the $\mathcal{O}(p^2)$ BChPT result of eq.(2.46) and the fit parameters of table 2.4, we obtain a first estimate for the contribution of u and d quarks to the spin of a nucleon [DGH08]:

$$J_{u+d}(t=0,\, m_\pi = 0.14\,\text{GeV}) \approx 0.24 \pm 0.05, \tag{2.47}$$

[8] The contribution $\sim a_{(q)\pi}$ is already contained in the function $\Delta B_{h.o.}(t=0, m_\pi)$ discussed in subsection 2.5.3.

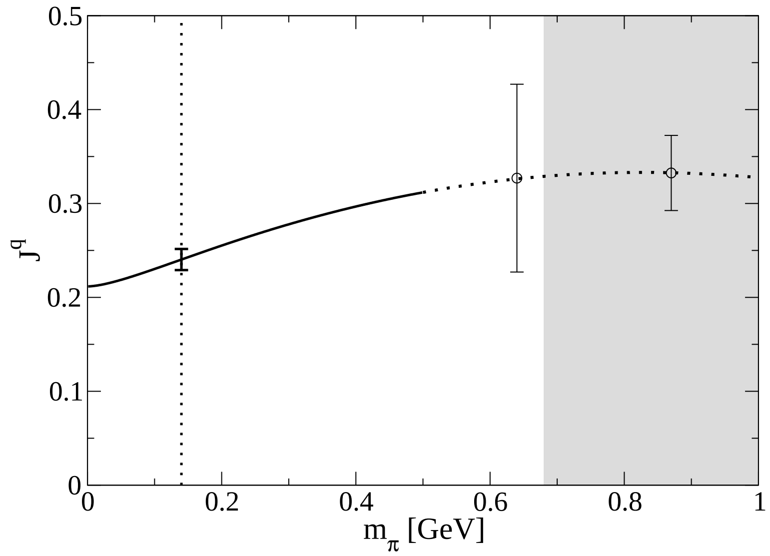

Figure 2.7: The contribution of $u+d$ quarks to the spin of a nucleon as function of the effective pion mass. The $\mathcal{O}(p^2)$ BChPT result shown as the solid line is a prediction of eq.(2.46) which utilizes the fit-parameters of table 2.4. For comparison we have also plotted simulation data from QCDSF [G$^+$04] in the figure. At the physical point one can read off $J_{u+d} \approx 0.24$. (The error bar shown at the physical point is only statistical, i.e. arises due to the errors attached to the parameters in table 2.4, and does not reflect any systematic uncertainties.)

which is only about half of the total spin of the nucleon! We emphasize that this number is just a rough estimate, as we are assuming that the true error is dominated by *systematic* errors from both the lattice input to our analysis and the possible higher order corrections to our chiral calculation[9]. Anyway, the authors of references [H$^+$08]-LHPC collaboration- and [B$^+$07b]-QCDSF/UKQCD collaboration- have showed that the same chiral analysis performed on a different set of lattice data leads to a result which is consistent with the value given here, even within statistical errors only. The values for this quantity are given in table 2.6; one can observe that the results obtained from the three chiral extrapolations are consistent with each other.
Based on the same input, we can also predict the quark mass dependence of J_{u+d}. The result is displayed as the solid line in fig.2.7.
Note that in contrast to the analysis given in ref.[G$^+$04], we do not obtain a flat chiral extrapolation function between the lattice data and the physical point. The $\mathcal{O}(p^2)$ BChPT analysis suggests that the value at the physical point lies *lower* than the values obtained in the QCDSF simulation at large quark

[9]The size of the statistical error read off from the fit of table 2.4 is ± 0.01 and therefore negligible. The small value of this statistical error is of course heavily influenced by the error assigned to the phenomenological value of $\langle x \rangle_{u+d}$ given in table 2.4. Note, however, that the size of statistical errors will increase once we extend our analysis to next-to-leading one loop order due to the presence of several so far unknown counter-terms at this order.

masses. Following the above discussion, this curve obviously can only be a first estimate of the true result.

2.5.3 A first glance at the generalized isoscalar form factors of the nucleon

In this section we present the results for the momentum- and quark mass dependence of the generalized form factors of the nucleon for the isoscalar flavor combination $u+d$ at $\mathcal{O}(p^2)$ in BChPT. We note that at this order the only nonzero loop contributions to the three isoscalar form factors arise from diagrams c) and e) of figure 2.1, as the coupling of the isoscalar tensor field to the nucleon is not affected by chiral rotations. One obtains

$$A^s_{2,0}(t) \;=\; A^s_{2,0}(0) - \frac{a^s_{2,0} g_A^2}{64\pi^2 F_\pi^2} F^s_{2,0}(t) + \frac{c_{13}}{M_0^2}\, t + \mathcal{O}(p^3)\,, \qquad (2.48)$$

with $A^s_{2,0}(0)$ given in eq.(2.41) and $F^s_{2,0}(t=0)=0$ by construction. Interestingly, to the order we are working here, the t dependence of this isoscalar form factor $A^s_{2,0}(t)$ is given by the same function

$$F^s_{2,0}(t) \;=\; F^v_{2,0}(t) + \mathcal{O}(p^3), \qquad (2.49)$$

that controls its isovector analogue eq.(2.38) albeit with *larger* numerical prefactors (compare eq.(2.48) to eq.(2.37)). We note that $F^s_{2,0}(t)$ does not depend on the scale λ of dimensional regularization for the loop diagrams. The chiral coupling c_{13} is therefore also scale-independent[10], parametrizing the quark mass independent short-distance contributions to the slope of $A^s_{2,0}(t)$. The unknown contributions from higher orders in the chiral expansion can be estimated from a calculation of the triangle diagram displayed in figure 2.2. Due to the coupling of the tensor field to the (long-range) pion-cloud of the nucleon, among all the contributions at the next chiral order this diagram should give the most important t dependent correction to the covariant $\mathcal{O}(p^2)$ result of eq.(2.48), resulting in the estimate $\mathcal{O}(p^3) \sim \Delta A^s_{h.o.}(t, m_\pi)$. The explicit expression for the function $\Delta A^s_{h.o.}(t, m_\pi)$ can be found in appendix A.5.2. For completeness, we also note that in the limit $1/M_0 \to 0$ we obtain the corresponding $\mathcal{O}(p^2)$ HBChPT result

$$A^s_{2,0}(t)|^{p^2}_{HBChPT} \;=\; a^s_{2,0} + 4m_\pi^2 \frac{c_9}{M_0^2} + \frac{c_{13}}{M_0^2}\, t + \mathcal{O}(1/(16\pi^2 F_\pi^2 M_0))\,, \quad (2.50)$$

[10] We note again that this scale-independence refers to the UV-scales of the ChEFT calculation, not to be confused with the scale- and scheme dependence of the quark-operators on the left hand side of eq.(2.7) which is completely outside the framework of ChEFT.

which is just a string of tree level couplings.
To order $\mathcal{O}(p^2)$ in covariant BChPT the two other isoscalar form factors read

$$\begin{aligned}B_{2,0}^s(t) &= b_{2,0}^s\frac{M_N(m_\pi)}{M_0} - \frac{a_{2,0}^s g_A^2 M_0^2}{16\pi^2 F_\pi^2}\int_{-\frac{1}{2}}^{\frac{1}{2}}\frac{du}{\tilde{M}^8}\Bigg\{\left(M_0^2-\tilde{M}^2\right)\tilde{M}^6 + 9m_\pi^2 M_0^2\tilde{M}^4 \\ &\quad -6m_\pi^4 M_0^2\tilde{M}^2 + 6m_\pi^2 M_0^2\left(m_\pi^4 - 3m_\pi^2\tilde{M}^2+\tilde{M}^4\right)\log\frac{m_\pi}{\tilde{M}} \\ &\quad -\frac{6m_\pi^3 M_0^2}{\sqrt{4\tilde{M}^2-m_\pi^2}}\left[m_\pi^4 - 5m_\pi^2\tilde{M}^2+5\tilde{M}^4\right]\arccos\left(\frac{m_\pi}{2\tilde{M}}\right)\Bigg\} \\ &\quad +\Delta B_{h.o.}^s(t,m_\pi), \end{aligned} \tag{2.51}$$

$$\begin{aligned}C_{2,0}^s(t) &= c_{2,0}^s\frac{M_N(m_\pi)}{M_0} - \frac{a_{2,0}^s g_A^2 M_0^2}{16\pi^2 F_\pi^2}\int_{-\frac{1}{2}}^{\frac{1}{2}}\frac{du\,u^2}{\tilde{M}^8}\Bigg\{2\left(M_0^2-\tilde{M}^2\right)\tilde{M}^6 - 3m_\pi^2 M_0^2\tilde{M}^4 \\ &\quad +6m_\pi^4 M_0^2\tilde{M}^2 - 6m_\pi^4 M_0^2\left(m_\pi^2 - 2\tilde{M}^2\right)\log\frac{m_\pi}{\tilde{M}} \\ &\quad +\frac{6m_\pi^3 M_0^2}{\sqrt{4\tilde{M}^2-m_\pi^2}}\left[m_\pi^4 - 4m_\pi^2\tilde{M}^2+2\tilde{M}^4\right]\arccos\left(\frac{m_\pi}{2\tilde{M}}\right)\Bigg\} \\ &\quad +\Delta C_{h.o.}^s(t,m_\pi), \end{aligned} \tag{2.52}$$

with $\tilde{M}$ defined in appendix A.4 (c.f. eq.(A.37)). $M_N(m_\pi)$ again denotes the (quark mass dependent) mass function of the nucleon eq.(2.10), introduced via eq.(2.8). In the limit $1/M_0 \to 0$ we obtain the corresponding $\mathcal{O}(p^2)$ HBChPT results for $B_{2,0}^s(t)$ and $C_{2,0}^s(t)$ which at this order only consist of the tree level coupling constants $b_{2,0}^s$ and $c_{2,0}^s$. As in the case of $A_{2,0}^s(t)$ we have estimated the contributions from higher orders via $\mathcal{O}(p^3) \sim \Delta B_{h.o.}^s(t,m_\pi)$, $\Delta C_{h.o.}^s(t,m_\pi)$, assuming that the dominant t dependent higher order corrections to our covariant $\mathcal{O}(p^2)$ BChPT results of eqs.(2.51,2.52) originate from the $\mathcal{O}(p^3)$ triangle diagram displayed in fig.2.2. Explicit expressions are given in appendix A.5.2. We note that the nonanalytic quark mass dependent terms in $A_{2,0}^s(t)$, $B_{2,0}^s(t)$ and $C_{2,0}^s(t)$ calculated in reference [BJ02] with the help of the HBChPT formalism[11] correspond to the leading terms in a $1/M_0$ expansion of the $\mathcal{O}(p^3)$ BChPT corrections $\Delta X_{h.o.}(t,m_\pi)$ where $X = A, B, C$ of appendix A.5.2.
At this point we refrain from a detailed numerical analysis of the t dependence of the generalized isoscalar form factors $A_{2,0}^s(t)$ and $B_{2,0}^s(t)$. On the one hand very few lattice data for this flavor combination have been published so far for pion masses below 600 MeV. Moreover, available lattice data neglect contributions from disconnected diagrams and are therefore accompanied by

[11] In refs.[ACK06, DMS06, DMS07] additional terms have been calculated within the HBChPT approach. While some terms correspond to $\mathcal{O}(p^3)$ and $\mathcal{O}(p^4)$ contributions according to our power-counting, expanding the covariant $\mathcal{O}(p^2)$ result of eq.(2.51) to the order $\frac{1}{(4\pi F_\pi)^2 M_0^1}$ one can e.g. also recognize a term $\sim a_{2,0}^s\, m_\pi^2$ present in ref.[ACK06]. However, as far as we can see, neither ref.[ACK06] nor ref.[DMS06, DMS07] presents a *complete* $\mathcal{O}(p^3)$ HBChPT calculation of the *matrix element* eq.(2.8).

an unknown systematic uncertainty which is very hard to estimate. On the other hand, in the t dependence both of $A^s_{2,0}(t)$ and of $B^s_{2,0}(t)$ we encounter chiral couplings (c_{13} at $\mathcal{O}(p^2)$ in eq.(2.48) and B_{34} at $\mathcal{O}(p^3)$ in $\Delta B^s_{h.o.}(t, m_\pi)$ of eq.(A.45)) connected with presently unknown short-distance physics and systematic uncertainties can therefore not be guaranteed to be small at the order we are working here. We are therefore postponing this discussion until further information is available, in particular from a calculation of the effects of next-to-leading one loop order. Despite of these considerations, the authors of reference [H$^+$08] have afterwards attempted a study of the t dependence of the $\mathcal{O}(p^2)$ BChPT results for the isoscalar generalized form factors and have reported quite satisfactory findings. In the meantime we are preparing a full (next-to-leading one loop) $\mathcal{O}(p^3)$ BChPT analysis of the isoscalar moments of the GPDs which (in addition to several other diagrams!) also contains the contributions from the triangle diagram shown in fig.2.2, already presented in appendix A.5.2.

Before finally proceeding to the summary of this chapter, we take a look at the third generalized isoscalar form factor $C^s_{2,0}(t)$ of eq.(2.52). According to our power-counting, short distance contributions to the radius of this form factor are suppressed and only start to enter at $\mathcal{O}(p^4)$, both in HBChPT and in BChPT. After adding the $\mathcal{O}(p^3)$ *estimate* $\Delta C^s_{h.o.}(t, m_\pi)$ to the $\mathcal{O}(p^2)$ BChPT result of eq.(2.52), we can hope to catch a first glance of the t dependence of this elusive nucleon structure. Utilizing $a^s_{2,0}$ of table 2.4 and assuming $x^0_\pi \approx \langle x\rangle^s_\pi \approx 0.5$ at a renormalization scale $\mu^2 = 4$ GeV^2 [GRS99] we can determine its slope

$$
\begin{aligned}
\rho^s_C &= \frac{d\,C^s_{2,0}(t)}{d\,t}\Big|_{t=0} \\
&= \frac{g_A^2}{640\pi^2 F_\pi^2 M_0^6}\Bigg\{ 4a^s_{2,0} m_\pi^4 \left(2m_\pi^2 - 3M_0^2\right)\log\frac{m_\pi}{M_0} + a^s_{2,0}\Bigg(m_\pi^6 - 8m_\pi^4 M_0^2 \\
&\quad + 2m_\pi^2 M_0^4 - \frac{2}{3}M_0^6\Bigg) + x^0_\pi M_0 M_N(m_\pi)\Bigg(7m_\pi^4 - 27m_\pi^2 M_0^2 + \frac{20}{3}M_0^4\Bigg) \\
&\quad - 2x^0_\pi \frac{M_N(m_\pi)}{M_0}\Bigg(4m_\pi^6 - 21m_\pi^4 M_0^2 + 20m_\pi^2 M_0^4 + 5M_0^6\Bigg)\log\frac{m_\pi}{M_0} \\
&\quad - \frac{1}{m_\pi^3 - 4m_\pi M_0^2}\Bigg[a^s_{2,0}\Bigg(m_\pi^8 - 4M_0^2 m_\pi^6 + 2M_0^4 m_\pi^4\Bigg) \\
&\quad - x^0_\pi M_0 M_N(m_\pi)\Bigg(m_\pi^6 - 7m_\pi^4 M_0^2 + 9m_\pi^2 M_0^4 + 8M_0^6\Bigg) \\
&\quad - \frac{1}{\sqrt{4M_0^2 - m_\pi^2}}\Bigg(4a^s_{2,0} m_\pi^4\left(-2m_\pi^6 + 15m_\pi^4 M_0^2 - 30m_\pi^2 M_0^4 + 10M_0^6\right) \qquad (2.53) \\
&\quad + 2x^0_\pi \frac{M_N(m_\pi)}{M_0}\left(4m_\pi^{10} - 45M_0^2 m_\pi^8 + 170m_\pi^6 M_0^4 - 225m_\pi^4 M_0^6\right. \\
&\quad \left. + 30m_\pi^2 M_0^8 + 32M_0^{10}\right)\Bigg)\Bigg]\arccos\left(\frac{m_\pi}{2M_0}\right)\Bigg\}.
\end{aligned}
\tag{2.54}
$$

At the physical point this would give us $\rho_C^s(m_\pi = 0.14\text{GeV}) = -0.77\ \text{GeV}^{-2}$. Truncating eq.(2.54) in $1/M_0$ we reproduce the chiral singularity $\sim m_\pi^{-1}$ found in ref.[BJ02][12]

$$\rho_C^s = -\frac{g_A^2 x_\pi^0 M_N(m_\pi)}{160\pi F_\pi^2 m_\pi} - \frac{g_A^2}{960\pi^2 F_\pi^2}\left[a_{2,0}^s + x_\pi^0 \frac{M_N(m_\pi)}{M_0}\left(-13 + 15\log\frac{m_\pi}{M_0}\right)\right] + ... \tag{2.55}$$

This amounts to a slope of $\rho_C^s|_\chi = -1.12\ \text{GeV}^{-2}$ which is 45% larger than the BChPT estimate of eq.(2.54). Interestingly, among the terms $\sim m_\pi^0$ shown in eq.(2.55) it is the $1/M_0$ suppressed corrections to the leading HBChPT result of ref.[BJ02] that dominate numerically. This gives a strong indication that a *covariant* calculation of $\Delta C_{h.o.}^s(t, m_\pi)$ as given in appendix A.5.2 is advisable, automatically containing *all* associated $(1/M_0)^i$ corrections.
The quark mass dependence of the slope function ρ_C^s of eq.(2.54) already suggests that one obtains an interesting variation of the t dependence of this form factor as a function of the quark mass! We therefore close this discussion with a look at fig.2.8. There we have fixed the only unknown parameter $c_{2,0}^s = -0.41 \pm 0.1$ such that the BChPT result coincides with the dipole parametrization of the QCDSF collaboration at $t = 0$ [G$^+$04] for the lightest pion mass in the simulation, i.e. $m_\pi = 640$ MeV. We note explicitly that this coupling only affects the overall normalization of this form factor but does not impact its momentum dependence. It is therefore quite remarkable to observe that the resulting t dependence of this form factor according to this ChEFT estimate agrees quite well with the phenomenological dipole-parametrization of the QCDSF data at this large quark mass, even over quite a long range in four-momentum transfer. The result is rather close to a straight line which is a consequence of the fact that in the renormalization scheme introduces and applied in this work, pion-cloud effects are switched off at large pion masses (see fig.2.8). We remind the reader that the value of this form factor at $t = 0$ and $m_\pi = 0.14$ GeV determines the strength of the so-called D-term of the nucleon, playing a decisive role in the analysis of DVCS experiments [Ji98, Die03, BR05]. Utilizing the extracted value of $c_{2,0}^s$, we can now study the C-form factor also at the physical point, with the result also shown in fig.2.8. At this low value of the pion mass one can suddenly observe a nonlinear t dependence for low values of four-momentum transfer *due to the pion-cloud of the nucleon.* This is a very interesting observation because such a mechanism would allow for a more negative value of $C_{2,0}^s(t = 0)$ at the physical point than previously extracted from lattice QCD analyses via dipole extrapolations (e.g. see ref.[G$^+$04]). We obtain [DGH08]

$$C_{2,0}^s\,(t = 0, m_\pi = 140\,\text{MeV}) \approx\ -0.36 \pm 0.1, \tag{2.56}$$

The assigned error corresponds to the fit error of $C_{2,0}^s(m_\pi = 0.64\,\text{GeV}, \text{t} = 0)_{\text{QCDSF}}$ given in ref.[G$^+$04], as it directly influences our unknown coupling

[12]Note that due to a different definition of the covariant derivative in the quark-operator on the left hand side of eq.(2.7) our definition of the third isoscalar form factor differs from ref.[BJ02] by a factor of 4: $C_2^{BJ}(t) = 4C_{2,0}^s(t)$.

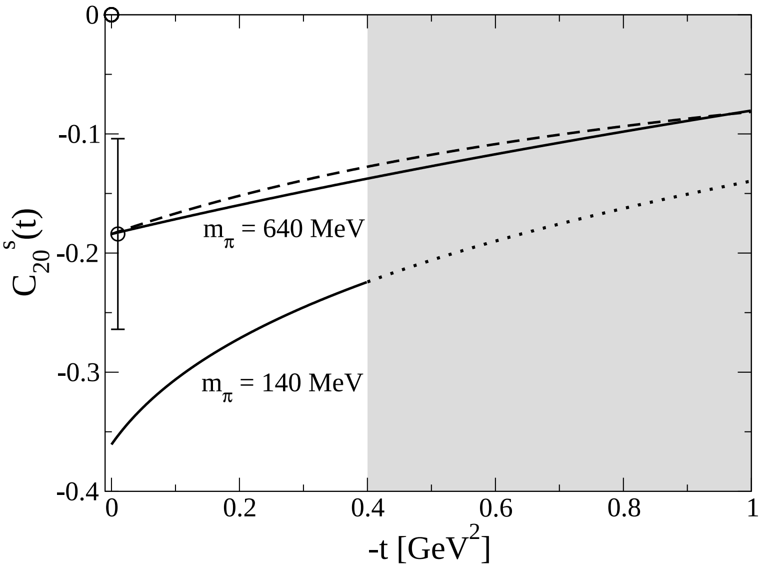

Figure 2.8: Momentum dependence of the form factor $C^s_{2,0}(t)$ [DGH08]. The quasi linear t dependence of the $\mathcal{O}(p^2)$ BChPT result at $m_\pi = 640$ MeV (solid line) has been normalized to the dipole parametrization of the QCDSF data of ref.[G$^+$04] (dashed line), the shown data point gives the result of this reference at $t = 0$ and $m_\pi = 640$ MeV. The resulting nonlinear t dependence in this form factor for smaller values of m_π is then due to the coupling of the tensor field to the pion-cloud, providing an interesting mechanism to obtain "large" negative values at $t = 0$ and $m_\pi = 140$ MeV.

$c^s_{2,0}$. However, we note that the (unknown) systematic uncertainties, both from the quenched simulation results of ref.[G$^+$04] and possible further $\mathcal{O}(p^3)$ contributions (beyond $\Delta C^s_{h.o.}(t, m_\pi)$) to our BChPT results are not accounted for in this error bar.
Despite of these considerations we observe that the value of eq.(2.56) is consistent with the value $C^s_{2,0}\,(t = 0, m_\pi = 140\,\text{MeV}) = -0.267 \pm 0.036$ derived in the subsequently published ref.[H$^+$08], where our formula of eq.(2.52) has been fitted to dynamical lattice data from the LHPC collaboration.

With the value of eq.(2.56) we can finally obtain the first estimate for the radius of this elusive form factor [DGH08]:

$$\begin{aligned} (r^s_C)^2 &= \frac{6}{C^s_{2,0}(t=0,\, m_\pi = 140\,\text{MeV})}\,\rho^s_C(m_\pi = 140\,\text{MeV}) \\ &\approx (0.5 \pm 0.1)\,\text{fm}^2. \end{aligned} \tag{2.57}$$

We compare this result with the radii of the isovector Dirac- and Pauli form factors of the nucleon which are also dominated by pion-cloud effects. Interestingly, with $(r^v_1)^2 = 0.585$ fm^2 and $(r^v_2)^2 = 0.80$ fm^2 [MMD96] the estimated value for $(r^s_C)^2$ seems to lie in the same order of magnitude! We note, however, that our numerical estimate for the slope ρ^s_C of eq.(2.54) given above is *significantly* smaller than the corresponding slopes of the isovector Pauli- and Dirac form factors, as expected from general arguments and as already observed in

lattice QCD simulations with dynamical fermions [L+04] (at very large quark masses).
However, before we can go into a more detailed numerical analysis of these interesting new form factors of the nucleon, one should first complete the $\mathcal{O}(p^3)$ calculation of the generalized isoscalar form factors, as there are additional diagrams next to fig.2.2 possibly also affecting the t dependence, albeit presumably in a weaker fashion [DHH].

2.6 The First Moments of axial GPDs

In full analogy to the vector case, we now extend our analysis to the first moments of the axial Generalized Parton Distributions [DGH07, DH]. Axial GPDs have been first introduced together with the vector ones and are usually referred to as $\tilde{H}^q(x,\xi,t)$ and $\tilde{E}^q(x,\xi,t)$ [Ji98, Die03, BP07].
In eq.(2.3) of the introduction of this chapter we have seen how x-integrals of vector GPDs are related to the Dirac and Pauli form factors. Integrating the functions $\tilde{H}^q(x,\xi,t)$ and $\tilde{E}^q(x,\xi,t)$ over the momentum fraction x we find

$$\int_{-1}^{1} dx\, \tilde{H}^q(x,\xi,t) = G_A^q(t), \tag{2.58}$$

$$\int_{-1}^{1} dx\, \tilde{E}^q(x,\xi,t) = G_P^q(t), \tag{2.59}$$

where $G_A^q(t)$, $G_P^q(t)$ are the axial and the pseudoscalar form factors of the nucleon, respectively. These definitions come clear if we consider the matrix element of the axial current between two nucleons. Lorentz invariance and parity conservation require this matrix element to take the form

$$\langle p_2|\overline{q}\gamma^\mu\gamma_5 q|p_1\rangle = \overline{u}(p_2)\left[G_A^q(t)\gamma^\mu\gamma_5 + G_T^q(t)\frac{i\sigma^{\mu\nu}\gamma_5 q_\nu}{2M_N} + G_P^q(t)\frac{\gamma_5 q^\mu}{2M_N}\right]u(p_1), \tag{2.60}$$

where the three form factors $G_A^q(t)$, $G_T^q(t)$ and $G_P^q(t)$ reflect the structure of the nucleon when probed by an external axial field.
The axial tensor term, $G_T^q(t)$, is often omitted because it has the opposite behaviour under charge symmetry from the other two terms. The case $G_T \neq 0$ would imply the existence of the so called second-class currents [Wei58], which are experimentally excluded to a high precision [Wil00]. We therefore choose to work in absence of second-class terms, assuming $G_T = 0$.
Furthermore, the axial-vector form factor $G_A(t)$ admits the following expansion for small momentum transfer

$$G_A(t) = g_A\left(1 + \frac{1}{6}\langle r_A^2\rangle t + \mathcal{O}(t^2)\right), \tag{2.61}$$

with $\langle r_A^2\rangle$ the nucleon axial radius and g_A the axial-vector coupling constant, which defines the limit for vanishing momentum transfer of the nucleon axial form factor $G_A(t)$. We have already encountered the chiral limit of the axial-vector coupling constant g_A in section 1.1.2, when we have introduced the leading-order pion-nucleon Lagrangian $\mathcal{L}^{(1)}_{\pi N}$. A calculation of the quark-mass dependence of g_A in Chiral Perturbation Theory is obtained via studying the response of the nucleon to the presence of an external axial field.

An analysis of the axial form factors of the nucleon $G^q_A(t)$, $G^q_P(t)$ in the framework of HBChPT can be found in references [BKM92a, BKLM94, FLMS97, BFHM98], while a calculation in the covariant scheme has been presented in reference [SFGS07]. Further discussion on data and theoretical prediction can be found in refs. [BEM02, GF04]. In the last years a lot of chiral extrapolation studies of lattice data of the coupling constant g_A have been also performed, for details we refer to [HPW03b, E$^+$06b, PMHW07, K$^+$06].
As for the vector case we focus on the *first* moments in x of the axial GPDs

$$\int_{-1}^{1} dx\, x\, \tilde{H}^q(x,\xi,t) = \tilde{A}^q_{2,0}(t), \tag{2.62}$$

$$\int_{-1}^{1} dx\, x\, \tilde{E}^q(x,\xi,t) = \tilde{B}^q_{2,0}(t). \tag{2.63}$$

A study of the leading one loop order contributions to the first moments of axial GPDs in the framework of Heavy Baryon ChPT has been presented in references [CJ01, ACK06, DMS06, DMS07], while no calculation in covariant BChPT can be found in literature sofar.
For each quark q one has *two generalized form factors* $\tilde{A}^q_{2,0}(t)$ and $\tilde{B}^q_{2,0}(t)$, defined by the form factor decomposition of the matrix element [Ji98, Die03, BP07]

$$\begin{aligned} &i\langle p_2|\ \bar{q}\gamma_{\{\mu}\gamma_5 \overleftrightarrow{D}_{\nu\}}q|p_1\rangle = \\ &\quad \tilde{A}^{\,q}_{\,2,0}(q^2)\ \overline{u}(p_2)\gamma_{\{\mu}\gamma_5\,\overline{p}_{\nu\}}u(p_1) + \frac{\tilde{B}^{\,q}_{\,2,0}(q^2)}{2M_N}\,\overline{u}(p_2)\gamma_5\, q_{\{\mu}\,\overline{p}_{\nu\}}u(p_1), \end{aligned} \tag{2.64}$$

where we make use of the same notation of eq.(2.7). Like in the vector case, the form factors $\tilde{A}^q_{2,0}(q^2)$, $\tilde{B}^q_{2,0}(q^2)$ are real functions of the momentum transfer squared. Notice that no $\tilde{C}^q(q^2)$ form factor appears in the matrix element of eq.(2.64); time reversal invariance indeed constraints the current $\overline{u}(p_2)\gamma_5 u(p_1)$ to come with odd powers of q^α, allowing for a *two-form factors* decomposition of the matrix element.

Working within the methods of ChPT, we concentrate on the triplet (v) contribution of the quarks to the two axial generalized form factors:

$$\begin{aligned} &i\langle p_2|\bar{q}\gamma_{\{\mu}\gamma_5 \overleftrightarrow{D}_{\nu\}}q|p_1\rangle_{u-d} = \\ &\quad \overline{u}(p_2)\left[\tilde{A}^{\,v}_{\,2,0}(q^2)\ \gamma_{\{\mu}\gamma_5\,\overline{p}_{\nu\}} + \frac{\tilde{B}^{\,v}_{\,2,0}(q^2)}{2M_N}\,\gamma_5\, q_{\{\mu}\,\overline{p}_{\nu\}}\right]u(p)\times \eta^\dagger\frac{\tau^a}{2}\eta. \end{aligned} \tag{2.65}$$

In analogy to the vector case, in the limit of vanishing four-momentum transfer the isovector form factor $\tilde{A}^v_{2,0}(t\to 0)$ is directly connected to the spin dependent analogue of the mean momentum fraction $\langle \Delta x \rangle_{u-d}$.

$$\begin{aligned} \tilde{A}^v_{2,0}(0) &= \langle \Delta x \rangle_{u-d} = \int_0^1 dx\ x\,(q_\downarrow(x) - q_\uparrow(x))\big|_{u-d} \\ &= \Delta a^v_{2,0} + \mathcal{O}(p^2), \end{aligned} \tag{2.66}$$

where $q_{\downarrow(\uparrow)}(x)$ is the distribution with quark spin parallel (antiparallel) to the spin of the nucleon and $\Delta a^v_{2,0}$ corresponds to the chiral limit value of $\langle \Delta x \rangle_{u-d}$. Because of this helicity dependence, which becomes explicit in the forward limit case, axial GPDs are sometimes referred to as "polarized" generalized parton distributions, where the adjective off course refers to the spin of the parton and not of the target.

2.6.1 The Generalized Axial Form Factors in $\mathcal{O}(p^2)$ BChPT

Like in the vector case, we stop our analysis of the generalized axial form factors to the leading one loop order in BChPT, i.e to the $\mathcal{O}(p^2)$ level according to the power counting scheme presented in section 2.3.3.
The loop diagrams contributing to the first moments of the axial GPDs are again the ones discussed for the vector GPDs (see fig.2.1), off course after appropriate interchange of the couplings.
We remark that the results presented in this section still have to be published.

The relevant parts of the chiral Lagrangian we need for the evaluation of the tensor current of eq.(2.65) have been given in section 2.3. Note that since we are now dealing with an external axial source ($a^i_{\alpha\beta}$), the leading term to the form factor $\tilde{A}_{2,0}(q^2)$, i.e the coupling $\Delta a^v_{2,0}$, appears in the order p^0 Lagrangian of eq.(2.18), while the leading term to the form factor $\tilde{B}_{2,0}(q^2)$, i.e $\Delta b^v_{2,0}$, now belongs to the next-to-leading order Lagrangian $\mathcal{L}^{(2)}_{t\pi N}$ of eq.(2.24), which has to be extended as follows:

$$\begin{aligned} \mathcal{L}^{(2)}_{t\pi N} &= \ldots \\ &\quad + \overline{\psi}_N \Big\{ - \frac{\Delta b^v_{2,0}}{8M_0} \Big(i\gamma_5 \left[\overrightarrow{D}_\mu, V^{\mu\nu}_-\right] \overrightarrow{D}_\nu + h.c. \Big) \Big\} \psi_N \\ &\quad + \frac{c_{14}}{4M_0^2}\, \overline{\psi}_N \Big\{ Tr(\chi_+) V^-_{\mu\nu} \gamma^\mu \gamma_5 i \overrightarrow{D}^\nu + h.c. \Big\} \psi_N \\ &\quad + \frac{c_{16}}{4M_0^2} \overline{\psi}_N \Big\{ [\overrightarrow{D}^\alpha, [\overrightarrow{D}_\alpha, V^-_{\mu\nu}]] \gamma^\mu \gamma_5 i \overrightarrow{D}^\nu + h.c. \Big\} \psi_N \\ &\quad + \ldots \end{aligned} \tag{2.67}$$

The new coupling c_{14} governs the leading quark-mass insertion in $\langle \Delta x_{u-d} \rangle$, while c_{16} parametrizes the contributions of short-distance physics to the slopes of the generalized form factor $\tilde{A}^v_{2,0}(t)$.

In the following we present the $\mathcal{O}(p^2)$ BChPT expressions for both the generalized form factors, obtained again in the $\overline{\text{IR}}$ regularization scheme. The $\mathcal{O}(p^2)$ amplitudes corresponding to the diagrams of fig.2.1 can be found in appendix A.6.1.

We first start providing results for the form factors $\tilde{A}^v_{2,0}(t)$ at $t=0$. In this limit, the PDF-moment reads

$$
\begin{aligned}
\tilde{A}^v_{2,0}(0) \;=\; & \langle \Delta x \rangle_{u-d} \\
& \Delta a^v_{2,0} + \frac{\Delta a^v_{2,0} m_\pi^2}{3(4\pi F_\pi)^2}\Bigg\{ -3(2g_A^2+1)\log\frac{m_\pi^2}{\lambda^2} - 3g_A^2 \\
& + g_A^2 \frac{m_\pi^2}{M_0^2}\left(1 + 9\log\frac{m_\pi}{M_0}\right) - g_A^2 \frac{m_\pi^4}{M_0^4}\log\frac{m_\pi}{M_0} \\
& + \frac{g_A^2 m_\pi}{\sqrt{4M_0^2 - m_\pi^2}}\left(22 - 11\frac{m_\pi^2}{M_0^2} + \frac{m_\pi^4}{M_0^4}\right)\arccos\left(\frac{m_\pi}{2M_0}\right)\Bigg\} \\
& + \frac{a^v_{2,0}\, g_A\, m_\pi^2}{3(4\pi F_\pi)^2}\Bigg\{ 2\frac{m_\pi^2}{M_0^2}\left(1 + 3\log\frac{m_\pi^2}{M_0^2}\right) - \frac{m_\pi^4}{M_0^4}\log\frac{m_\pi^2}{M_0^2} \\
& + \frac{2m_\pi(4M_0^2 - m_\pi^2)^{\frac{3}{2}}}{M_0^4}\arccos\left(\frac{m_\pi}{2M_0}\right)\Bigg\} \\
& + 4m_\pi^2\frac{c_{14}^{(r)}(\lambda)}{M_0^2} + \mathcal{O}(p^3). \qquad (2.68)
\end{aligned}
$$

The truncation of this expression at leading order in $1/M_0$ gives the exact $\mathcal{O}(p^2)$ HBChPT limit of our result, in agreement with the findings of references [CJ01, ACK06, DMS06, DMS07]:

$$
\begin{aligned}
\tilde{A}^v_{2,0}(0)|^{p^2}_{HBChPT} \;=\; & \Delta a^v_{2,0}\left\{1 - \frac{m_\pi^2}{(4\pi F_\pi)^2}\left(g_A^2 + (2g_A^2+1)\log\frac{m_\pi^2}{\lambda^2}\right)\right\} \\
& + 4m_\pi^2\frac{c_{14}^{(r)}(\lambda)}{M_0^2} + \mathcal{O}(p^3). \qquad (2.69)
\end{aligned}
$$

Looking at the covariant expression for $\langle \Delta x \rangle_{u-d}$ of eq.(2.68) and at the result for $\langle x \rangle_{u-d}$ given in eq.(2.25), one can easily observe that each isovector moment ($\langle x \rangle_{u-d}$ and $\langle \Delta x \rangle_{u-d}$) depends on 3 unknown parameters: 2 couplings ($a^v_{2,0}$, $\Delta a^v_{2,0}$) and one counterterm (c_8 and c_{14}, respectively). As the same couplings contribute in both moments, it is hoped that a simultaneous fit of our BChPT results to the lattice data of ref.[H$^+$08] can considerably reduce the statistical errors. As one can see from Fig.2.9, the results of this procedure are pretty outstanding, given that the values at the physical pion mass were not included in the fit! The chiral curvature in both observables naturally bends down to the phenomenological value for lighter quark masses, leading to a very satisfactory extrapolation curve.

The input values used in the fit and the resulting values for the fit parameters, together with their statistical errors are given in table 2.7. We make the choice to adopt the same input values used throughout the present work and in ref.[DGH08], i.e. we choose to set the nucleon axial-vector coupling and the pion decay constant equal to their physical values $g_A = 1.2$ and $F_\pi = 0.0924$ MeV. Strictly speaking, these quantities should be taken in the chiral limit ($g_A = 1.267$, $F_\pi = 0.0862$[CD04]). The difference resulting from this choice is

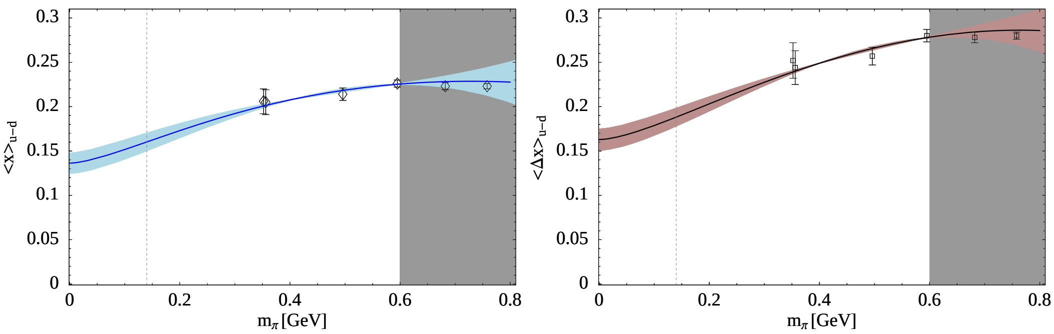

Figure 2.9: Combined FIT of the $\mathcal{O}(p^2)$ results given in eq.(2.25) and eq.(2.68) to the LHPC data of reference [H$^+$08] for $\langle x\rangle_{u-d}$ and $\langle \Delta x\rangle_{u-d}$ respectively. The fit is a 4-parameter fit; only data points for $m_\pi < 600$ MeV have been included in the fit, for a total of 8 data points. Note that the phenomenological values at physical pion mass were not included in the fit. The bands shown indicate estimate of higher order possible corrections.

Input values		Values from the fit	
g_A	1,2	$a^v_{2,0}$	0.14± 0.01
M_0 [GeV]	0.889	$\Delta a^v_{2,0}$	0.16 ± 0.01
F_π [GeV]	0.0924	$c^r_8(1\text{ GeV})$	-0.23 ± 0.06
		$c^r_{14}(1\text{ GeV})$	-0.16 ± 0.05

Table 2.7: The input values and the values of the coupling constants resulting from the *combined fit* of the $\mathcal{O}(p^2)$ BChPT results of eqs.(2.25) and (2.68) to the LHPC data of reference [H$^+$08] for $\langle x\rangle_{u-d}$ and $\langle \Delta x\rangle_{u-d}$ respectively. The corresponding plot is showed in fig.2.9.

not significant, because the ratio g_A/F_π is very insensitive to this change. The choice of the physical values instead of the chiral ones does not lead to any significant changes in the final results.
The extrapolated values of the two observables are reported in table 2.8, where the corresponding phenomenological values $\langle x\rangle^{phen}_{u-d}$ and $\langle \Delta x\rangle^{phen}_{u-d}$ are also given. As one can easily see, the obtained ChPT values from this $\mathcal{O}(p^2)$ leading-one-loop analysis are clearly consistent with the experimental ones! The table shows also the extrapolated values for $\langle x\rangle_{u-d}$ at the physical pion mass, as obtained by LHPC and QCDSF/UKQCD collaborations from chiral extrapolation studies on recent lattice data. Both analysis make use of the covariant chiral expression for $\langle x\rangle_{u-d}$ given in ref.[DGH08](i.e. eq.(2.25)). Differently to our case, where all couplings are free-parameters in the fit, the analysis performed by both lattice QCD collaborations implies an input value for the coupling constant $\Delta a^v_{2,0}$. The LHPC collaboration uses the value $\Delta a^v_{2,0} = 0.17$ obtained from a chiral fit to own lattice results for $\langle \Delta x\rangle_{u-d}$ [E$^+$06a], while the QCDSF/UKQCD collaboration follows ref.[DGH08] and constraints the coupling $\Delta a^v_{2,0}$ from the phenomenological value of $\langle \Delta x\rangle_{u-d} = 0.21$. The LHPC

		$\langle x\rangle_{u-d}$	$\langle \Delta x\rangle_{u-d}$
Phenomenology		0.16 ± 0.006[xpr]	0.21 ± 0.025 [BB02]
Extrapolated	BChPT	0.16 ± 0.01	0.19 ± 0.01
values	LHPC	0.157 ± 0.006 [H+08]	
at m_π^{phys}	QCDSF/UKQCD	0.198 ± 0.008 [B+07b]	

Table 2.8: The phenomenological values of the observables $\langle x\rangle_{u-d}$ and $\langle \Delta x\rangle_{u-d}$, together with the extrapolated values obtained from the combined fit showed in fig.2.9 and from the chiral extrapolation studies on lattice data from LHPC [H+08] and QCDSF/UKQCD [B+07b] collaborations. All three analysis make use of the chiral results of ref.[DGH08].

result agrees very well with both phenomenology and the value obtained from our combined fit. On the contrary, the QCDSF/UKQCD value is definitely too high, it is not consistent with the experimental result. We want to stress once again that such comparisons between the results from the different collaborations have to be done with due caution, reminding of the serious differences on normalization between the two sets of lattice data. Therefore, before firm conclusions can be drawn, the combined fit described in this section will have to be newly performed once QCDSF/UKQCD lattice data for $\langle \Delta x\rangle_{u-d}$ will also be available.

We would like to stress that this efficient cross-talk between the ChPT results for $\langle x\rangle_{u-d}$ and $\langle \Delta x\rangle_{u-d}$ occurs only in the covariant framework, while as clearly showed by eqs.(2.27),(2.69)in the non-relativistic approach both observables are completely independent at this order.

We now turn to the full t-dependence of $\tilde{A}^v_{2,0}(t)$, which can be written as

$$\tilde{A}^v_{2,0}(t) = \tilde{A}^v_{2,0}(0) + \frac{\Delta a^v_{2,0}\, g_A^2}{96\pi^2 F_\pi^2}\tilde{F}^v_{2,0}(t) + \frac{c_{16}}{M_0^2}t + \mathcal{O}(p^3), \tag{2.70}$$

with $\tilde{A}^v_{2,0}(0)$ given above and

$$\begin{aligned}\tilde{F}^v_{2,0}(t) \;=\; & -\frac{1}{6}t - \frac{m_\pi^3(4m_\pi^4 - 17m_\pi^2 M_0^2 + 10M_0^4)}{M_0^4\sqrt{4M_0^2 - m_\pi^2}}\arccos\left(\frac{m_\pi}{2M_0}\right)\\ & + \frac{m_\pi^4}{M_0^4}(4m_\pi^2 - 9M_0^2)\log\frac{m_\pi}{M_0}\\ & + \int_{-\frac{1}{2}}^{\frac{1}{2}} du\,\left\{\frac{m_\pi^3 M_0^2(4m_\pi^4 - 17m_\pi^2\tilde{M}^2 + 10\tilde{M}^4)}{\tilde{M}^6\sqrt{4\tilde{M}^2 - m_\pi^2}}\arccos\left(\frac{m_\pi}{2\tilde{M}}\right)\right.\\ & + 3\frac{m_\pi^2}{\tilde{M}^2}(\tilde{M}^2 - M_0^2) + \frac{4m_\pi^4}{M_0^2\tilde{M}^4}(M_0^4 - \tilde{M}^4) \qquad (2.71)\\ & \left. + 2M_0^2\log\frac{M_0}{\tilde{M}} + \frac{m_\pi^4 M_0^2}{\tilde{M}^6}(9\tilde{M}^2 - 4m_\pi^2)\log\frac{m_\pi}{\tilde{M}}\right\}, \qquad (2.72)\end{aligned}$$

where we use the same notation introduced for the vector channel in section 2.4.3.
Furthermore, we provide the expression of the slope of $\tilde{A}^v_{2,0}(t)$:

$$\begin{aligned}\rho^v_{\tilde{A}} &= \frac{c_{16}}{M_0^2} - \frac{\Delta a^v_{2,0}\, g_A^2}{6(4\pi F_\pi)^2}\frac{m_\pi^2}{(4M_0^2 - m_\pi^2)}\Bigg\{ -2 + 4\frac{m_\pi^2}{M_0^2}\left(2 + 3\log\frac{m_\pi}{M_0}\right) \qquad (2.73)\\ &-\frac{m_\pi^4}{M_0^4}\left(2 + 11\log\frac{m_\pi}{M_0}\right) + 2\frac{m_\pi^6}{M_0^6}\log\frac{m_\pi}{M_0}\\ &+\frac{m_\pi}{\sqrt{4M_0^2 - m_\pi^2}}\left(10 - 30\frac{m_\pi^2}{M_0^2} + 15\frac{m_\pi^4}{M_0^4} - 2\frac{m_\pi^6}{M_0^6}\right)\arccos\left(\frac{m_\pi}{2M_0}\right)\Bigg\}\\ &+\frac{m_\pi \delta^t_{\tilde{A}}}{(4\pi F_\pi)^2 M_0}, \qquad (2.74)\end{aligned}$$

where we have introduced the term $\frac{m_\pi \delta^t_{\tilde{A}}}{(4\pi F_\pi)^2 M_0}$ to indicate the size of possible higher order corrections coming from $\mathcal{O}(p^3)$, in complete analogy to the vector case (see section 2.4.2). A first numerical analysis of the formula given above reveals a very small contribution of the pion-cloud to the slope of the axial generalized form factor, suggesting again that the physics which governs the size of this object is hidden in the counter-term contribution c_{16}.

Finally, we present the $\mathcal{O}(p^2)$ BChPT results for $\tilde{B}^v_{2,0}(t)$:

$$\begin{aligned}\tilde{B}^v_{2,0}(t) &= \Delta b^v_{2,0}\frac{M_N}{M_0} + \frac{\Delta a^v_{2,0}\, g_A^2 M_0^2}{24\pi^2 F_\pi^2}\int_{-\frac{1}{2}}^{\frac{1}{2}}\frac{du}{\tilde{M}^8}\,u^2\Bigg\{\tilde{M}^6(\tilde{M}^2 - M_0^2)\\ &+3m_\pi^2 M_0^2\tilde{M}^4\left(4 + \log\frac{m_\pi^2}{\tilde{M}^2}\right) - 3m_\pi^4 M_0^2\tilde{M}^2\left(4 + 5\log\frac{m_\pi^2}{\tilde{M}^2}\right)\\ &+6m_\pi^6 M_0^2\log\frac{m_\pi^2}{\tilde{M}^2} - \frac{6m_\pi^3 M_0^2}{\sqrt{4\tilde{M}^2 - m_\pi^2}}(7\tilde{M}^4 - 9m_\pi^2\tilde{M}^2 + 2m_\pi^4)\\ &\times\arccos\frac{m_\pi}{2\tilde{M}}\Bigg\} + \mathcal{O}(p^3), \qquad (2.75)\end{aligned}$$

which in the limit $t \to 0$ reduces to

$$\begin{aligned}\tilde{B}^v_{2,0}(0) &= \Delta b^v_{2,0}\frac{M_N}{M_0} + \frac{\Delta a^v_{2,0}\, g_A^2\, m_\pi^2}{6(4\pi F_\pi)^2}\Bigg\{4 + \log\frac{m_\pi^2}{M_0^2} - \frac{m_\pi^2}{M_0^2}\left(4 + 5\log\frac{m_\pi^2}{M_0^2}\right)\\ &+2\frac{m_\pi^4}{M_0^4}\log\frac{m_\pi^2}{M_0^2} - \frac{2m_\pi}{\sqrt{4M_0^2 - m_\pi^2}}\left(7 - 9\frac{m_\pi^2}{M_0^2} + 2\frac{m_\pi^4}{M_0^4}\right)\arccos\left(\frac{m_\pi}{2M_0}\right)\Bigg\}\\ &+\mathcal{O}(p^3). \qquad (2.76)\end{aligned}$$

Furthermore, the relative slope reads:

$$
\begin{aligned}
\rho^v_{\tilde{B}} \;=\; & \frac{\Delta a^v_{2,0}\, g_A^2}{180(4\pi F_\pi)^2}\frac{1}{(4M_0^2-m_\pi^2)}\Bigg\{-4M_0^2+m_\pi^2\left(109+24\log\frac{m_\pi^2}{M_0^2}\right) \\
& -6\frac{m_\pi^4}{M_0^2}\left(35+31\log\frac{m_\pi^2}{M_0^2}\right)+3\frac{m_\pi^6}{M_0^4}\left(16+47\log\frac{m_\pi^2}{M_0^2}\right) \\
& -24\frac{m_\pi^8}{M_0^6}\log\frac{m_\pi^2}{M_0^2}+\frac{6m_\pi^3}{\sqrt{4M_0^2-m_\pi^2}}\left(-70+140\frac{m_\pi^2}{M_0^2}-63\frac{m_\pi^4}{M_0^4}+8\frac{m_\pi^6}{M_0^6}\right) \\
& \times\arccos\left(\frac{m_\pi}{2M_0}\right)\Bigg\}+\delta_{\tilde{B}}\frac{M_N}{(4\pi F_\pi)^2 M_0}. \qquad (2.77)
\end{aligned}
$$

Again, $\delta_{\tilde{B}}$ is not part of the $\mathcal{O}(p^2)$ result but it is given to indicate the size of possible higher order corrections from $\mathcal{O}(p^3)$.
Since the axial generalized form factor $\tilde{B}^v_{2,0}(t)$ is not directly connected to any phenomenological observables, unfortunately a numerical analysis of the expressions given above does not provide us with new interesting physical information. Anyway, lattice data for this quantity are available and $\tilde{B}^v_{2,0}(t)$ shows dependence on the coupling constant $\Delta a^v_{2,0}$ which appears in several quantities discussed in this chapter. Future lattice simulations, together with higher order calculations from ChPT will then allow to perform simultaneous fits of many observables with consequent minimization of the statistical errors, paving the way to even more reliable chiral predictions.

2.7 Summary

The pertinent results of this chapter can be summarized as follows:

1. We have constructed the effective chiral Lagrangian for symmetric, traceless tensor fields of positive parity up to $\mathcal{O}(p^2)$ in the covariant framework of Baryon Chiral Perturbation Theory for two light quark flavors.

2. Within this covariant framework we have calculated the generalized isovector and isoscalar form factors of the nucleon $A_{2,0}^{v,s}(t, m_\pi^2)$, $B_{2,0}^{v,s}(t, m_\pi^2)$ and $C_{2,0}^{v,s}(t, m_\pi^2)$ up to $\mathcal{O}(p^2)$ which corresponds to leading one loop order. Our results were $\overline{\text{IR}}$ renormalized and we can thus exactly reproduce the corresponding nonrelativistic $\mathcal{O}(p^2)$ results previously obtained in Heavy Baryon ChPT by taking the limit $1/M_0 \to 0$. Several HBChPT results published recently could not yet be reproduced, as they correspond to partial, nonrelativistic results from the higher orders $\mathcal{O}(p^3, p^4, p^5)$.

3. According to our numerical analysis of the quark mass dependence of the generalized form factors, we have noted that for $B_{2,0}^{s,v}(t)$ and $C_{2,0}^{s,v}(t)$ the observable quark mass dependencies could be dominated by the (well known) quark mass dependence of the mass of the nucleon $M_N(m_\pi)$. This mass function appears in several places in the chiral results due to kinematical factors in the matrix element used in the definition of the generalized form factors. Such a "trivial" but numerically significant effect is already known from the analysis of lattice QCD data for the Pauli form factors of the nucleon.

4. The pion-cloud contributions to all three generalized *isovector* form factors only show a very weak dependence on t. The momentum transfer dependence of these structures seems to be dominated by presently unknown short distance contributions. The situation in this isovector channel reminds us of an analogous role played by chiral dynamics in the isoscalar Dirac- and Pauli form factors of the nucleon. At this point we are therefore not able to give predictions for the *numerical* size of the slopes of these interesting nucleon structure quantities. It is hoped that a global fit to new lattice QCD data at small pion masses and small values of t – extrapolated to the physical point with the help of the formulae presented in this work – will lead to first insights into this new field of baryon structure physics. A first step on this way has already been achieved in reference [H^+08, B^+07b].

5. The leading one loop order covariant BChPT results for the generalized *isoscalar* form factors $A_{2,0}^{s}(t)$ and $B_{2,0}^{s}(t)$ are quite surprising. As far as the topology of possible Feynman diagrams is concerned, one is reminded of the isovector Dirac- and Pauli form factors of the nucleon. A power-counting analysis, however, told us that those diagrams (e.g see fig.2.2) which one would naively expect to strongly depend on both the momentum transfer and the quark mass only start to contribute at

next-to-leading one loop order. Our analysis therefore suggests that the momentum dependence at low values of t is dominated by short-distance physics.

6. It is the value of $\langle x\rangle_\pi$ of a pion in the chiral limit that controls the magnitude of those long-distance pion-cloud effects in the generalized isoscalar form factors of the nucleon, pointing to the need of a simultaneous analysis of pion- and nucleon structure on the lattice and in ChEFT.

7. In the forward limit, the isovector form factor $A^v_{2,0}(t \to 0)$ reduces to $\langle x\rangle_{u-d}$. Our covariant $\mathcal{O}(p^2)$ BChPT result for this isovector moment provides a smooth chiral extrapolation function between the high values at large quark masses from the LHPC collaboration and the lower value known from phenomenology. The required chiral curvature according to this new analysis does *not* originate from the chiral logarithm of the leading nonanalytic quark mass dependence of this moment – as had been speculated in the literature for the past few years – but is due to an infinite tower of terms $(m_\pi/M_0)^i$ with well constrained coefficients (see eq.(2.25)). The well known leading one loop HBChPT result for $\langle x\rangle_{u-d}$ of eq.(2.27) was found to be not applicable for chiral extrapolations above the physical pion mass, as expected.

8. Judging from the (quenched) lattice data of the QCDSF collaboration, our $\mathcal{O}(p^2)$ BChPT result of eq.(2.41) for $A^s_{2,0}(t = 0) = \langle x\rangle_{u+d}$ also provides a very stable chiral extrapolation function out to quite large values of effective pion masses.

9. A study of the forward limit in the isoscalar sector has led to a first estimate of the contribution of the u and d quarks to the total spin of a nucleon $J_{u+d} \approx 0.24$. This low value compared to previous determinations arises from the possibility of a small negative contribution of $B^s_{2,0}(t = 0) \approx -0.06$ at the physical point, driven by pion-cloud effects. However, at the moment the uncertainty in such a determination is rather large.

10. In a first glance at the third generalized isoscalar form factor $C^s_{2,0}(t)$ the quark mass dependence was found to be qualitatively different from $A^s_{2,0}(t)$ and $B^s_{2,0}(t)$. Its slope contains a chiral singularity $\sim m_\pi^{-1}$ and the influence of short distance contributions is suppressed. A first numerical estimate of its slope gives $\rho^s_C \approx -0.75$ GeV^2 which is much smaller than the slopes of the corresponding Dirac- or Pauli form factors. At low t we have also observed significant changes in the momentum dependence of this form factor as a function of the quark mass, resulting in the estimate $C^s_{2,0}(t = 0) \approx -0.35$ at the physical point.

11. The extension of our analysis to the axial generalized form factors is straightforward. The covariant $\mathcal{O}(p^2)$ results of eq.(2.27) for $A^v_{2,0}(t = 0) = \langle x\rangle_{u-d}$ and of eq.(2.69) for $\tilde{A}^v_{2,0}(t = 0) = \langle \Delta x\rangle_{u-d}$ reveals the cross-talk of coupling constants between the two observables, suggesting the

idea of a simultaneous fit to the available lattice data. The result of the procedure is extremely convincing and we conclude that combined fits of several observables characterized by a common subset of ChEFT couplings are the winning strategy towards the most reliable chiral extrapolations of lattice QCD results.

12. Throughout this chapter we have indicated how to estimate possible corrections of higher orders to our leading one loop order BChPT results. The associated theoretical uncertainties of our $\mathcal{O}(p^2)$ calculation have been discussed in detail. Ultimately, in order to judge the stability of our results it is mandatory that we analyse the complete next-to-leading one loop order. In appendix A.7 we provide preliminary results of this calculation, referring to a later communication for a more complete and detailed discussion.

Finally we refer that the BChPT results for the generalized isovector and isoscalar vector form factors presented in this chapter have already been applied in a detailed analysis of recent lattice data in references [H$^+$08, B$^+$07b].

Chapter 3

Doubly Virtual Compton Scattering with Δ's

3.1 Introduction and summary

In the last decades much effort has been done in theoretical as well as in experimental physics in order to better understand the spin structure of the nucleon [FJ01]. A lot of interesting results have been obtained but at the same time new exciting challenges have arisen. Nowadays facilities make possible to work with polarized beams and polarized targets, paving the way to the study of the spin structure of the nucleon encoded in the spin structure functions g_1 and g_2. In particular, the high luminosity available at Jefferson Laboratory has allowed to extract the spin structure functions and their moments with an unprecedented precision over a wide kinematic range [Che08]. Results for Q^2 values as low as $0.1\,\mathrm{GeV}^2$ have been reported and the plan for a future 12 GeV upgrade at Jefferson Lab has already been proposed. On the theoretical side, such a low energy region can be investigated in Chiral Perturbation Theory, while the region of intermediate momentum transfer can be analysed by means of quark or resonance models or using dispersion relations. Dispersion relations represent the link which can bridge the gap between the high energy and the low energy domain, between perturbative and non-perturbative QCD. To this purpose, a very interesting topic is represented by certain sum rules, such as the Gerasimov-Drell-Hearn (GDH) sum rule and its generalization to finite photon virtuality or the Burkhardt-Cottingham (BC) sum rule. These sum rules have validity at all energy scales and can be written in terms of the spin structure functions g_1 and g_2. They are very special tools because thanks to dispersion relations they can be related to the spin-dependent structure functions $S_{1,2}(\nu, Q^2)$ which appear in the spin amplitude of the process of doubly virtual Compton scattering (VVCS), where ν is the energy transfer and Q^2 the negative of the photon virtuality. A set of sum rules in terms of the structure functions $S_{1,2}(\nu, Q^2)$ can be derived: in addition to the BC sum rule and the GDH sum rule at finite virtuality, of particular interest are also the sum rules for the forward ($\gamma_0(Q^2)$) and longitudinal-transverse ($\delta_0(Q^2)$) gen-

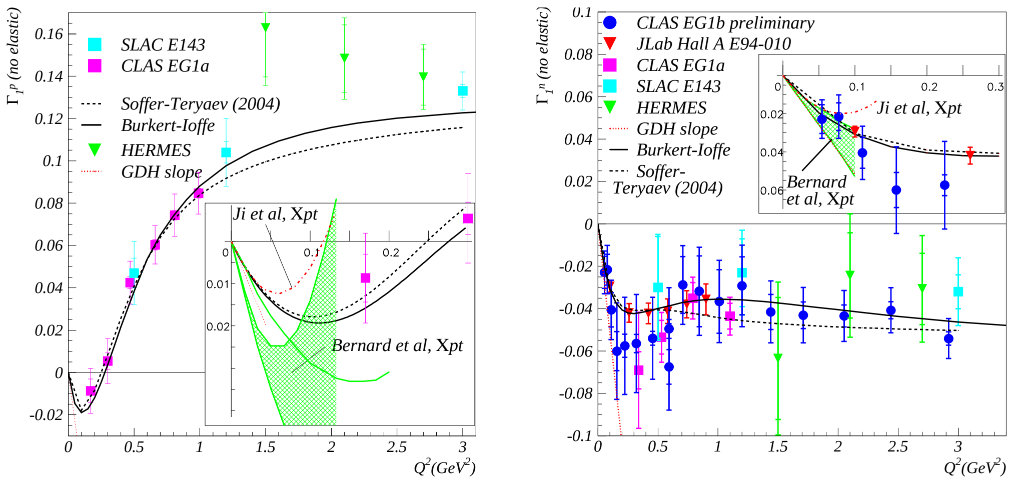

Figure 3.1: The left and right panels show the first moments $\Gamma_1(Q^2)$ for proton and neutron, in the order [Che08]. Experimental results are compared to model predictions (full and dashed lines). In the insets the region at low Q^2 is selected, where the next-to-leading order ChPT calculations of refs. [JKO00] and [BHM03b] gain validity.

eralized spin polarizabilities. Chiral Perturbation Theory represents the most suitable framework to study the process of doubly virtual Compton scattering in the low energy domain. Indeed, a determination of the two building blocks $S_{1,2}(\nu = 0, Q^2)$ in ChPT allows to make predictions for all desired spin-related sum rules and for all moments of the nucleon's spin structure functions for $Q^2 \lesssim 0.3\,\text{GeV}^2$. Such a calculation has been done at the next-to-leading one loop order ($\mathcal{O}(p^4)$) within HBChPT [KSV03, Bur01, JKO00] and in infrared regularization [BHM02, BHM03b]. At that order of the chiral expansion, no unknown low-energy constants appear and thus the calculation owns the big advantage to allow parameter-free predictions for the spin structure functions $S_{1,2}(\nu, Q^2)$. Starting from there one can derive all the desired results for the sum rules and the spin polarizabilities.
The calculation has gained relevance because of new recent experimental results. In particular, new data are available for the first moments $\Gamma_1(Q^2)$ of the spin structure function g_1. They are shown for both proton and neutron in fig.3.1, where experimental data from JLAB Hall A [A^+02, M^+05], CLAS EG1A [F^+03, Y^+03] and EG1B [D^+06, P^+08, B^+07a], SLAC [A^+03] and HERMES [A^+07] are compared to model predictions and to the ChPT results of reference [JKO00] and [BHM03b]. As one can observe, the calculations are in reasonable agreements with the data at the lowest Q^2 values up to ~ 0.07 GeV^2, that is within the range of applicability of the theory. In particular, a better convergence is achieved by the covariant calculation of Bernard et al. [BHM03b], thanks to higher order terms which appear in the covariant but not in the heavy baryon calculation. The authors of ref.[BHM03b] have also made a first attempt to estimate the influence of the $\Delta(1232)$ resonance on

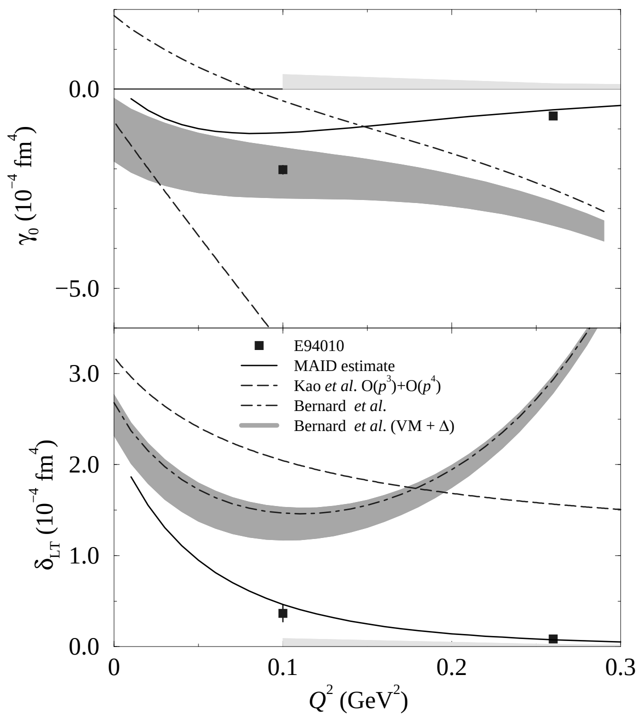

Figure 3.2: Experimental results for the neutron generalized spin polarizabilities γ_0 and δ_0 [Che08]. The squares represent the results with statistical uncertainties, while systematic uncertainties are shown by the light bands. The results are compared to the heavy baryon predictions of ref.[KSV03] and to the relativistic baryon ChPT calculation of ref.[BHM03b], without (dot-dashed curves) and with (dark bands) Δ and vector meson resonances contributions.

the values of the VVCS matrix element. The Δ plays a very important role in the spin sector of the nucleon and it is reasonable to think that its inclusion on the calculation can bring about a considerable change of value to the spin structure fucntions. To this purpose, the Δ relativistic Born graphs have been evaluated. Furthermore, in order to consider the fact that the $N \to N\gamma$ transition occurs at finite Q^2 and in order to get an input from phenomenology, a transition form factor has been introduced and its value has been fixed from pion electroproduction in the $\Delta(1232)$ resonance region. Finally, a modelling of the less important vector mesons resonances has also been included. Going back to the analysis of the first moment Γ_1, we observe that the calculations are in reasonable agreement with the data at the lowest Q^2 settings of $0.05 - 0.1\,\mathrm{GeV}^2$. Anyway, the evaluation of the only Born graphs is definitely not enough for a reliable estimation of the effect of the Δ resonance. Thus our challenge consists in performing the leading one loop order calculation of the structure function $S_{1,2}(\nu, Q^2)$ in covariant small scale expansion. We are confident that the $\mathcal{O}(\epsilon^3)$ calculation presented in this work will throw light upon the role played by the Δ resonance within this sector of nuclear physics.

Besides the case of the first moments Γ_1, another very interesting topic is represented by the generalized spin polarizabilities γ_0 and δ_0. They are defined in such a way that they represent the ideal objects to test ChPT at low Q^2. Thus the need for their investigation towards a better understanding of the dynamics of QCD in the low energy domain of chiral EFT. From ChPT it turns out that γ_0 is sensitive to the nucleon resonances, while δ_0 is not much affected by the Δ resonance. Figure 3.2 shows the first results for the neutron spin polarizabilities obtained at JLAB Hall A [A$^+$02, M$^+$05], together with predictions from the next-to-leading one loop order ($\mathcal{O}(p^4)$) calculation in HBChPT [KSV03] and from the corresponding relativistic calculation of the already cited reference [BHM03b], with and without a first rough estimantion of the vector meson and Δ resonance contributions. We observe how at low Q^2 the relativistic calculation including the resonance contributions are in very good agreement with the experimental data, while the heavy baryon curve shows big discrepancies even at $Q^2 = 0.1\text{GeV}^2$. This can be explained by either a bad behaviour of the non-relativistic approximation or the significant role played by the resonances. A calculation of the same quantities with inclusion of the Δ as explicit degree of freedom will help us solve out the issue. Since the δ_0 polarizability is not influenced by the nucleon resonances, we could think that it is more suitable to the comparison to the experimental results. On the contrary, even at the lowest $Q^2 = 0.1\text{GeV}^2$ the ChPT predictions are in significant disagreement with the experimental data, thus the name of "δ_0 puzzle" to this problem of chiral EFT. The reason of this disagreement can be related to problems generated by the Infrared regularization (IR) scheme: as discussed in section 1.2, this scheme suffers from the deficiency of presenting unphysical cuts in the infrared regular part R of the loop integrals. This cuts first appears at values of Q^2 already outside the range of applicability of the chiral theory but they can influence in a significant way the behaviour of the curve at lower values of Q^2. A possible solution to the "δ_0 puzzle" could be the use of the modified infrared ($\overline{\text{IR}}$) instead of the infrared regularization (IR) scheme. Finally, the calculation of this quantity in the covariant SSE scheme will confirm or disprove the statement that the δ_0 is not sensitive to the Δ resonance.

Concluding, our aim is to present a leading one loop order calculation of the spin amplitude of the process of doubly virtual Compton scattering (VVCS) off nucleons in the SSE scheme of covariant Chiral Perturbation Theory.

The chapter is organized as follows: In the next section we firstly introduced the formalism needed to describe the process of doubly virtual Compton scattering, we derive the pertinent sum rules and we define the nucleon polarizabilities. In section 3.3 we explain how to extract the spin structure functions in ChPT, while section 3.4 is devoted to the description of the effective Lagrangians required for the calculation and to the application of the power-counting scheme for the identification of the Feynman diagrams contributing to the VVCS matrix element. Finally, section 3.5 collects the main results of our $\mathcal{O}(\epsilon^3)$ calculation. For a more detailed and complete analysis of the material portrayed in this work we refer to a later communication [BDKM].

3.2 Compton scattering and nucleon polarizabilities

In this section we provide an overview on spin dependent doubly virtual Compton scattering, focusing in particular on its connection to inelastic electroproduction and on the derivation of the corresponding sum rules.
We first discuss the forward scattering of a real photon by a nucleon and afterwards we generalize the formalism to the case of finite virtuality. For a more detailed introduction to the subject we refer to references [DPV03, DT04, CDM05]

3.2.1 Real Compton scattering

Working in the laboratory frame, we consider the photon scattering process

$$\gamma(q,\varepsilon) + N(p) \rightarrow \gamma(q',\varepsilon') + N(p') \tag{3.1}$$

on a nucleon with initial and final momentum p and p'. The incident photon has four-momentum $q = (\nu, \vec{q})$ and polarization four-vector $\varepsilon = (0, \vec{\varepsilon})$, with $q^2 = 0$ (real photon) and $\vec{\varepsilon} \cdot \vec{q} = 0$ (transerve polarization). The corresponding primed quantities characterize the kinematics of the scattered photon. For forward scattering with $\vec{q} = \vec{q}\,'$ the Compton amplitude reduces to

$$T(\nu,\, \theta = 0) = \vec{\varepsilon}\,' \cdot \vec{\varepsilon}\, f(\nu) + i\, \vec{\sigma} \cdot (\vec{\varepsilon}\,' \times \vec{\varepsilon})\, g(\nu), \tag{3.2}$$

where θ is the nucleon scattering angle and ν has been defined above and corresponds to the photon laboratory energy. The two functions $g(\nu)$,$f(\nu)$ are the spin-flip and non-flip amplitude, respectively. Because of crossing symmetry, the amplitude T of eq.(3.2) has to be invariant under exchange of the in- and outgoing photons, that is under the transformation $\varepsilon' \leftrightarrow \varepsilon$ and $\nu \leftrightarrow -\nu$. This implies that the two amplitudes $f(\nu)$ and $g(\nu)$ have opposite parity:

$$f(\nu) = f(-\nu)\ , \quad g(\nu) = -g(-\nu). \tag{3.3}$$

The amplitudes f and g can be separated by measurement of scattering processes of circularly polarized photons off nucleons polarized parallel or antiparallel with respect to the photon momentum. Let us take for example a right-handed photon (helicity $\lambda = 1$): one can have 1/2 or 3/2 helicity intermediate states, whose corresponding amplitudes are linear combination of f and g:

$$f_{\frac{1}{2}} = f + g \quad f_{\frac{3}{2}} = f - g. \tag{3.4}$$

Similarly, we define the total absorption cross section as the spin average over the two helicity cross sections,

$$\sigma_T = \frac{1}{2}\,(\sigma_{3/2} + \sigma_{1/2}), \tag{3.5}$$

and the transverse-transverse interference term by the helicity difference,

$$\sigma'_{TT} = \frac{1}{2}\,(\sigma_{3/2} - \sigma_{1/2}). \tag{3.6}$$

By the optical theorem, we are able to relate the just defined absorption cross sections to the imaginary part of the respective forward scattering amplitudes:

$$\begin{aligned}
\text{Im } f(\nu) &= \frac{\nu}{8\pi}(\sigma_{1/2}(\nu) + \sigma_{3/2}(\nu)) = \frac{\nu}{4\pi}\,\sigma_T(\nu), &(3.7)\\
\text{Im } g(\nu) &= \frac{\nu}{8\pi}(\sigma_{1/2}(\nu) - \sigma_{3/2}(\nu)) = -\frac{\nu}{4\pi}\,\sigma'_{TT}(\nu). &(3.8)
\end{aligned}$$

Unitarity, crossing symmetry and the analytic properties of the forward Compton scattering amplitude allow us to set up dispersion relations for $f(\nu)$ and $g(\nu)$. Replacing the imaginary part by the cross sections, as given in eq.(3.7) and (3.8), the dispersion integrals then read

$$\begin{aligned}
\text{Re } f(\nu) &= f(0) + \frac{\nu^2}{2\pi^2}\,\mathcal{P}\int_{\nu_0}^{\infty} \frac{\sigma_T(\nu')}{\nu'^2 - \nu^2}\,d\nu', &(3.9)\\
\text{Re } g(\nu) &= \frac{\nu}{4\pi^2}\,\mathcal{P}\int_{\nu_0}^{\infty} \frac{\sigma_{1/2}(\nu') - \sigma_{3/2}(\nu')}{\nu'^2 - \nu^2}\,\nu' d\nu', &(3.10)
\end{aligned}$$

where $\mathcal{P}$ denotes the principal value integral and ν_0 is the threshold energy for pion production. We point out that for the function $f(\nu)$ a subtraction at $\nu = 0$ has been made in order to ensure the convergence of the integral also in case of a divergent cross section. If we subtract at $\nu = 0$, i.e., we consider the difference $f(\nu) - f(0)$, we also remove the nucleon pole terms at $\nu = 0$. For the function $g(\nu)$ we are able to write an unsubtracted dispersion relation, since we assume that the difference of cross sections $\sigma_{1/2}(\nu') - \sigma_{3/2}(\nu')$ decreases sufficiently fast at large ν' to make the integral converge even without subtraction.
At this point we expand the relations of eq.(3.9) and (3.10) in powers of ν

$$\begin{aligned}
\text{Re } f(\nu) &= f(0) + \sum_{n=1}\left(\frac{1}{2\pi^2}\int_{\nu_0}^{\infty} \frac{\sigma_T(\nu')}{\nu'^{2n}}\,d\nu'\right)\nu^{2n}, &(3.11)\\
\text{Re } g(\nu) &= \sum_{n=1}\left(\frac{1}{4\pi^2}\int \frac{\sigma_{1/2}(\nu') - \sigma_{3/2}(\nu')}{(\nu')^{2n-1}}\,d\nu'\right)\nu^{2n-1}, &(3.12)
\end{aligned}$$

and we recall the historical low energy theorem (LET) of Low [Low54], Gell-Mann and Goldberger [GMG54]. According to this theorem the leading and next-to-leading terms of the expansion are determined by the global properties of the system:

$$\begin{aligned}
f(\nu) &= -\frac{e^2\,e_N^2}{4\pi M_0} + (\alpha + \beta)\,\nu^2 + \mathcal{O}(\nu^4), &(3.13)\\
g(\nu) &= -\frac{e^2\kappa_N^2}{8\pi M_0^2}\,\nu + \gamma_0\nu^3 + \mathcal{O}(\nu^5). &(3.14)
\end{aligned}$$

Here $e\,e_N$ is the charge ($e_p = 1$,$e_n = 0$), M_0 the nucleon mass and $(e/2M_0)\kappa_N$ the anomalous magnetic moment. Moreover, $f(0) = -\frac{e^2\,e_N^2}{4\pi M_0} = -\alpha_{QED}/M_0$ is the well-known Thomson limit and it indicates that, if the photon wavelength is very large with respect to any dimension of the proton, the scattering amplitude can only involve the fine structure constant α_{QED} and the proton mass. The spin-flip amplitude $g(\nu)$ vanishes at $\nu = 0$ and because of crossing symmetry $f(\nu)$ has no $\mathcal{O}(\nu)$ term. The terms of order ν^2 introduce the *electric* (α) and *magnetic* (β) *polarizabilities*; they yield information on the internal nucleon structure providing a measure of the global resistance of the nucleon's internal degrees of freedom against displacement in an external electric or magnetic field. The term $\mathcal{O}(\nu^3)$ provide information on the spin structure through the *forward spin polarizability* γ_0.
By matching eq.(3.11),(3.12) and the just discussed low energy expansions of eq.(3.13),(3.14) we can write a set of sum rules; in particular we obtain the so-called Baldin's sum rule for the electric and magnetic polarizabilities[Bal60]

$$\alpha + \beta = \frac{1}{2\pi^2} \int_{\nu_0}^{\infty} \frac{\sigma_T(\nu')}{\nu'^2}\, d\nu', \tag{3.15}$$

the sum rule of Gerasimov[Ger66], Drell and Hearn[DH66] (GDH sum rule)

$$\frac{\pi e^2 \kappa_N^2}{2M_0^2} = \int_{\nu_0}^{\infty} \frac{\sigma_{3/2}(\nu') - \sigma_{1/2}(\nu')}{\nu'}\, d\nu' \;\equiv I, \tag{3.16}$$

and the forward spin polarizability can be expressed via[GMG54]

$$\gamma_0 = -\frac{1}{4\pi^2} \int_{\nu_0}^{\infty} \frac{\sigma_{3/2}(\nu') - \sigma_{1/2}(\nu')}{\nu'^3}\, d\nu'. \tag{3.17}$$

Several experiment of Compton scattering off protons have been performed over the last decades (for example at MAMI, ELSA, GRAAL, CEBAF and Spring-8). A weighted average over various measurements give the following values for the static electric and magnetic polarizabilities of the proton[Sch05]:

$$\begin{aligned} \overline{\alpha}_p &= (12.0 \pm 0.6)\cdot 10^{-4}\,\text{fm}^3, \\ \overline{\beta}_p &= (1.9 \mp 0.6)\cdot 10^{-4}\,\text{fm}^3. \end{aligned} \tag{3.18}$$

Taking into account the typical size of a proton ($\sim 1\text{fm}^3$), these findings indicate that the protons are rather stiff objects.
Since stable single-nucleon targets do not exist, corresponding values for the neutron are much harder to access; a weighted average over several experiment gives[Sch05]:

$$\begin{aligned} \overline{\alpha}_n &= (12.5 \pm 1.7)\cdot 10^{-4}\,\text{fm}^3, \\ \overline{\beta}_n &= (2.7 \mp 1.8)\cdot 10^{-4}\,\text{fm}^3. \end{aligned} \tag{3.19}$$

On the theory side, reference [BKM92b] provides a calculation of the electric and magnetic nucleon polarizabilities at the leading one loop order in

HBChPT, while an extension to the next-to-leading order has been presented in references [BKSM93] and [BKMS94]. The authors of reference [HGHP04] have proposed a multipole analysis of the so called dynamical polarizabilities with inclusion of the $\Delta(1232)$ as explicit degree of freedom and have successfully compared their results to a dispersion relations analysis. A calculation of the spin polarizability in HBChPT can be found in reference [BKKM92], while an analysis in the SSE scheme has been presented in reference [HGHP05]. Finally, for a study in Chiral Effective Field Theory of Compton scattering from deuteron as an alternative way to extract the neutron polarizabilities we refer to [HGH04].

3.2.2 Doubly virtual Compton scattering

In analogy to the case of real Compton scattering, we now proceed to discuss the process of doubly virtual Compton scattering (VVCS); we derive the relative sum rules and we generalize the concept of polarizabilities to virtual photons. As already stated in the introduction of this chapter, of particular interest are the sum rules for the spin-dependent VVCS amplitudes, which will be the object of our chiral analysis.
Let us consider the following reaction:

$$\gamma^\star(q,\varepsilon) + N(p,s) \to \gamma^\star(q,\varepsilon') + N(p,s'), \tag{3.20}$$

describing a process of doubly virtual Compton scattering off nucleons in the forward direction. The virtual photon is now characterized by a negative photon virtuality, that is $q^2 = q_0^2 - \vec{q}^{\,2} = -Q^2 < 0$, and $s(s')$ defines the nucleon spin polarization four-vector of the incoming (outgoing) nucleon.
The absorption of a virtual photon on a nucleon is related to inelastic electroproduction, $e + N \to e' +$ anything, where e and e' are the electrons in the initial and final states, respectively. The differential cross section for this process can be written in terms of four virtual photoabsorption cross sections: $\sigma_T(\nu,Q^2)$, $\sigma_L(\nu,Q^2)$ of eq.(3.5) and (3.6), $\sigma'_L(\nu,Q^2)$ and $\sigma'_{LT}(\nu,Q^2)$. Because of the longitudinal polarization of the virtual photon, in addition to the cross sections introduced for the real Compton scattering in section 3.2.1 one has the longitudinal cross section σ'_L and a longitudinal-transverse interference σ'_{LT}.
The four virtual photoabsorption cross sections are related to the quark structures functions measured in (un)polarized deep inelastic electron scattering via[DKT01]

$$\sigma_T(\nu,Q^2) = \frac{4\pi^2\alpha_{QED}}{mK} F_1(\nu,Q^2), \tag{3.21}$$

$$\sigma_L(\nu,Q^2) = \frac{4\pi^2\alpha_{QED}}{K}\Big[\frac{F_2(\nu,Q^2)}{\nu}(1+\gamma^2) - \frac{F_1(\nu,Q^2)}{M_0}\Big], \tag{3.22}$$

$$\sigma'_{LT}(\nu,Q^2) = -\frac{4\pi^2\alpha_{QED}}{K}\,\gamma\,\nu\,\Big[(G_1(\nu,Q^2) + \frac{\nu}{M_0}G_2(\nu,Q^2))\Big], \tag{3.23}$$

$$\sigma'_{TT}(\nu,Q^2) = -\frac{4\pi^2\alpha_{QED}}{K}\,\nu\,\Big[(G_1(\nu,Q^2) - \frac{\nu}{M_0}\gamma^2 G_2(\nu,Q^2))\Big], \tag{3.24}$$

where $\nu = E - E'$ is the virtual photon energy in the lab frame, $\alpha_{QED} = e^2/4\pi = 1/137.036$ is the fine structure constant, $\gamma = Q/\nu$ and K is a flux factor.
The VVCS amplitude for forward scattering reads:

$$\begin{aligned} T(\nu,\, Q^2,\, \theta = 0) \;&=\; \vec{\varepsilon}\,' \cdot \vec{\varepsilon}\, f_T(\nu,\, Q^2) + f_L(\nu,\, Q^2) \\ &\quad + i\, \vec{\sigma} \cdot (\vec{\varepsilon}\,' \times \vec{\varepsilon})\, g_{TT}(\nu,\, Q^2) \\ &\quad - i\, \vec{\sigma} \cdot [(\vec{\varepsilon}\,' - \vec{\varepsilon}) \times \hat{q}\,]\, g_{LT}(\nu,\, Q^2). \end{aligned} \tag{3.25}$$

We note that in the limit $Q^2 = 0$ the amplitude reduces to the one of eq.(3.2) for the real scattering, with $f(\nu) = f_T(\nu, Q^2 = 0)$ and $g(\nu) = g_{TT}(\nu, Q^2 = 0)$, while the remaining two structures vanish because they are related to the longitudinal polarization of the photon.
We focus our attention on the spin dependent VVCS amplitudes $g_{TT}(\nu,\, Q^2)$, $g_{LT}(\nu,\, Q^2)$ and we choose to work on eq.(3.25) in its covariant form,

$$\begin{aligned} T^{[\mu\nu]}(p, q, s) \;&=\; -i\, \frac{M_0}{N}\, \epsilon^{\mu\nu\alpha\beta} q_\alpha \Big\{ s_\beta\, S_1(\nu, Q^2) \\ &\quad + [p \cdot q\, s_\beta - s \cdot q\, p_\beta]\, \frac{S_2(\nu, Q^2)}{M_0^2} \Big\}, \end{aligned} \tag{3.26}$$

where s^ν is a spin-polarization four-vector satisfying $s \cdot p = 0$, $s^2 = -1$ and $\epsilon^{\mu\nu\alpha\beta}$ is the totally antisymmetric Levi-Civita tensor with $\epsilon^{0123} = 1$. Again, $\nu = p \cdot q/M_0$ defines the energy transfer and $Q^2 = -q^2$ is the negative of the photon virtuality. The normalization factor N depends on the convention used. As suggested in references [BHM02, BHM03b] we set $S_1(0, 0) = -e^2\kappa_N^2/M_0^2$ and as usual in baryon ChPT we use the spinor normalization $\overline{u}u = 1$, so that $N = 2M_0$. We note that crossing symmetry ($\nu \to -\nu$) implies that $S_1(\nu, Q^2)$ and $S_2(\nu, Q^2)$ have opposite parity, even and odd respectively.
The optical theorem relates the imaginary parts of the spin amplitudes to the structure functions $G_i(\nu, Q^2)$ introduced in eq.(3.23),(3.24):

$$\text{Im}\; S_i(\nu, Q^2) = 2\pi\; G_i(\nu, Q^2), \quad (i = 1, 2)\;. \tag{3.27}$$

Assuming an appropriate behaviour at high energy, the functions $S_1(\nu, Q^2)$ and $S_2(\nu, Q^2)$ satisfy the following dispersion relations [IKL, JO01]:

$$\begin{aligned} S_1(\nu, Q^2) \;&=\; 4e^2 \int_{Q^2/2M_0}^{\infty} \frac{d\nu' \nu' G_1(\nu', Q^2)}{\nu'^2 - \nu^2}\;, \\ \nu S_2(\nu, Q^2) \;&=\; 4e^2 \int_{Q^2/2M_0}^{\infty} \frac{d\nu' \nu^2 G_2(\nu', Q^2)}{\nu'^2 - \nu^2}. \end{aligned} \tag{3.28}$$

Note that from now on we will refer to $S_i(\nu, Q^2)$ as to the real part of the relative functions.
Before we proceed with the derivation of the sum rules we have to make an important remark. As we will better see in section 3.5, the contribution of the

corresponding diagrams to the spin amplitudes can be separated into elastic and inelastic parts. The elastic part is given by the single nucleon exchange (pole) terms and does not contribute to the nucleon spin structure. In particular, the sum rules we are talking about involve uniquely the contributions from inelastic processes for $\nu > \nu_0$. In what follows we will therefore consider a subtracted version of the Compton amplitudes, i.e. we subtract the contribution from the elastic intermediate state:

$$\bar{S}_i(\nu, Q^2) = S_i(\nu, Q^2) - S_i^{\rm el}(\nu, Q^2). \tag{3.29}$$

We next perform a low energy expansion around $\nu = 0$ for the subtracted structure functions [JO01, BHM03b],

$$\begin{aligned} \bar{S}_1(\nu, Q^2) &= \bar{S}_1^{(0)}(0, Q^2) + \nu^2\, \bar{S}_1^{(2)}(0, Q^2) + \nu^4\, \bar{S}_1^{(4)}(0, Q^2) + \dots\ , \\ \nu\bar{S}_2(\nu, Q^2) &= \bar{S}_2^{(-1)}(0, Q^2) + \nu^2\, \bar{S}_2^{(1)}(0, Q^2) + \nu^4\, \bar{S}_2^{(3)}(0, Q^2) \\ &\quad + \nu^5\, \bar{S}_2^{(5)}(0, Q^2) + \dots, \end{aligned} \tag{3.30}$$

where the ellipses indicate terms of higher order in ν. Let us first concentrate on the expansion of the structure function $\overline{S}_1(\nu, Q^2)$; recalling eq.(3.28) one gets the following sum rules for $\overline{S}_1(0, Q^2)$:

$$\bar{S}_1^{(0)}(0, Q^2) = 4e^2 \int_{\nu_0}^{\infty} \frac{d\nu' G_1(\nu', Q^2)}{\nu'} = \frac{4e^2}{M_0^2} I_1(Q^2)\ , \tag{3.31}$$

$$\bar{S}_1^{(2i)}(0, Q^2) = 4e^2 \int_{\nu_0}^{\infty} \frac{d\nu' G_1(\nu', Q^2)}{\nu'^{(2i+1)}} \quad (i = 1, 2, 3, \dots), \tag{3.32}$$

where

$$I_1(Q^2) = \frac{2M_0^2}{Q^2} \int_0^{x_0} g_1(x, Q^2) dx \tag{3.33}$$

is the generalization of the GDH sum rule to finite Q^2[AIL89]. Here, $x = Q^2/2\nu M_0$ is the standard scaling variable, x_0 corresponds to the pion production threshold and the spin structure function g_1 is related to G_1 via $G_1 = g_1/(M_0\nu)$. At zero virtuality $I_1(0) = -\kappa_N^2/4$ and one recovers the GDH sum rule of eq.(3.16), while in the limiting case $Q^2 \to \infty$ the often used first moment $\Gamma_1(Q^2) \equiv \int_0^1 dx\, g_1(x, Q^2)$ appears:

$$I_1(Q^2) \to \begin{cases} -\frac{1}{4}\kappa_N^2 & \text{for} \quad Q^2 \to 0, \\ \frac{2M_0^2}{Q^2}\Gamma_1 + \mathcal{O}(Q^{-4}) & \text{for} \quad Q^2 \to \infty. \end{cases} \tag{3.34}$$

From the second line of eq.(3.30) we obtain the sum rules for the second spin dependent amplitude $\overline{S}_2(0, Q^2)$:

$$\bar{S}_2^{(-1)}(0, Q^2) = 4e^2 \int_{\nu_0}^{\infty} d\nu' G_2(\nu', Q^2) = \frac{4}{M_0^2} I_2(Q^2)\ , \tag{3.35}$$

$$\bar{S}_2^{(i)}(0, Q^2) = 4e^2 \int_{\nu_0}^{\infty} \frac{d\nu' G_2(\nu', Q^2)}{\nu'^{(i+1)}} \quad (i = 1, 3, 5, \dots), \tag{3.36}$$

where $I_2(Q^2)$ is defined by the sum rule for the spin structure function g_2, known as Burkhardt-Cottingham sum rule[BC70]:

$$\begin{aligned} I_2(Q^2) &= \frac{2M_0^2}{Q^2}\int_0^{x_0} g_2(x,Q^2)dx && (3.37)\\ &= \frac{1}{4}\frac{G_M(Q^2)-G_E(Q^2)}{(1+\frac{Q^2}{4M_0^2})}G_M(Q^2) \\ &= \frac{1}{4}F_2(Q^2)\left(F_1(Q^2)+F_2(Q^2)\right). && (3.38)\end{aligned}$$

Here, $G_E(Q^2)$ and $G_M(Q^2)$ are the electric and magnetic nucleon Sachs form factors, in order, while $F_1(Q^2)$ and $F_2(Q^2)$ are the Dirac and Pauli nucleon form factors already encountered in the introduction of chapter 2. Moreover, $G_2 = g_2/\nu^2$.

The Burkhardt-Cottingham sum rule has the limiting cases

$$I_2(Q^2) \rightarrow \begin{cases} \frac{1}{4}\mu_N\kappa_N & \text{for} \quad Q^2 \rightarrow 0, \\ \mathcal{O}(Q^{-10}) & \text{for} \quad Q^2 \rightarrow \infty. \end{cases} \tag{3.39}$$

In addition to the GDH and the BC sum rules, the following sum rules have been introduced in the past year[DKT01]:

$$\begin{aligned} I_A(Q^2) &= \frac{M_0^2}{8\pi^2\alpha}\int_{\nu_0}^{\infty}(1-x)\,(\sigma_{1/2}-\sigma_{3/2})\frac{d\nu}{\nu} \\ &= \frac{2M_0^2}{Q^2}\int_0^{x_0}(g_1-\gamma^2\,g_2)\,dx\,, && (3.40)\\ &= \frac{2M_0^2}{Q^2}\int_0^{x_0}\frac{1}{\sqrt{1+\gamma^2}}\,(g_1-\gamma^2\,g_2)\,dx\,, && (3.41)\\ I_C(Q^2) &= \frac{M_0^2}{8\pi^2\alpha}\int_{\nu_0}^{\infty}(\sigma_{1/2}-\sigma_{3/2})\frac{d\nu}{\nu} \\ &= \frac{2M_0^2}{Q^2}\int_0^{x_0}\frac{1}{1-x}\,(g_1-\gamma^2\,g_2)\,dx. && (3.42)\end{aligned}$$

While $I_A(Q^2)$ can be easily expressed in terms of $\overline{S}_i(\nu,Q^2)$ via

$$I_A(Q^2) = \frac{M_0^2}{4}\left(\bar{S}_1^{(0)}(0,Q^2) - Q^2\bar{S}_2^{(1)}(0,Q^2)\right), \tag{3.43}$$

in order to calculate $I_C(Q^2)$ we have to introduce the new quantity $\overline{S}_R(\nu,Q^2)$ defined as:

$$\bar{S}_R(\nu,Q^2) = 4e^2\int_{\nu_0}^{\infty} d\nu'\frac{G_1(\nu',Q^2)-Q^2G_2(\nu',Q^2)/\nu}{\nu'-\nu}. \tag{3.44}$$

This leads to the relation

$$I_C(Q^2) = \frac{M_0^2}{4}\bar{S}_R(Q^2/2M_0, Q^2). \tag{3.45}$$

We now turn to the *forward spin polarizability* defined in eq.(3.17) and we write it as

$$\begin{aligned}\gamma_0 &= \frac{1}{4\pi^2}\int \frac{d\nu}{\nu^3}(x-1)\sigma'_{TT} \\ &= \frac{1}{4\pi^2}\int \frac{d\nu}{\nu^3}(1-x)(\sigma_{1/2}(\nu,0)-\sigma_{3/2}(\nu,0)).\end{aligned} \tag{3.46}$$

We observe that in case of zero virtuality, i.e. $Q^2 = 0$, we have that x vanishes and we go back to the dispersion integral of eq.(3.17).
Finally, considering the longitudinal-transverse interference cross section σ_{LT} we can define the so called *longitudinal-transverse spin polarizability* δ_0:

$$\delta_0 = -\frac{1}{2\pi^2}\int \frac{d\nu}{\nu^3}\lim_{Q^2\to 0}\left(\frac{\nu}{Q}\sigma'_{LT}\right), \tag{3.47}$$

We note that the definitions of the spin polarizabilities present an extra $1/\nu^2$ weighting compared to the GDH and BC sum rules. As a consequence the integrals defining the generalized spin polarizabilities receive less contributions from the large-ν region and converge much faster, representing the most suitable object to test ChPT in the low energy domain.
The generalization to finite Q^2 can be performed also for these quantities, yielding to the following relations between $\gamma_0(Q^2)$, $\delta_0(Q^2)$ and the $\overline{S}_i(\nu, Q^2)$:

$$\gamma_0(Q^2) = \frac{1}{8\pi}\left(\bar{S}_1^{(2)}(0,Q^2) - \frac{Q^2}{M_0}\bar{S}_2^{(3)}(0,Q^2)\right), \tag{3.48}$$

$$\delta_0(Q^2) = \frac{1}{8\pi}\left((\bar{S}_1^{(2)}(0,Q^2) + \bar{S}_2^{(1)}(0,Q^2)\right). \tag{3.49}$$

3.3 The spin polarizabilities in covariant ChPT

We now fix the most suitable kinematical and gauge conditions for the evaluation of the VVCS forward matrix element of eq.(3.26). Working in the rest frame of the nucleon, i.e. $p = (M_0, \vec{0})$ and adopting the Coulomb gauge condition $\varepsilon_0 = \varepsilon'_0 = 0$, the matrix element $T^{[\mu\nu]}$ takes the form:

$$\begin{aligned}T^{[ij]}_{\mathrm{rest}}(\nu,Q^2) &= \frac{i}{2}\,\epsilon_{ijk}\chi^\dagger\Big\{\nu\,\sigma^k S_1(\nu,Q^2) \\ &\qquad + \left[\nu^2\sigma^k - \sigma\cdot q\,q^k\right]\frac{S_2(\nu,Q^2)}{M_0}\Big\}\chi,\end{aligned} \tag{3.50}$$

where ν now corresponds to the photon energy in the laboratory system, $\sigma_k(k=1,2,3)$ denote the Pauli spin matrices and χ is a two component spinor. Making use of the following identity

$$\begin{aligned}\vec{\sigma}\cdot\vec{q}\,\vec{\varepsilon}'\cdot(\vec{\varepsilon}\times\vec{q}) &= \left(\nu^2+Q^2\right)[\vec{\sigma}\cdot(\vec{\varepsilon}'\times\vec{\varepsilon}) \\ &\quad -(\vec{\sigma}\cdot(\vec{\varepsilon}'\times\hat{q})\,\vec{\varepsilon}\cdot\hat{q}-\vec{\sigma}\cdot(\vec{\varepsilon}\times\hat{q})\,\vec{\varepsilon}'\cdot\hat{q})] \qquad (3.51)\end{aligned}$$

we obtain

$$\begin{aligned}\varepsilon'\cdot T\cdot\varepsilon|_{\text{rest}} &= \frac{1}{2M_0}\chi^\dagger\left\{i\vec{\sigma}\cdot(\vec{\varepsilon}'\times\vec{\varepsilon})\left[M_0\nu S_1(\nu,Q^2)-Q^2S_2(\nu,Q^2)\right]\right. \\ &\quad \left.+i\left[\vec{\sigma}\cdot(\vec{\varepsilon}'\times\hat{q})\,\vec{\varepsilon}\cdot\hat{q}-\vec{\sigma}\cdot(\vec{\varepsilon}\times\hat{q})\,\vec{\varepsilon}'\cdot\hat{q}\right]\left(\nu^2+Q^2\right)S_2(\nu,Q^2)\right\}\chi. \\ &\qquad (3.52)\end{aligned}$$

The VVCS forward matrix element is now written in the most convenient way for the calculation in the framework of relativistic Baryon Chiral Perturbation Theory.
We recall that the amplitude T has to fulfill crossing symmetry. Calling $s=(p+q)^2$ the standard Mandelstam variable, one can express the photon energy by $\nu=(s-M_0^2+Q^2)/2M_0$. Therefore, for each diagram, the corresponding crossed one can be derived very easily by operating the transformations $\varepsilon\leftrightarrow\varepsilon'$ and $s\to(2M_0^2-2Q^2-s)$. These considerations enable us to write eq.(3.52) via two invariant functions $A(\nu,Q^2)$ and $B(\nu,Q^2)$:

$$\begin{aligned}\varepsilon'\cdot T\cdot\varepsilon|_{\text{rest}} &= \chi^\dagger\left\{i\vec{\sigma}\cdot(\vec{\varepsilon}'\times\vec{\varepsilon})\left[A(s,Q^2)-A(2M_0^2-2Q^2-s,Q^2)\right]\right. \\ &+ i\left[\vec{\sigma}\cdot(\vec{\varepsilon}'\times\hat{q})\,\vec{\varepsilon}\cdot\hat{q}-\vec{\sigma}\cdot(\vec{\varepsilon}\times\hat{q})\,\vec{\varepsilon}'\cdot\hat{q}\right]|\vec{q}|^2\left[B(s,Q^2)\right. \\ &\quad \left.\left.-B(2M_0^2-2Q^2-s,Q^2)\right]\right\}\chi. \\ &\qquad (3.53)\end{aligned}$$

In this expression both contributions from a diagram and its crossed partner are therefore taken into account. As we have seen in section 3.2.2, we are mainly interested in quantities evaluated at $\nu=0$, i.e. at $s=M_0^2+q^2=M_0^2-Q^2$. Let us for example concentrate on the term proportional to $i\vec{\sigma}\cdot(\vec{\varepsilon}'\times\vec{\varepsilon})$ in eq.(3.53): as required by crossing symmetry, if we write the equation in terms of ν the quantity $[A(M_0^2-Q^2+2M_0\nu,Q^2)-A(M_0^2-Q^2-2M_0\nu,Q^2)]$ appear, that is the difference of a quantity taken at ν with the same quantity taken at $-\nu$. As a consequence, comparing eq.(3.53) with eq.(3.52) we deduce that the $\overline{S}_i(0,Q^2)$ we need in order to evaluate the sum rules can be simply obtained as derivatives of order $n\geq 1$ of the functions A and B.

For example:

$$\bar{S}_1(0,Q^2) - \frac{Q^2}{M_0}\lim_{\nu\to 0}\frac{\underline{\bar{S}}_2(0,Q^2)}{\nu} = 4\left.\frac{dA(\nu,Q^2)}{d\nu}\right|_{\nu=0}, \tag{3.54}$$

$$Q^2\lim_{\nu\to 0}\frac{\underline{\bar{S}}_2(0,Q^2)}{\nu} = 4\left.\frac{dB(\nu,Q^2)}{d\nu}\right|_{\nu=0}, \tag{3.55}$$

where $\nu\underline{\bar{S}}_2(\nu,Q^2) \equiv \nu\bar{S}_2(\nu,Q^2) - \nu\bar{S}_2(\nu,Q^2)\big|_{\nu=0}$ and we recall that the contribution from the elastic intermediate state is subtracted, i.e. $\bar{S}_i(\nu,Q^2) \equiv S_i(\nu,Q^2) - S_i^{\mathrm{el}}(\nu,Q^2)$.

3.4 Effective Field Theory with Δ's.

In section 1.3 we have summarized the basic concepts of a Chiral Effective Field Theory with inclusion of the Δ resonance as explicit degree of freedom. Here we give the Lagrangians we need in order to perform a leading one loop order calculation of the Compton amplitudes within the covariant framework of SSE.
The full relativistic Lagrangian we require for a system of nucleon, Δ and pion degrees of freedom consists of four terms:

$$\mathcal{L}^{\mathrm{eff}} = \mathcal{L}_{\pi N} + \mathcal{L}_{\pi\pi} + \mathcal{L}_{\pi N\Delta} + \mathcal{L}_{\pi\Delta}. \tag{3.56}$$

Concerning the first two pieces of Lagrangians listed, the ones relevant for our study are the well known $\mathcal{L}^{(1)}_{\pi N}$ of eq.(1.7) and the meson $\mathcal{L}^{(2)}_{\pi\pi}$ of eq.(1.2). These terms are responsible for the interaction of a photon with nucleons and pions, respectively.
The leading chiral pion-Δ-nucleon Lagrangian is given by [HHK98]:

$$\mathcal{L}^{(1)}_{\pi N\Delta} = g_{\pi N\Delta}\,\overline{\psi}^{\mu}_i\,\Theta^{\mu}_{\nu}(Z)\,w^{\nu}_i\,\psi_N + \mathrm{h.c.}, \tag{3.57}$$

with

$$w^{\nu}_i = \langle\tau^i u_\nu\rangle/2, \tag{3.58}$$

$$\Theta_{\mu\nu}(Z) = g_{\mu\nu} + (Z-1/2)\gamma_\mu\gamma_\nu, \tag{3.59}$$

where τ^i,$i=1,2,3$ denote as usual the Pauli matrices in isospin space and Z is the so-called off-shell parameter. Since this parameter does not appear in the spin 3/2 contributions, the physical observables we are interested in do not depend on it. In our calculation we choose to put $Z=1/2$. Here, the LEC $g_{\pi N\Delta}$ represents the leading axial-vector $N\Delta$ coupling constant, frequently called c_A in the literature. We recall that ψ^{μ}_i denotes the spin-3/2 field in the Rarita-Schwinger notation.
The coupling of the Δ resonance to the photon is described through the following Lagrangian[HHK98],

$$\mathcal{L}^{(1)}_{\pi,\Delta} = -\overline{\psi}^{\mu}_{i}\left[\left(i\not{D}^{ij} - m^0_\Delta\delta^{ij}\right)g_{\mu\nu} - \frac{1}{4}\gamma_\mu\gamma^\lambda\left(i\not{D}^{ij} - m^0_\Delta\delta^{ij}\right)\gamma_\lambda\gamma_\nu\right]\psi^\nu_j, \quad (3.60)$$

where we have listed only the terms pertinent to the calculation. Here, the minimally coupled covariant derivative reads:

$$D^{ij}_\mu = D_\mu\delta^{ij} - i\epsilon^{ijk}\mathrm{Tr}[\tau^k D_\mu], \quad (3.61)$$

with D_μ the standard derivative defined in eq.(1.5), whereas m^0_Δ refers to the $\Delta(1232)$ mass in the chiral limit.
Finally, we introduce the relativistic counterterm Lagrangian[HHK97]

$$\mathcal{L}^{(2)}_{\gamma N\Delta} = i\frac{b_1}{2M_N}\,\overline{\psi}^{\mu}_{i}\,g_{\mu\nu}\gamma_\rho\gamma_5\,\frac{1}{2}\mathrm{Tr}[f^{\rho\nu}_{+}]\,\psi_N + \text{h.c.}, \quad (3.62)$$

which defines the $\gamma N\Delta$ transition at order ϵ^2 and will be employed for the evaluation of the Born graphs contributing to the process at order $\mathcal{O}(\epsilon^3)$. For the numerical estimate of the magnitude of the finite $\mathcal{O}(\epsilon^2)$ counterterm b_1 we follow reference [HHK97], where the authors employ the phenomenological findings of reference [DMW91] to get $b_1 = -(2.5 \pm 0.35)$. The chiral tensor field appearing above is defined as[BKM95]

$$f^{+}_{\mu\nu} = u^\dagger F^R_{\mu\nu}u \pm uF^L_{\mu\nu}u^\dagger,$$

with

$$F^{L,R}_{\mu\nu} = \partial_\mu F^{L,R}_\nu - \partial_\nu F^{L,R}_\mu - i[F^{L,R}_\mu, F^{L,R}_\nu]$$

and $F^L_\mu = v_\mu - a_\mu$, $F^R_\mu = v_\mu + a_\mu$, where v_μ and a_μ denote external vector and axial vector sources, in the order.

We are now provided with the building blocks of the theory underlying the process of VVCS at the one-loop level in relativistic SSE. Finally, we remind that our choice for the Δ propagator corresponds to the following general form in the Rarita-Schwinger formalism (see section 1.3 for further details):

$$G^{ij}_{\mu\nu}(p) = -i\frac{\not{p} + m_\Delta}{p^2 - m^2_\Delta}\left\{g_{\mu\nu} - \frac{1}{d-1}\gamma_\mu\gamma_\nu - \frac{(d-2)}{(d-1)}\frac{p_\mu p_\nu}{m^2_\Delta} + \frac{p_\mu\gamma_\nu - p_\nu\gamma_\mu}{(d-1)m_\Delta}\right\}\xi^{ij}_{3/2}. \quad (3.63)$$

3.4.1 Power-counting

In this section we draw the Feynman diagrams contributing to the VVCS process up to the leading one loop order. Similarly to the study of the generalized

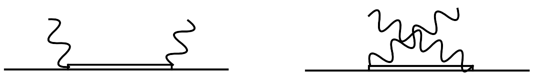

Figure 3.3: Born graphs at order $\mathcal{O}(\varepsilon^3)$.

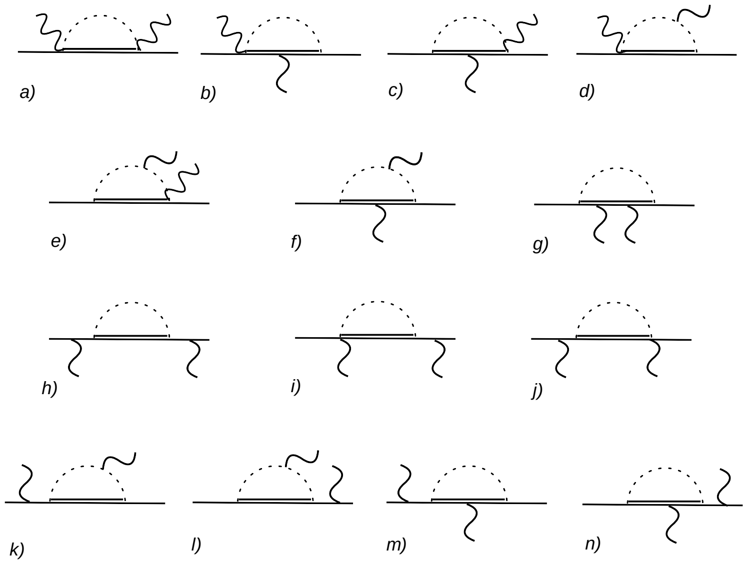

Figure 3.4: Loop diagrams contributing to the spin structure functions at third order. Solid, dashed and wiggly lines denote nucleons, pions and photons, respectively. A double solid line denotes the Δ resonance. Crossed diagrams are not shown.

form factors (see section 2.3.3), we start from the general power-counting formula of Baryon ChPT:

$$D = 2N_L + 1 + \sum_d (d-2) N_d^M + \sum_d (d-1) N_d^{MB}. \qquad (3.64)$$

Now the variable N_d^{MB} counts the number of vertices of chiral dimension d from pion-nucleons but also from pion-Δ and pion-nucleon-Δ Lagrangians. The chiral order $D = 1$ is given by a zero number of loops $N_L = 0$ and an arbitrary number of insertions from the leading meson and meson-baryon Lagrangian. Since the leading $\gamma N \Delta$ coupling is of chiral dimension $d = 2$ (its generated by the $\mathcal{L}^{(2)}_{\gamma N \Delta}$ of eq.(3.62)), there is no $D = 1$ diagram contribution to the spin amplitudes.

The first contribution with $N_L = 0$ occurs at the chiral order $D = 3$ and it corresponds to the Born graph depicted in figure 3.3, where the crossed partner is also shown. We observe that no two-photon seagull term contributes to the spin-dependent structure at this order. Indeed, in order to construct a two-photon observable, one needs two factors of the electromagnetic field strength tensor $F_{\mu\nu}$, that is a term of fourth order. Since such terms always appear together with structures of the type $\epsilon' \cdot \epsilon$, i.e. they just contribute to

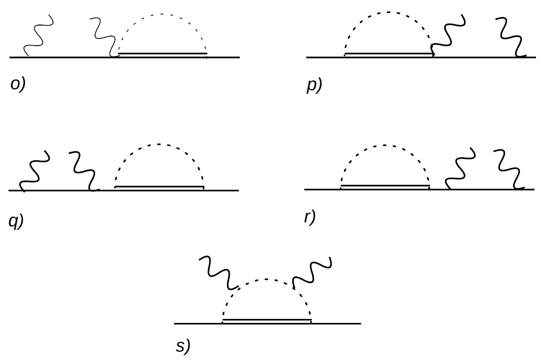

Figure 3.5: Loop diagrams giving a zero contribution to the inelastic structure of the spin dependent VVCS amplitudes.

the spin-independent amplitudes, the terms of our interest will firstly appear at the fifth order.
In addition to the Born graphs, at the chiral order $D = 3$ starts the $N_L = 1$ topology. One has the two possibilities $N_1^{MB} = 2$,$N_2^M = 0$ and $N_1^{MB} = 1$,$N_2^M = 1$, corresponding to the diagrams of figures 3.4 and 3.5. Only the diagrams depicted in figure 3.4 actually give non zero contribution to the spin amplitudes. The diagrams o),p),q),r) do provide no inelastic structure: after subtraction of the elastic part they reduce to constant terms, which consequently vanish once the corresponding crossed diagrams are added. Finally, diagrams s) provides neither elastic nor inelastic contributions, it simply does not contribute to the spin dependent amplitude of the process. Explicit expressions for the diagrams in terms of the pertinent loop functions will be discussed in the next section.
The Feynman rules pertinent to the calculation can be found in appendix B.

As already stated in the introduction of this chapter, a very important feature of this calculation is the possibility to make parameter free predictions. For the observables considered here no contributions from $\mathcal{L}_{\pi N}^{(3)}$ exist, so that no LECs will appear in the results for the structure functions $\overline{S}_{1,2}(\nu, Q^2)$, where again the bar reminds us of the subtraction of the elastic intermediate states.

3.5 Covariant $\mathcal{O}(\epsilon^3)$ results for the VVCS amplitudes.

We apply the methods of Chiral EFT in the scheme of small scale expansion to derive the leading one loop order contributions to the functions $A(s,Q^2)$ and $B(s,Q^2)$ as defined in eq.(3.53). Strictly speaking, we make use of the Feynman rules listed in appendix B to calculate the diagrams of chiral order $D = 3$, according to the power-counting scheme presented in section 3.4.1. As already mentioned in the last section, the only tree level contributions are given by the Born graphs of fig.3.3, which have to be summed up to the loop diagrams of figs.3.4,3.5.
We are interested exclusively in the inelastic contributions and we therefore subtract the terms from the elastic intermediate states. In practise, this implies the subtraction of the contributions from the single nucleon exchange, the nucleon poles of the kind $1/(s - M_0^2)$ generated by the nucleon propagators which appear in the diagrams.
In what follows we provide the expressions for $A(s,Q^2)$ and $B(s,Q^2)$ after subtraction of the nucleon pole terms.

We first start our discussion with the Born graphs, the sum of the both (direct plus crossed terms) is given by:

$$
\begin{aligned}
A(s,Q^2) &= \left(\frac{e\,b_1}{2M_0}\right)^2 \frac{1}{(s-m_\Delta^2)} \\
&\quad \left\{\frac{M_0^2-Q^2-s}{6M_0 m_\Delta^2}\left[M_0^4+s^2+5Q^4+Q^2(6m_\Delta^2+2s)\right.\right. \\
&\quad \left.\left. -2M_0^2(5Q^2+2s)\right]\right\}
\end{aligned} \tag{3.65}
$$

$$
B(s,Q^2) = \left(\frac{e\,b_1}{2M_0}\right)^2 \frac{1}{(s-m_\Delta^2)}\left\{-\frac{2M_0-m_\Delta}{3m_\Delta^2}(M_0^2-Q^2-s)\right\}. \tag{3.66}
$$

Unlike the case with the nucleon as only baryonic degree of freedom [BHM03b], the Born graphs now obviously show no elastic contributions, so that the whole amplitude survives the subtraction from the elastic intermediate states. We recall that the value of the LEC b_1 coming from the Lagrangian $L^{(2)}_{\gamma N\Delta}$ of eq.(3.62) can be fixed by fitting resonant multipoles and its values is about $-(2.5 \pm 0.35)$.

We now proceed with the analysis of the diagrams which give a non-zero contribution to the spin amplitudes. The crossed partners are easily obtained by taking the negative of the direct ones once the substitution $s \to (2M_0^2 - 2Q^2 - s)$ has been made. In particular, in this section we provide the expressions for the diagrams involving only one Δ propagator, while the more complicated ones, involving two or three Δ propagators are reported in appendix B.3 for practical reasons. Those longer and more difficult diagrams are calculated in an automatic way by using Form and Mathematica. The program, realized by Hermann Krebs, provides results in terms of the general loop

function $\mathrm{II}[d+n][\{\{p_1, m_1\}, k_1\}, \ldots \{\{p_N, m_N\}, k_N\}]$ of eq.(B.10). As shown in reference [Dav91], such integrals can be rewritten by reduction in terms of the scalar loop functions $\{J_0,\gamma_0,\Gamma_0\}$ defined in appendix B.2.
The following results still show the dependence on the dimension d and are obtained assuming we make use of either the infrared (IR) or the modified-infrared ($\overline{\mathrm{IR}}$) regularization scheme. This means that when the graphs exclusively involve nucleon or Δ propagators, no infrared singularities appear and the integrals can be set equal to zero. The choice of the regularization procedure will be of crucial importance for the analysis of the results, in particular to check the vanishing of the divergences, essential requirement of the calculation at the order we are working. The expression of the scalar loop functions appearing in the results will therefore depend on the scheme of regularization adopted.
The results are given in terms of the two variables s and Q^2, where $s = (p+q)^2 = 2M_0\nu + M_0^2 - Q^2$ is the standard Mandelstam variable and $Q^2 = -q^2 \geq 0$ is the negative of the photon virtuality. In order to extract the spin structure functions $\overline{S}_{1,2}(0, Q^2)$ we will have to make a change of variable and to operate via derivative with respect to the energy transfer ν, as shown in eq.(3.55). An expansion around $\nu = 0$ will then provide us with the expressions for the $\overline{S}^{(i)}_{1,2}(0, Q^2)$, allowing for numerical estimations for the sum rules and the spin polarizabilities.
Diagram a) is the simplest diagram contributing to the spin structures, it is purely inelastic and gives the following amplitudes:

$$
\begin{aligned}
A(s,Q^2) &= \frac{1}{3(d-1)^2 s\, m_\Delta M_0} \left(\frac{e\, g_{\pi N\Delta}}{F_\pi}\right)^2 \\
&\times\Big\{ \left[-2(s+m_\pi^2)M_0 + 2m_\Delta^2 M_0 + (d-1)m_\Delta(Q^2+s+M_0^2)\right] \Delta_\pi \\
&- \left[2\left(m_\pi^4 + (s-m_\Delta^2)^2 - 2m_\pi^2(s+m_\Delta^2)\right) M_0 \right. \\
&-(d-1)m_\Delta(s+m_\pi^2-m_\Delta^2)(Q^2+s+M_0^2) \\
&\left. +2(d-1)s\, m_\Delta(Q^2+s+2m_\Delta M_0+M_0^2)\right] J_0[s] \Big\},
\end{aligned} \tag{3.67}
$$

$$
\begin{aligned}
B(s,Q^2) &= -\frac{1}{3(d-1)^2 s^2\, m_\Delta} \left(\frac{e\, g_{\pi N\Delta}}{F_\pi}\right)^2 \\
&\times\Big\{ \left[d(-m_\pi^2+m_\Delta^2+3s) - 4s\right] \Delta_\pi \\
&+ \left[-4s\, m_\Delta^4 + d(-m_\pi^2+m_\Delta^2+s)^2\right] J_0[s] \Big\}.
\end{aligned} \tag{3.68}
$$

Diagram h) contributes exclusively to the structure $A(s, Q^2)$ and it is characterized by two nucleon poles. The subtraction of the elastic intermediate states is made by subtracting the power series expansion for the function about $s = M_0^2$ up to the power $(s - M_0^2)^{-1}$. Besides the scalar two-point function $J_0[s]$, the result thus presents also the scalar loop function evaluated at $s = M_0^2$ and the function $J_0'[M_0^2]$, where the prime denotes differentiation with respect to s. One has:

$$
\begin{aligned}
A(s,Q^2) &= \frac{(d-2)}{16(d-1)F_\pi^2 m_\Delta^2 M_0}\left(\frac{e\,g_{\pi N\Delta}}{F_\pi}\right)^2(\tau_3+1) \qquad (3.69)\\
&\times\Bigg\{-\frac{1}{d\,sM_0^2}\left[4m_\pi^2 M_0^2 s + d\left(m_\pi^4(M_0^2+Q^2) - M_0^2 s(Q^2+s)\right.\right.\\
&+(M_0^2+Q^2)m_\Delta^4 + 2m_\pi^2(2M_0^2 s - (M_0^2+Q^2)m_\Delta^2)\big)\big]\,\Delta_\pi\\
&+\frac{Q^2}{M_0^2(M_0^2-s)^2}(-m_\pi^2+(M_0+m_\Delta)^2)\left[m_\pi^4(s-3M_0^2)\right.\\
&+(M_0-m_\Delta)(M_0+m_\Delta)(3M_0^4 - 2M_0^2 m_\Delta + 2M_0 s\,m_\Delta - s\,m_\Delta^2\\
&+M_0^2(-5s+3m_\Delta^2)) + m_\pi^2(-2M_0^3 m_\Delta + 2M_0 s\,m_\Delta - 2s\,m_\Delta^2\\
&+M_0^2(4s+6m_\Delta^2))\big]\,J_0[M_0^2]\\
&+\frac{1}{s(M_0^2-s^2)^2}\left[\left((s-m_\pi^2)(M_0^4+M_0^2(Q^2-2s)+s(Q^2+s))\right.\right.\\
&+4M_0Q^2 s\,m_\Delta + (M_0^4+M_0^2(Q^2-2s)+s(Q^2+s))m_\Delta^2\big)\left(m_\pi^4\right.\\
&+(s-m_\Delta^2)^2 - 2m_\pi^2(s+m_\Delta^2)\big]\,J_0[s]\\
&+\frac{2Q^2}{(s-M_0^2)}\left((M_0-m_\Delta)^2-m_\pi^2\right)\left(m_\pi^2-(M_0+m_\Delta)^2\right)^2 J_0'[M_0^2]\Bigg\},\\
B(s,Q^2) &= 0. \qquad (3.70)
\end{aligned}
$$

Diagram i)+j). Whenever appropriate, the contributions from different diagrams are added together in order to reproduce the proper structures appearing in eq.(3.53). The calculation of these two diagrams give:

$$
\begin{aligned}
A(s,Q^2) &= \frac{(2-d)}{24(d-1)^2 m_\Delta^2 M_0}\left(\frac{e\,g_{\pi N\Delta}}{F_\pi}\right)^2(\tau_3+1) \qquad (3.71)\\
&\times\Bigg\{\frac{1}{d\,sM_0^2}\left[d\,Q^2(m_\pi^2-m_\Delta^2)^2 + 2d\,Q^2 m_\Delta(-m_\pi^2+m_\Delta^2)M_0\right.\\
&+\left(d\,s(Q^2+s) - d(m_\pi^4+m_\Delta^2) + m_\pi^2(-4(d-1)s+2d\,m_\Delta^2)\right)M_0^2\\
&-2d\,m_\Delta(s+m_\pi^2-m_\Delta^2)M_0^3\big]\,\Delta_\pi\\
&+\left[\frac{Q^2((m_\Delta-M_0)^2-m_\pi^2)(m_\pi^2-(m_\Delta+M_0)^2)^2}{M_0^2(M_0^2-s^2)}\right]J_0[M_0^2]\\
&+\frac{1}{s(M_0^2-s)}\left[\left(m_\pi^4+(s-m_\Delta^2)^2-2m_\pi^2(s+m_\Delta^2)\right)\left((s-m_\pi^2)\right.\right.\\
&(Q^2+s-M_0^2) + m_\Delta M_0(Q^2-s-M_0^2)\big)\big]\,J_0[s]\Bigg\},
\end{aligned}
$$

$$
\begin{aligned}
B(s,Q^2) &= \frac{(d-2)}{24(d-1)^2 s^2\, m_\Delta^2 M_0^3}\left(\frac{e\, g_{\pi N\Delta}}{F_\pi}\right)^2 (\tau_3+1) \qquad (3.72)\\
&\times\Big\{\, \big[d\, m_\pi^4 (s+M_0^2) + d\, m_\pi^2 (2sM_0 + m_\Delta(s+M_0^2)) - 2m_\pi^2 (d\, s\, m_\Delta M_0 \\
&+2(d-1)sM_0^2 + d\, m_\pi^4 (s+M_0^2))\big]\,\Delta_\pi \\
&+\frac{d\, s^2}{(s-M_0^2)}\Big[\big((m_\Delta - M_0)^2 - m_\pi^2\big)\big(m_\Delta + M_0)^2 + m_\pi^2\big)^2\Big] J_0[M_0^2] \\
&-\frac{d\, M_0^3}{(s-M_0^2)}\big[\big(m_\pi^4 + (s-m_\Delta^2)^2 - m_\pi^2(s+m_\Delta^2)\big)\big(2sm_\Delta \\
&+M_0(s-m_\pi^2+m_\Delta^2)\big)\big] J_0[s]\Big\}.
\end{aligned}
$$

In **Diagram d)+e)** the interaction of a photon with the pion loop occurs. This implies the appearance of the three-point functions $\gamma_i[s]$ in addition to the two-point function $J_0[s]$ already encountered in the previous diagrams:

$$
\begin{aligned}
A(s,Q^2) &= \frac{2}{3(d-1)^2 s\, m_\Delta}\left(\frac{e\, g_{\pi N\Delta}}{F_\pi}\right)^2 \qquad (3.73)\\
&\times\Big\{-\big(3s+m_\pi^2-m_\Delta^2\big)\Delta_\pi + s(Q^2+4m_\pi^2)J_0^{\pi\pi}[Q^2] \\
&-\big[m_\pi^4 + (s-m_\Delta^2)^2 - 2m_\pi^2(s+m_\Delta^2)\big] J_0[s] \\
&+4s\big[(m_\Delta+M_0)^2 - m_\pi^2\big]\gamma_3[s]\Big\}, \\
B(s,Q^2) &= \frac{1}{3(d-1)^2 s^2 m_\Delta}\left(\frac{e\, g_{\pi N\Delta}}{F_\pi}\right)^2 \Big\{ s^2(Q^2+4m_\pi^2)J_0^{\pi\pi}[Q^2] \qquad (3.74)\\
&-\big[s(s-m_\pi^2-3m_\Delta^2) + d(m_\pi^2-m_\Delta^2)(-s+m_\pi^2-m_\Delta^2)\big] J_0[s] \\
&+\big[s(3Q^2-4s) + d(2s^2+Q^2 m_\pi^2 - Q^2(2s+m_\Delta^2))\big]\Delta_\pi \\
&+2s^2\big[(m_\Delta+M_0)^2-m_\pi^2\big]\big(\gamma_1[s]-2\gamma_4[s]\big)\Big\}.
\end{aligned}
$$

We remark that the remaining scalar loop function, namely the three-point function Γ_0, is defined according to the diagrams of fig.3.4 where the photon interacts with the Δ propagator and thus a two-Δ's, 1-π system is created.

We now turn to the diagrams of fig.3.5, which do not give contribution to the spin structures of the nucleon.

Diagram o)+p) show no dependence on the Mandelstam variable s, so that the addition of the crossed partner make the result vanish:

$$\begin{aligned} A(s,Q^2) &= \frac{1}{24(d-1)^2 m_\Delta^2 M_0^3}\left(\frac{e\,g_{\pi N\Delta}}{F_\pi}\right)^2(\tau_3+1) \\ &\times\Big\{(d-2)\left[\left((M_0-m_\Delta)^2-m_\pi^2\right)\left((M_0+m_\Delta)^2-m_\pi^2\right)^2\right]J_0[M_0^2] \\ &-\frac{1}{d}\left[dm_\pi^4+d(m_\Delta-M_0)(m_\Delta+M_0)^3-2m_\pi^2\left(d\,m_\Delta^2\right.\right. \\ &\left.\left.+d\,m_\Delta M_0-2(d-1)M_0^2\right)\right]\Delta_\pi\Big\} \end{aligned} \quad (3.75)$$

$$B(s,Q^2) = 0 \quad (3.76)$$

The same happens for **Diagram q)+r)**, whose contributions read:

$$\begin{aligned} A(s,Q^2) &= \frac{(d-2)}{4(d-1)m_\Delta^2 M_0(M_0^2-p^2)}\left(\frac{e\,g_{\pi N\Delta}}{F_\pi}\right)^2(\tau_3+1) \\ &\times\left\{\left[\left(m_\pi^2-(m_\Delta-M_0)^2\right)\left(m_\pi^2-(m_\Delta+M_0)^2\right)^2\right]J_0[M_0^2]\right. \\ &+\frac{1}{d}\left[4m_\pi^2M_0^2+d\left(m_\pi^4-(M_0^2-m_\Delta^2)(M_0+m_\Delta)^2\right.\right. \\ &\left.\left.\left.+m_\pi^2(-2m_\Delta^2+6m_\Delta M_0+4M_0^2)\right)\right]\Delta_\pi\right\}, \end{aligned} \quad (3.77)$$

$$B(s,Q^2) = 0. \quad (3.78)$$

Finally, **Diagram s)**'s contribution to the spin dependent terms of the matrix element of the process is equal to zero:

$$A(s,Q^2) = 0, \quad (3.79)$$

$$B(s,Q^2) = 0. \quad (3.80)$$

The final expressions for the functions $A(\nu,Q^2)$ and $B(\nu,Q^2)$ are then obtained via addition of all the contributions coming from the different diagrams, the ones reported in this section and the ones given in the appendix. The next step will be the analysis of the divergences; given that at $\mathcal{O}(\epsilon^3)$ no coutertterms are involved, the vanishing of the divergences represent a very important check for the calculation. Afterwards numerical parameter-free prediction for the generalized sum rules and the spin polarizabilities will be available, giving indication about the role of the Δ resonance for these quantities. For a more complete and detailed discussion of the topic we refer to a future communication [BDKM].

Summary and future perspectives

The aim of the present work was to provide a deeper understanding about the structure of the nucleon by electromagnetic probes. When limited to the low energy region, the most suitable framework for this study is Chiral Perturbation Theory. After an introduction on the formalism, we have applied the acquired knowledge in Baryon Chiral Perturbation Theory to a leading one loop order calculation of the generalized isovector and isoscalar form factors of the nucleon $A^{v,s}_{2,0}(t, m_\pi^2)$, $B^{v,s}_{2,0}(t, m_\pi^2)$ and $C^{v,s}_{2,0}(t, m_\pi^2)$ in the $\overline{\text{IR}}$ renormalization scheme. To this purpose, we had to construct the effective chiral Lagrangian necessary to describe the interaction between external tensor fields and a strongly interacting system. The calculation has provided us with both the quark-mass and momentum transfer dependence of the six generalized form factors. These objects are under investigation also in lattice QCD; thanks to recent simulations, lattice data at values of the pion mass as low as 350 MeV are nowadays available, paving the way to reliable studies of chiral extrapolation. Of particular interest was the chiral extrapolation of the lattice data for the moment of the parton distribution function $A^v_{2,0}(0) = \langle x \rangle_{u-d}$. Our covariant $\mathcal{O}(p^2)$ BChPT result for this isovector moment provides a smooth chiral extrapolation function between the high values at large quark masses from lattice QCD and the lower value known from phenomenology. A fist attempt to use our results to make chiral extrapolations of lattice data of the spin-dependent quantity $A^s_{2,0}(0) = \langle \Delta x \rangle_{u-d}$ and of the isoscalar form factor $B^s_{2,0}(0)$ has also been done, together with a first estimation of the contribution of u and d quarks to the total spin of the nucleon J_{u+d}. Despite of the surprisingly nice results provided by these analysis, the values given in the chapter have to be taken with due caution and to be considered as a first rough estimate. Indeed, the lattice data used as input for the chiral extrapolation of the isoscalar generalized form factors correspond to very high values of the pion mass, they are characterized by unknown systematic uncertainties since disconnected diagrams are not included in the simulations. As pointed out throughout chapter 2, in order to judge the stability of our results an analysis of the complete next-to-leading one loop order calculation is definitely called for. Despite of this considerations, a first comparison to the values obtained by chiral extrapolation studies of lattice data subsequent to our publication and based on our formulae show that our BChPT calculation is definitely improving the quality of chiral extrapolation.

In the last section of chapter 2, the possibility to perform simultaneous fits of several observables characterized by a common subset of ChEFT couplings has been explored, with particular emphasis on the advantages which such a procedure can bring about in the studies of chiral extrapolations of lattice QCD data.

The second part of this work was dedicated to the possibility to extract new information about the spin structure of the nucleon from the analysis of the process of doubly virtual Compton scattering (VVCS) off nucleons. Starting from the connection between the spin structure functions in VVCS and the ones probed in inelastic electroproduction experiments and making use of the optical theorem and dispersion relations, we have written a set of sum rules in terms of the spin-dependent structure functions $S_{1,2}(\nu, Q^2)$ appearing in the spin amplitude of the process. A calculation of these structure functions in Chiral Perturbation Theory allows to make predictions for all spin-related sum rules in the low energy region. Although the several calculations on the topic present in the literature, the issue still owns big relevance in theoretical as well as experimental physics research. Indeed, given the key role played by the Δ resonance in the spin sector of the nucleon and the very active experimental activity at many laboratories e.g. at Jefferson Lab, a calculation of the derived sum rules with inclusion of the Δ resonance as explicit degree of freedom is definitely called for, in order to improve the agreement between theoretical predictions and experimental data. For this purpose we have performed a leading one loop order calculation of the matrix element of VVCS in the small scale expansion scheme of covariant Chiral Perturbation Theory and we have provided the results for all the Feynman diagrams contributing to the process at this order of working. The technicalities about the calculation have been given, together with first indications regarding how to derive numerical results for the sum rules and the spin polarizabilities starting from the provided ChPT expressions. The analysis of such results is still on the way. We postpone the complete and definitive discussion of the obtained results to a later communication [BDKM].

Appendix **A**

Appendix to GPDs

A.1 Feynman Rules for GPDs

Isovector vector channel

In this section we collect the expressions of the vertices we need in order to calculate the $\mathcal{O}(p^2)$ amplitudes in the isovector channel corresponding to the five diagrams of fig.2.1. In particular for each diagram we give the pertinent Feynman rule generated by the $\mathcal{O}(p^0)$ Lagrangian of eq.(2.18).

We make use of the following notation:
l^μ pion four-momentum
p^μ nucleon four-momentum
τ^i Pauli matrices
$a, b, c, \ldots$ isospin indices

From $\mathcal{L}^{(1)}_{\pi N}$ of eq.(1.7)

◇ nucleon propagator

$$\frac{i}{\not{p} - M_0 + i\epsilon} \tag{A.1}$$

◇ N in, π out, N out:

$$\frac{1}{2}\frac{g_A}{F_\pi}\,\not{l}\gamma_5\,\tau^a \tag{A.2}$$

◇ N in, π in, N out:

$$-\frac{1}{2}\frac{g_A}{F_\pi}\,\not{l}\gamma_5\,\tau^a \tag{A.3}$$

From $\mathcal{L}_{\pi\pi}^{(2)}$ of eq.(1.2)

◇ pion propagator

$$\frac{i\delta^{ab}}{l^2 - m_\pi^2 + i\epsilon} \tag{A.4}$$

From $\mathcal{L}_{t\pi N}^{(0)}$ of eq.(2.18)

◇ diagram a)

N in (momentum p), tensor field (isospin a), π out (momentum l, isospin c), N out (momentum $(p' - l)$):

$$i\,\frac{\Delta a_{2,0}^v}{4F_\pi}\,\epsilon^{cad}\tau^d\,\gamma_5\gamma^{\{\mu}(l - 2\overline{p})^{\nu\}} \tag{A.5}$$

◇ diagram b)

N in (momentum $(p - l)$), π in (momentum l, isospin c), tensor field (isospin a), N out (momentum p'):

$$i\,\frac{\Delta a_{2,0}^v}{4F_\pi}\,\epsilon^{cad}\tau^d\,\gamma_5\gamma^{\{\mu}(l - 2\overline{p})^{\nu\}} \tag{A.6}$$

◇ diagram c):

N in (momentum $(p - l)$), tensor field (isospin a), N out (momentum $(p' - l)$):

$$i\,\frac{a_{2,0}^v}{2}\,\tau^a\,\gamma^{\{\mu}(\overline{p} - l)^{\nu\}} \tag{A.7}$$

◇ diagram d):

π in (isospin c), tensor field (isospin a), π out (isospin d):

$$i\,\frac{a_{2,0}^v}{4F_\pi^2}\,(\delta^{ca}\tau^d + \delta^{ad}\tau^c - 2\delta^{cd}\tau^a)\,\gamma^{\{\mu}\overline{p}^{\nu\}} \tag{A.8}$$

Isoscalar vector channel

As reported in section 2.5, to $\mathcal{O}(p^2)$ the only nonzero loop contributions to the isoscalar channel arise from diagram c) and e) in fig.2.1. The pertinent Feynman rule, generated by the $\mathcal{O}(p^0)$ Lagrangian of eq.2.18, reads:

◇ diagram c):

N in (momentum $(p-l)$), tensor field (isospin a), N out (momentum $(p'-l)$):

$$i\,\frac{a^s_{2,0}}{2}\,\tau^a\,\gamma^{\{\mu}(\overline{p}-l)^{\nu\}} \tag{A.9}$$

A.2 Basic Integrals

The integrals required for one-loop calculations in BChPT can be reduced to two basis integrals in d-dimensions:

$$\Delta_\pi\left(m\right) \equiv \frac{1}{i}\int \frac{d^d l}{(2\pi)^d}\,\frac{1}{m^2 - l^2 - i\epsilon}, \tag{A.10}$$

$$H_{11}\left(M^2, m^2, p^2\right) \equiv \frac{1}{i}\int \frac{d^d l}{(2\pi)^d}\,\frac{1}{\left(m^2 - l^2 - i\epsilon\right)\left(M^2 - (l-p)^2 - i\epsilon\right)}, \tag{A.11}$$

where m (M) is a mass function involving the mass of the Goldstone Boson (of the Baryon) and p^μ denotes a four-momentum determined by the kinematics.The propagators are shifted into the complex energy-plane by a small amount ϵ to ensure causality. Utilizing the $\overline{\text{MS}}$-renormalization scheme of ref.[GSS88] one obtains the dimensionally-regularized results

$$\Delta_\pi = 2m^2\left(L + \frac{1}{16\pi^2}\ln\frac{m}{\lambda}\right) + \mathcal{O}(d-4), \tag{A.12}$$

$$\begin{aligned} H_{11}(M^2, m^2, p^2) = -2L - \frac{1}{16\pi^2}\Bigg[&-1 + \log\frac{M^2}{\lambda^2} + \frac{p^2 - M^2 + m^2}{p^2}\log\frac{m}{M} \\ &+ \frac{2mM}{p^2}\sqrt{1 - \left(\frac{p^2 - M^2 - m^2}{2mM}\right)^2}\arccos\left(\frac{m^2 + M^2 - p^2}{2mM}\right)\Bigg] \\ &+ \mathcal{O}(d-4), \end{aligned} \tag{A.13}$$

with

$$L = \frac{\lambda^{d-4}}{16\pi^2}\left[\frac{1}{d-4} + \frac{1}{2}\left(\gamma_E - 1 - \ln 4\pi\right)\right] \tag{A.14}$$

which is divergent as $d \to 4$. λ is the scale parameter introduced in dimensional regularization and $\gamma_E = 0.577215\ldots$ the Euler-Mascheroni constant.

More complicated integral expressions needed during the calculation are defined via

$$\begin{aligned} &\frac{1}{i}\int \frac{d^d l}{(2\pi)^d}\,\frac{\{l^\mu, l^\mu l^\nu\}}{(m^2 - l^2 - i\epsilon)(M^2 - (l-p)^2 - i\epsilon)} = \\ &\qquad\qquad \{p^\mu H_{11}^{(1)}, g^{\mu\nu} H_{11}^{(2)} + p^\mu p^\nu H_{11}^{(3)}\}, \end{aligned} \tag{A.15}$$

$$\begin{aligned} &\frac{1}{i}\int \frac{d^d l}{(2\pi)^d}\,\frac{l^\mu l^\nu l^\alpha}{(m^2 - l^2 - i\epsilon)(M^2 - (l-p)^2 - i\epsilon)} = \\ &\qquad\qquad \left(p^\mu g^{\mu\alpha} + p^\nu g^{\mu\alpha} + p^\alpha g^{\mu\nu}\right) H_{11}^{(4)} + p^\mu p^\nu p^\alpha\, H_{11}^{(5)}\ . \end{aligned} \tag{A.16}$$

The integrals $H_{11}^{(i)}$ are related to the three basis integrals of eqs.(A.10-A.11) via the tensor-identities

$$H_{11}^{(1)}(M,p,m) = \frac{1}{2p^2}\Big[\Delta_\pi - \Delta_N + \left(m^2+p^2-M^2\right)H_{11}(M,p,m)\Big], \tag{A.17}$$

$$H_{11}^{(2)}(M,p,m) = \frac{1}{2(d-1)}\Big[-\Delta_N + 2m^2 H_{11}(M,p,m) - \left(m^2+p^2-M^2\right)H_{11}^{(1)}\Big], \tag{A.18}$$

$$H_{11}^{(3)}(M,p,m) = \frac{1}{p^2(d-1)}\Big[\left(1-\frac{d}{2}\right)\Delta_N - m^2 H_{11}(M,p,m) + \frac{d}{2}\left(m^2+p^2-M^2\right)H_{11}^{(1)}(M,p,m)\Big], \tag{A.19}$$

$$H_{11}^{(4)}(M,p,m) = \frac{1}{2p^2}\Big[\left(m^2+p^2-M^2\right)H_{11}^{(2)}(M,p,m) + \frac{1}{d}\left(m^2\Delta_\pi - M^2\Delta_N\right)\Big], \tag{A.20}$$

$$H_{11}^{(5)}(M,p,m) = \frac{1}{2p^2}\Big[\left(m^2+p^2-M^2\right)H_{11}^{(3)}(M,p,m) - 4H_{11}^{(4)}(M,p,m) - \Delta_N\Big] \tag{A.21}$$

Finally, we note that the integrals involving more than one baryon propagator can be related to the ones defined above via

$$H_{11}^{(i)\prime}(M^2,m^2,p^2) \equiv \frac{\partial}{\partial M^2}H_{11}^{(i)}(M^2,m^2,p^2). \tag{A.22}$$

A.3 Regulator Functions

The basis regulator function is defined as

$$R_{11}\left(M^2,m^2,p^2\right) \equiv \int_{x=1}^{\infty} dx \int \frac{d^d l}{(2\pi)^d} \left[xM^2 + (x^2-x)p^2 + (1-x)m^2 - l^2\right]^{-2}. \tag{A.23}$$

More complicated regulator functions $R_{11}^{(i)}$, $i = 1\ldots5$ can be defined in analogy to Eqs.(A.15, A.16). We note that for our $\mathcal{O}(p^2)$ calculation of the moments of the GPDs we only need to know these functions up to the power[1] of m_π^2, t in order to obtain a properly renormalized, scale-independent result, which at the same time is also consistent with the requirements of power-counting. The regulator functions needed for our $\mathcal{O}(p^2)$ BChPT calculation (see Appendix A.4) read

$$\begin{aligned}
R_{11}(M_0^2,m_\pi^2,p^2) &= \left(2-\frac{m_\pi^2}{M_0^2}\right)L + \frac{1}{16\pi^2}\left[2\log\frac{M_0}{\lambda}\right. \\
&\left. -1-\frac{1}{2}\frac{m_\pi^2}{M_0^2}\left(2\log\frac{M_0}{\lambda}+3\right)\right]+..., & \text{(A.24)}\\
R_{11}^{(1)}(M_0^2,m_\pi^2,p^2) &= \left(1+\frac{m_\pi^2}{M_0^2}\right)L + \frac{1}{16\pi^2}\left[\log\frac{M_0}{\lambda}+\frac{1}{2}\frac{m_\pi^2}{M_0^2}\left(2\log\frac{M_0}{\lambda}-1\right)\right] \\
&+..., & \text{(A.25)}\\
R_{11}^{(2)}(M_0^2,m_\pi^2,p^2) &= \left(\frac{1}{3}M_0^2+\frac{1}{2}m_\pi^2\right)L + \frac{1}{48\pi^2}\left[\frac{M_0^2}{3}\left(3\log\frac{M_0}{\lambda}-1\right)\right. \\
&\left.+\frac{3}{2}m_\pi^2\left(\log\frac{M_0}{\lambda}-1\right)\right]+..., & \text{(A.26)}\\
R_{11}^{(3)}(M_0^2,m_\pi^2,p^2) &= \frac{2}{3}L + \frac{1}{48\pi^2}\left[2\log\frac{M_0}{\lambda}+\frac{1}{3}+\frac{3}{2}\frac{m_\pi^2}{M_0^2}\right]+..., & \text{(A.27)}
\end{aligned}$$

[1]Strictly speaking we need to know the regulator terms contributing to the generalized form factor $A_{2,0}^{s,v}(t)$ up to power $(m_\pi^2,t)^1$, whereas for $B_{2,0}^{s,v}(t)$, $C_{2,0}^{s,v}(t)$ only the leading terms $(m_\pi^2,t)^0$ are required [Gai07].

The derivatives of the regulator functions needed for the calculation (see Appendix A.4) read

$$R_{11}^{(2)\prime}(\tilde{M}^2, m_\pi^2, \tilde{p}^2) = \frac{1}{2}\left(1+\frac{m_\pi^2}{M_0^2}\right)L + \frac{1}{32\pi^2}\left[\log\frac{M_0}{\lambda} + \frac{m_\pi^2}{2M_0^2}\left(2\log\frac{M_0}{\lambda} - 1\right)\right] - \frac{t}{384\pi^2 M_0^2} + ..., \quad (A.28)$$

$$R_{11}^{(3)\prime}(\tilde{M}^2, m_\pi^2, \tilde{p}^2) = -\frac{m_\pi^2}{M_0^4}L + \frac{1}{16\pi^2 M_0^2}\left[\frac{1}{2} + \frac{m_\pi^2}{M_0^2}\left(-\log\frac{M_0}{\lambda} + 1\right)\right] + \frac{t}{192\pi^2 M_0^4} + ..., \quad (A.29)$$

$$R_{11}^{(4)\prime}(\tilde{M}^2, m_\pi^2, \tilde{p}^2) = \frac{1}{3}L + \frac{1}{16\pi^2}\left[\frac{1}{3}\log\frac{M_0}{\lambda} + \frac{1}{18} + \frac{m_\pi^2}{4M_0^2}\right] - \frac{t}{576\pi^2 M_0^2} + ..., \quad (A.30)$$

$$R_{11}^{(5)\prime}(\tilde{M}^2, m_\pi^2, \tilde{p}^2) = \frac{1}{16\pi^2 M_0^2}\left[\frac{1}{3} - \frac{m_\pi^2}{2M_0^2}\right] + \frac{t}{288\pi^2 M_0^4} + ... \quad (A.31)$$

One can clearly observe that all contributions are polynomial in m_π^2 (and therefore polynomial in the quark-mass [BL99]) or polynomial in t [Gai07], as expected. Their addition to the $\overline{\text{MS}}$-results therefore just amounts to a shift in the coupling constants [BL99, Gai07] of the effective field theory and does not affect the non-analytic quark-mass dependencies, which are the scheme-independent signatures of chiral dynamics.

A.4 Isovector vector Amplitudes in $\mathcal{O}(p^2)$ BChPT

The five $\mathcal{O}(p^2)$ amplitudes in the isovector channel corresponding to the five diagrams of Fig.2.1 written in terms of the basic integrals

$$I_{11}^{(i)}\left(M^2,m^2,p^2\right) = H_{11}^{(i)}\left(M^2,m^2,p^2\right) + R_{11}^{(i)}\left(M^2,m^2,p^2\right)\ , \quad i=0\dots5, \tag{A.32}$$

of Appendices A.2 and A.3 read

$$\begin{aligned} Amp^{a+b} =& - i\,\frac{\Delta a_{2,0}^v\ g_A}{F_\pi^2}\ \ \eta^\dagger\frac{\tau^a}{2}\eta\ \overline{u}(p')\,\gamma_{\{\mu}\overline{p}_{\nu\}}\,u(p)\,\Bigg[2m_\pi^2 I_{11}(M_0^2,m_\pi^2,p^2) \\ & - m_\pi^2 I_{11}^{(1)}(M_0^2,m_\pi^2,p^2) + 2I_{11}^{(2)}(M_0^2,m_\pi^2,p^2)\Bigg], \end{aligned} \tag{A.33}$$

$$\begin{aligned} Amp^c =& i\,\frac{a_{2,0}^v g_A^2}{4F_\pi^2}\ \ \eta^\dagger\frac{\tau^a}{2}\eta\ \overline{u}(p')\int_{-\frac{1}{2}}^{\frac{1}{2}} du \\ &\times\Bigg\{\gamma_{\{\mu}\overline{p}_{\nu\}}\Bigg[-\Delta_\pi + 4M_0^2\Bigg(I_{11}^{(1)}(M_0^2,m_\pi^2,p^2) - I_{11}^{(3)}(M_0^2,m_\pi^2,p^2) \\ & + (2M_0^2-\tilde{M}^2)\left(I_{11}^{(3)\prime}(\tilde{M}^2,m_\pi^2,\tilde{p}^2) - I_{11}^{(5)\prime}(\tilde{M}^2,m_\pi^2,\tilde{p}^2)\right) \\ & + (d-2)\left(I_{11}^{(4)\prime}(\tilde{M}^2,m_\pi^2,\tilde{p}^2) - I_{11}^{(2)\prime}(\tilde{M}^2,m_\pi^2,\tilde{p}^2)\right)\Bigg)\Bigg] \\ & + i\,\Delta^\alpha\sigma_{\alpha\{\mu}\overline{p}_{\nu\}}\,4M_0^3\Bigg(I_{11}^{(3)\prime}(\tilde{M}^2,m_\pi^2,\tilde{p}^2) - I_{11}^{(5)\prime}(\tilde{M}^2,m_\pi^2,\tilde{p}^2)\Bigg) \\ & - \Delta_{\{\mu}\Delta_{\nu\}}\Bigg(8M_0^3\,u^2\,I_{11}^{(5)\prime}(\tilde{M}^2,m_\pi^2,\tilde{p}^2)\Bigg)\Bigg\}\ u(p), \end{aligned} \tag{A.34}$$

$$Amp^d = -\ i\,\frac{a_{2,0}^v}{F_\pi^2}\ \ \eta^\dagger\frac{\tau^a}{2}\eta\ \overline{u}(p')\,\gamma_{\{\mu}\overline{p}_{\nu\}}\,u(p)\,\Delta_\pi, \tag{A.35}$$

$$Amp^e = i\,a_{2,0}^v\ \ \eta^\dagger\frac{\tau^a}{2}\eta\ \overline{u}(p')\,\gamma_{\{\mu}\overline{p}_{\nu\}}\,u(p)\,\mathcal{Z_N}. \tag{A.36}$$

Note that the various couplings and parameters are defined in section 2.3. η denotes the isospin doublet of proton and neutron. The variables in the integral functions are given as

$$\tilde{p}^2 \ = \ \tilde{M}^2 \ \equiv \ M_0^2 + \left(u^2 - \frac{1}{4}\right)t, \tag{A.37}$$

where $t=\Delta^2$ corresponds to the momentum transfer by the tensor fields.

$\mathcal{Z_N}$ denotes the Z-factor of the nucleon, calculated to the required $\mathcal{O}(p^3)$ accuracy in BChPT. It is obtained from the self-energy Σ_N at this order via the prescription

$$\mathcal{Z_N} = 1 + \frac{\partial\Sigma_N}{\partial \not{p}}\Bigg|_{\not{p}=M_0} + \mathcal{O}(p^4), \tag{A.38}$$

with

$$\Sigma_N = \frac{3g_A^2}{4F_\pi^2}(M_0+\not{p})\left[m_\pi^2 I_{11}(M_0^2, m_\pi^2, p^2)+(M_0-\not{p})\not{p} I_{11}^{(1)}(M_0^2, m_\pi^2, p^2)-\Delta_N\right]+\mathcal{O}(p^4). \tag{A.39}$$

A.5 BChPT Results in the Isoscalar vector Channel

A.5.1 Isoscalar vector Amplitudes in $\mathcal{O}(p^2)$ BChPT

To $\mathcal{O}(p^2)$ in BChPT the results in the isoscalar channel are quite simple. The amplitudes corresponding to the Feynman diagrams of Fig.2.1 can be simply expressed in terms of results already obtained in the isovector channel discussed in the previous section A.4. They read

$$\begin{aligned}
\overline{Amp}_{a+b} &= 0 + \mathcal{O}(p^3), && \text{(A.40)}\\
\overline{Amp}_{c} &= -3\,\frac{a^s_{2,0}\eta^\dagger \mathbf{1}\eta}{a^v_{2,0}\eta^\dagger\tau^a\eta}\,Amp^c + \mathcal{O}(p^3), && \text{(A.41)}\\
\overline{Amp}_{d} &= 0 + \mathcal{O}(p^3), && \text{(A.42)}\\
\overline{Amp}_{e} &= \frac{a^s_{2,0}\eta^\dagger \mathbf{1}\eta}{a^v_{2,0}\eta^\dagger\tau^a\eta}\,Amp^e + \mathcal{O}(p^3). && \text{(A.43)}
\end{aligned}$$

Note that the various couplings and parameters are defined in section 2.3.

A.5.2 Estimate of $\mathcal{O}(p^3)$ contributions

The contributions from the (higher order) Feynman diagram shown in figure 2.2 to the generalized isoscalar form factors read

$$\begin{aligned}
\Delta A^s_{h.o.}(t,m_\pi) = {} & \frac{g_A^2 x_\pi^0}{32\pi^2 F_{\pi^2}}\int_{-\frac{1}{2}}^{\frac{1}{2}}\frac{du}{\tilde{p}^8}\Bigg\{2\tilde{p}^8\left(3m_\pi^2+M_0^2\right)-t\tilde{p}^8\\
&+4M_0^2\left(2M_0^2+\tilde{m}^2\right)\tilde{p}^6-2M_0^2\left(11M_0^4-7\tilde{m}^2M_0^2+2\tilde{m}^4\right)\tilde{p}^4\\
&+12M_0^4\left(M_0^2-\tilde{m}^2\right)^2\tilde{p}^2+2M_0^2\Big[-\tilde{p}^8+3\tilde{p}^4\left(3M_0^4-\tilde{m}^4\right)\\
&-2\tilde{p}^2\left(M_0^2-\tilde{m}^2\right)\left(7M_0^4-2\tilde{m}^2M_0^2+\tilde{m}^4\right)+6M_0^2\left(M_0^2-\tilde{m}^2\right)^3\Big]\\
&\times\log\frac{\tilde{m}}{M_0}+\frac{2M_0^2}{\sqrt{2M_0^2\left(\tilde{p}^2+\tilde{m}^2\right)-\left(\tilde{p}^2-\tilde{m}^2\right)^2-M_0^4}}\\
&\times\Big[6M_0^{10}-4M_0^8\left(6\tilde{m}^2+5\tilde{p}^2\right)+M_0^6\left(36\tilde{m}^4+38\tilde{m}^2\tilde{p}^2+23\tilde{p}^4\right)\\
&-M_0^4\left(24\tilde{m}^6+18\tilde{p}^2\tilde{m}^4+11\tilde{p}^4\tilde{m}^2+9\tilde{p}^6\right)\\
&+M_0^2\left(6\tilde{m}^8+2\tilde{p}^2\tilde{m}^6-5\tilde{p}^4\tilde{m}^4-2\tilde{p}^6\tilde{m}^2-\tilde{p}^8\right)\\
&+\tilde{p}^2\left(\tilde{p}^2-\tilde{m}^2\right)^3\left(\tilde{p}^2+2\tilde{m}^2\right)\Big]\arccos\left(\frac{M_0^2+\tilde{m}^2-\tilde{p}^2}{2M_0\tilde{m}}\right)\Bigg\},
\end{aligned}$$

(A.44)

$$\begin{aligned}
\Delta B^s_{h.o.}(t,m_\pi) \;=\; & \frac{g_A^2 x_\pi^0}{96\pi^2 F_\pi^2}\,\frac{M_N(m_\pi)}{M_0}\int_{-\frac{1}{2}}^{\frac{1}{2}} du \Bigg\{ -6M_0^2 + 18m_\pi^2\left(2\log\frac{M_0}{\lambda}-3\right) \\
& + t\left(11 - 6\log\frac{M_0}{\lambda}\right) - \frac{36M_0^4}{\tilde{p}^8\sqrt{2\left(\tilde{m}^2+\tilde{p}^2\right)M_0^2 - \left(\tilde{p}^2-\tilde{m}^2\right)^2 - M_0^4}} \\
& \times\Big[M_0^8 - \left(4\tilde{m}^2+3\tilde{p}^2\right)M_0^6 + \left(6\tilde{m}^4+5\tilde{p}^2\tilde{m}^2+3\tilde{p}^4\right)M_0^4 \\
& - \left(4\tilde{m}^6+\tilde{p}^2\tilde{m}^4+\tilde{p}^4\tilde{m}^2+\tilde{p}^6\right)M_0^2 + \tilde{m}^8 - \tilde{m}^6\tilde{p}^2\Big] \\
& \times\arccos\left(\frac{M_0^2+\tilde{m}^2-\tilde{p}^2}{2M_0\tilde{m}}\right) \\
& - \frac{6M_0^4}{\tilde{p}^8}\Big[2\tilde{p}^6 - 3\left(3M_0^2-\tilde{m}^2\right)\tilde{p}^4 + 6\left(M_0^2-\tilde{m}^2\right)^2\tilde{p}^2 \\
& + 6\left(M_0^2\tilde{p}^4 - 2M_0^2\left(M_0^2-\tilde{m}^2\right)\tilde{p}^2 + \left(M_0^2-\tilde{m}^2\right)^3\right)\log\frac{\tilde{m}}{M_0}\Big]\Bigg\} \\
& + B_{33}^r(\lambda)\,\frac{M_N(m_\pi)}{M_0}\,\frac{4m_\pi^2}{\Lambda_\chi^2} + B_{34}^r(\lambda)\,\frac{M_N(m_\pi)}{M_0}\,\frac{t}{\Lambda_\chi^2},
\end{aligned} \tag{A.45}$$

$$\begin{aligned}
\Delta C^s_{h.o.}(t,m_\pi) \;=\; & \frac{g_A^2 x_\pi^0 M_0^2}{96\pi^2 F_\pi^2}\,\frac{M_N(m_\pi)}{M_0}\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{du}{\tilde{p}^8}\Bigg\{5\tilde{p}^8 - 9M_0^2\tilde{p}^6 - 6u^2\tilde{p}^2\Big[3\tilde{p}^6 - 2M_0^2\tilde{p}^4 \\
& - 3M_0^2\left(M_0^2-3\tilde{m}^2\right)\tilde{p}^2 - 6\left(M_0^3-M_0\tilde{m}^2\right)^2\Big] \\
& - 9\left(4u^2-1\right)\tilde{p}^8\log\frac{\tilde{m}}{M_0} + 9M_0^2\Big[4u^2\left(-\tilde{m}^2\tilde{p}^4 + 2\tilde{m}^2\tilde{p}^2\left(\tilde{m}^2-M_0^2\right)\right. \\
& \left. + \left(M_0^2-\tilde{m}^2\right)^3\right) - \tilde{p}^4\left(M_0^2-\tilde{m}^2\right)\Big]\log\frac{\tilde{m}}{M_0} \\
& + \frac{9M_0^2}{\sqrt{2M_0^2\left(\tilde{m}^2+\tilde{p}^2\right) - \left(\tilde{p}^2-\tilde{m}^2\right)^2 - M_0^4}}\Big[4u^2M_0^8 \\
& - 4\left(4\tilde{m}^2+\tilde{p}^2\right)u^2M_0^6 - \left(-24u^2\tilde{m}^4 + 4u^2\tilde{p}^2\tilde{m}^2 + \tilde{p}^4\right)M_0^4 \\
& + \left(-16u^2\tilde{m}^6 + 20u^2\tilde{p}^2\tilde{m}^4 + 2\left(1-2u^2\right)\tilde{m}^2\tilde{p}^4 + \tilde{p}^6\right)M_0^2 \\
& + \tilde{m}^2\left(\tilde{m}^2-\tilde{p}^2\right)\left(4u^2\tilde{m}^4 - 8u^2\tilde{m}^2\tilde{p}^2 + \left(4u^2-1\right)\tilde{p}^4\right)\Big] \\
& \times\arccos\left(\frac{M_0^2+\tilde{m}^2-\tilde{p}^2}{2M_0\tilde{m}}\right)\Bigg\},
\end{aligned} \tag{A.46}$$

with the new variable

$$\tilde{m}^2 = m_\pi^2 + \left(u^2 - \frac{1}{4}\right)t. \tag{A.47}$$

We note that the contributions of $\Delta A^s_{2,0}(t, m_\pi)$, $\Delta C^s_{2,0}(t, m_\pi)$ are finite at $\mathcal{O}(p^3)$ in BChPT, while $\Delta B^s_{2,0}(t, m_\pi)$ contains two new counterterms $B^r_{33}(\lambda)$, $B^r_{34}(\lambda)$ at this order [DHH].

A.6 Moments of axial GPDs in $\mathcal{O}(p^2)$ BChPT

A.6.1 Axial Amplitudes in $\mathcal{O}(p^2)$ BChPT

To $\mathcal{O}(p^2)$ the five amplitudes in the axial sector corresponding to the five diagrams of fig. 2.1 read

$$\begin{aligned}
Amp^{a+b} &= i\,\frac{a^v_{2,0}\;g_A}{F_\pi^2}\;\eta^\dagger\frac{\tau^a}{2}\eta\;\overline{u}(p')\,\gamma_{\{\mu}\gamma_5\,\overline{p}_{\nu\}}\,u(p)\,\Big[\Delta_N - 2m_\pi^2 I_{11}(M_0^2,m_\pi^2,p^2) \qquad \text{(A.48)}\\
&\quad + m_\pi^2 I_{11}^{(1)}(M_0^2,m_\pi^2,p^2) - 2I_{11}^{(2)}(M_0^2,m_\pi^2,p^2)\Big]\\
Amp^{c} &= i\,\frac{\Delta a^v_{2,0} g_A^2}{4F_\pi^2}\;\eta^\dagger\frac{\tau^a}{2}\eta\;\overline{u}(p')\int_{-\frac{1}{2}}^{\frac{1}{2}} du \qquad \text{(A.49)}\\
&\quad \times\Big\{\,\gamma_{\{\mu}\gamma_5\,\overline{p}_{\nu\}}\,\Big[-\Delta_\pi + 4M_0^2\Big(I_{11}^{(1)}(M_0^2,m_\pi^2,p^2) - I_{11}^{(3)}(M_0^2,m_\pi^2,p^2)\\
&\quad + (d-2)\Big(I_{11}^{(2)'}(\tilde{M}^2,m_\pi^2,\tilde{p}^2) - I_{11}^{(4)'}(\tilde{M}^2,m_\pi^2,\tilde{p}^2)\Big)\\
&\quad + \tilde{M}^2\Big(I_{11}^{(3)'}(\tilde{M}^2,m_\pi^2,\tilde{p}^2) - I_{11}^{(5)'}(\tilde{M}^2,m_\pi^2,\tilde{p}^2\Big)\Big]\\
&\quad + \gamma_5\,\overline{p}_{\{\mu}q_{\nu\}}\Big[16M_0^3\,u^2\,\Big(2I_{11}^{(5)'}(\tilde{M}^2,m_\pi^2,\tilde{p}^2) - I_{11}^{(3)'}(\tilde{M}^2,m_\pi^2,\tilde{p}^2)\Big)\Big]\Big\}\;u(p)\\
Amp^{d} &= -\,i\,\frac{\Delta a^v_{2,0}}{F_\pi^2}\;\eta^\dagger\frac{\tau^a}{2}\eta\;\overline{u}(p')\,\gamma_{\{\mu}\gamma_5\,\overline{p}_{\nu\}}\,u(p)\,\Delta_\pi \qquad \text{(A.50)}\\
Amp^{e} &= i\Delta a^v_{2,0}\;\eta^\dagger\frac{\tau^a}{2}\eta\;\overline{u}(p')\,\gamma_{\{\mu}\gamma_5\,\overline{p}_{\nu\}}\,u(p)\,\mathcal{Z}_{\mathcal{N}} \qquad \text{(A.51)}\\
& \qquad \text{(A.52)}
\end{aligned}$$

where we use the notation introduced in section A.4.

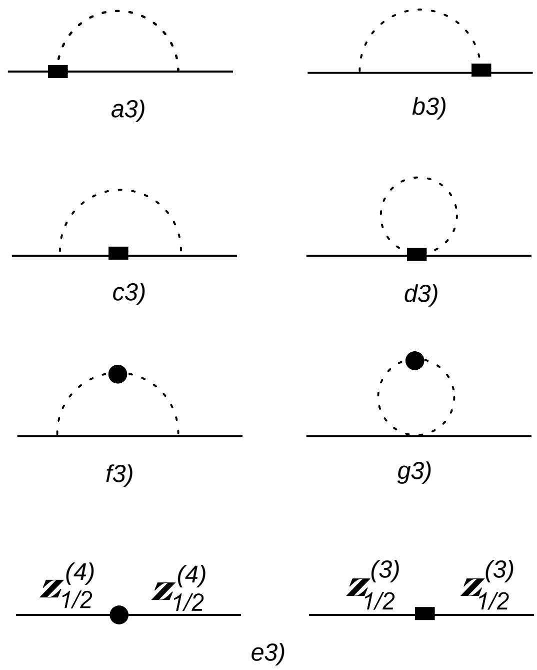

Figure A.1: The loop diagrams contributing to first moments of GPDs of a nucleon at next-to-leading one loop order in BChPT. The solid- and dashed lines represent nucleon and pion propagators, respectively. The solid dot and the solid square denote a coupling to a tensor field from the $\mathcal{O}(p^0)$ Lagrangian of eq.(2.18) and from the $\mathcal{O}(p^1)$ Lagrangian of eq.(2.19), respectively.

A.7 $\mathcal{O}(p^3)$ calculation of the generalized form factors: preliminary results

In this section we provide preliminary results of the first moments of GPDs at next-to-leading one loop order in BChPT. For an accurate analysis of the results we refer to a future communication [DHH].
Figure A.1 shows the diagrams contributing at the chiral order $D = 3$. The new couplings come from the $\mathcal{O}(p^1)$ Lagrangian of eq.(2.19) and are depicted as solid squares in order to distinguish them from the ones involved in the leading order calculation. We specify that the diagrams $f3$) and $g3$) of figure A.1 contribute only to the isoscalar generalized form factors, in particular a non-zero contribution comes only from diagram $f3$).
We observe that for a complete $\mathcal{O}(p^3)$ evaluation one has to consider also the contributions from the $\mathcal{O}(p^2)$ Feynman diagrams of figure 2.1 after appropriate p^2 nucleon-mass insertions.

A.7.1 Isovector vector channel

To $\mathcal{O}(p^3)$ the isovector vector amplitudes corresponding to the five diagrams of figure A.1 read:

$$\begin{aligned}
Amp^{a3+b3} &= i\,\frac{4\,h_A^v\,g_A}{F_\pi^2}\;\eta^\dagger\frac{\tau^a}{2}\eta\;\overline{u}(p')\,\gamma_{\{\mu}\overline{p}_{\nu\}}\,u(p)\,\Big[2I_{11}^{(2)}(M_0^2,m_\pi^2,p^2) \qquad (A.53)\\
&\quad - m_\pi^2 I_{11}^{(1)}(M_0^2,m_\pi^2,p^2)+\Delta_N\Big],\\
Amp^{c3} &= i\,\frac{b_{2,0}^v g_A^2}{8F_\pi^2}\;\eta^\dagger\frac{\tau^a}{2}\eta\;\overline{u}(p')\int_{-\frac{1}{2}}^{\frac{1}{2}}du \qquad (A.54)\\
&\quad\times\Bigg\{\gamma_{\{\mu}\overline{p}_{\nu\}}\Big[4M_0^3q^2\Big(I_{11}^{(3)\prime}(\tilde{M}^2,m_\pi^2,\tilde{p}^2)-I_{11}^{(5)\prime}(\tilde{M}^2,m_\pi^2,\tilde{p}^2)\Big)\Big]\\
&\quad + i\,\Delta^\alpha\sigma_{\alpha\{\mu}\overline{p}_{\nu\}}\Big[4M_0^2\Big(I_{11}^{(1)}(M_0^2,m_\pi^2,p^2)-I_{11}^{(3)}(M_0^2,m_\pi^2,p^2)\Big)\\
&\quad + m_\pi^2\Big(I_{11}'(\tilde{M}^2,m_\pi^2,\tilde{p}^2)+I_{11}^{(1)\prime}(\tilde{M}^2,m_\pi^2,\tilde{p}^2)\Big)\\
&\quad - 4I_{11}^{(2)\prime}(\tilde{M}^2,m_\pi^2,\tilde{p}^2)+6I_{11}^{(4)\prime}(\tilde{M}^2,m_\pi^2,\tilde{p}^2)\\
&\quad - 2\Big(\tilde{M}^2-M_0^2\Big)\Big(I_{11}^{(3)\prime}(\tilde{M}^2,m_\pi^2,\tilde{p}^2)-I_{11}^{(5)\prime}(\tilde{M}^2,m_\pi^2,\tilde{p}^2)\Big)-\Delta_\pi\Big]\\
&\quad + \Delta_{\{\mu}\Delta_{\nu\}}\Big(4M_0^2 I_{11}^{(4)\prime}(\tilde{M}^2,m_\pi^2,\tilde{p}^2)\Big)\Bigg\}\,u(p),\\
Amp^{d3} &= -\,\frac{b_{2,0}^v}{2M_0F_\pi^2}\;\eta^\dagger\frac{\tau^a}{2}\eta\,\overline{u}(p')\,\Delta^\alpha\sigma_{\alpha\{\mu}\overline{p}_{\nu\}}\,u(p)\,\Delta_\pi, \qquad (A.55)\\
Amp^{e3} &= i\;\eta^\dagger\frac{\tau^a}{2}\eta\;\overline{u}(p')\Big[-i\frac{b_{2,0}^v}{2M_0}\;\Delta^\alpha\sigma_{\alpha\{\mu}\overline{p}_{\nu\}}\,\mathcal{Z_N}^{(3)} \qquad (A.56)\\
&\quad + a_{2,0}^v\;\gamma_{\{\mu}\overline{p}_{\nu\}}\,\mathcal{Z_N}^{(4)}\Big]\;u(p).
\end{aligned}$$

We note the appearance of a new coupling called h_A^v in diagrams $a3$+$b3$. This coupling belongs to the first order Lagrangian $\mathcal{L}_{t\pi N}^{(1)}$ of eq.(2.19). Let us concentrate on the third term of that Lagrangian, namely $\overline{\psi}_N[\frac{g_A}{2}\gamma^\mu\gamma_5 u_\mu]\psi_N = \overline{\psi}_N[\frac{g_A}{2}\gamma^\mu\gamma_5 g^{\mu\nu}u_\nu]\psi_N$; in analogy to what has been done in section 2.3 (see eq.(2.17)), we introduce the structure $\widetilde{g_{\mu\nu}} = g_{\mu\nu}+\frac{h_A^v}{2}V_{\mu\nu}^+ + h_A^0 V_{\mu\nu}^0$ and we automatically get the new couplings h_A^v,h_A^0 appearing in the vertices of the diagrams $a3$,$b3$ of the isovector (h_A^v) and isoscalar (h_A^0) vector channels.
Again, $\mathcal{Z_N}$ denotes the Z-factor of the nucleon. Being a $\mathcal{O}(p^3)$ calculation, power counting indicates two different contributions to diagram $e3$: the p^1 coupling $b_{2,0}$ associated to the third order $\mathcal{Z_N}^{(3)}$ of eq.A.39, and the p^0 coupling $a_{2,0}$ associated to the fourth order $\mathcal{Z_N}^{(4)}$. The latter is obtained via:

$$\mathcal{Z_N}^{(4)} = 1+\frac{\partial\Sigma^{(3)}}{\partial \not{p}}\bigg|_{\not{p}=M_0-4c_1m_\pi^2}+\frac{\partial\Sigma^{(4)}}{\partial \not{p}}\bigg|_{\not{p}=M_0}. \qquad (A.57)$$

The fourth order self energy $\Sigma^{(4)}$ involves the two graphs of figure A.2, which

Figure A.2: The loop diagrams contributing to the $\mathcal{O}(p^4)$ nucleon self energy. The cross denotes a vertex from $\mathcal{L}_{\pi N}^{(2)}$.

include vertices generated by the Lagrangian $\mathcal{L}_{\pi N}^{(2)}$ [FMMS00]. One has:

$$\Sigma^{(4)} = \Sigma^{(4)}_{\text{tadpole}} + \Sigma^{(4)}_{c_1} \,, \tag{A.58}$$

with

$$\Sigma^{(4)}_{\text{tadpole}} = \frac{3m_\pi^2}{F_\pi^2}\Delta_\pi\left(2c_1 - \frac{p^2}{4M_0^2}c_2 - c_3\right), \tag{A.59}$$

$$\begin{aligned}\Sigma^{(4)}_{c_1} &= -\frac{3g_A^2}{4F_\pi^2}(4m_\pi^2 c_1)\Big\{m_\pi^2 I_{11}(M_0^2, m_\pi^2, p^2) + 2M_0 \not{p} I_{11}^{(1)}(M_0^2, m_\pi^2, p^2) \\ &\quad +\Delta_N + 2M_0\Big[(\not{p} + M_0)\frac{\partial}{\partial M_0^2}\left(m_\pi^2 I_{11}(M_0^2, m_\pi^2, p^2) - \Delta_N\right) \\ &\quad +(M_0^2 - p^2)\not{p}\frac{\partial}{\partial M_0^2} I_{11}^{(1)}(M_0^2, m_\pi^2, p^2)\Big]\Big\}.\end{aligned} \tag{A.60}$$

A.7.2 Isoscalar vector channel

As far as the isoscalar vector channel is concerned, the main contribution to the generalized form factors, namely the one due to the Feynman diagram shown in figure 2.2 and called $f3$) in figure A.1, has already been given in section A.5.2. Analogously to the leading order, the amplitudes of the remaining diagrams depicted in figure A.1 can be simply expressed in terms of the results obtained in the isovector channel discussed in the previous subsection. They read:

$$\overline{Amp}_{a3+b3} = \frac{3}{2}\frac{h_A^0\,\eta^\dagger \mathbf{1}\eta}{h_A^v\,\eta^\dagger\tau^a\eta}\,Amp_{a3+b3}, \tag{A.61}$$

$$\overline{Amp}_{c3} = -3\,\frac{b_{2,0}^s\,\eta^\dagger\mathbf{1}\eta}{b_{2,0}^v\,\eta^\dagger\tau^a\eta}\,Amp^{c3}, \tag{A.62}$$

$$\overline{Amp}_{d3} = 0, \tag{A.63}$$

$$\begin{aligned}\overline{Amp}_{e3} &= i\ \eta^\dagger\frac{\tau^a}{2}\eta\ \overline{u}(p')\Big[-i\frac{b_{2,0}^s}{2M_0}\ \Delta^\alpha\sigma_{\alpha\{\mu}\overline{p}_{\nu\}}\ \mathcal{Z}_N{}^{(3)} \\ &\quad + a_{2,0}^s\ \gamma_{\{\mu}\overline{p}_{\nu\}}\ \mathcal{Z}_N{}^{(4)}\Big]\,u(p).\end{aligned} \tag{A.64}$$

Appendix **B**

Appendix to VVCS

B.1 Feynman rules for VVCS

In this section we collect the expressions of the vertices used for the calculation of the $\mathcal{O}(\epsilon^3)$ amplitudes corresponding to the diagrams of figs. 3.3, 3.5 and 3.4.

We make use of the following notation:
l^μ: pion four-momentum
ε: photon polarization vector
α, β Dirac index
a, b, i, j isospin indices

From $\mathcal{L}^{(1)}_{\pi N}$ of eq.(1.7)

◇ photon:

$$i\frac{e}{2}(1+\tau^3)\not\varepsilon \qquad \text{(B.1)}$$

From $\mathcal{L}^{(2)}_{\pi\pi}$ of eq.(1.2)

◇ π in (momentum l,isospin i), photon, π out (momentum l', isospin j):

$$-\,e\,\epsilon^{3ij}\varepsilon\cdot(l+l') \qquad \text{(B.2)}$$

From $\mathcal{L}^{(1)}_{\pi N\Delta}$ of eq.(3.57)

◇ N in, Δ out (isospin i), π out (isospin a):

$$\frac{g_{\pi N\Delta}}{F_\pi}\,l^\mu\delta^{ia} \qquad \text{(B.3)}$$

◇ N out, Δ in (isospin i), π in (isospin a):

$$-\frac{g_{\pi N\Delta}}{F_\pi}\, l^\mu \delta^{ia} \tag{B.4}$$

◇ N in, photon, Δ out (isospin i), π out (isospin a):

$$i\,\frac{g_{\pi N\Delta}\, e}{F_\pi}\varepsilon^\alpha \epsilon^{a3i} \tag{B.5}$$

◇ photon, Δ in (isospin i), π in (isospin a), N out:

$$i\,\frac{g_{\pi N\Delta}\, e}{F_\pi}\varepsilon^\alpha \epsilon^{a3i} \tag{B.6}$$

From $\mathcal{L}^{(1)}_{\pi\Delta}$ of eq.(3.60)

◇ Δ propagator

$$G^{ij}_{\mu\nu}(p) = -i\frac{\not p + m_\Delta}{p^2 - m_\Delta^2}\big\{g_{\mu\nu} - \frac{1}{d-1}\gamma_\mu\gamma_\nu - \frac{(d-2)}{(d-1)}\frac{p_\mu p_\nu}{m_\Delta^2} + \frac{p_\mu\gamma_\nu - p_\nu\gamma_\mu}{(d-1)m_\Delta}\big\}\xi^{ij} \tag{B.7}$$

◇ Δ in (Dirac index β, isospin j), photon, Δ out (Dirac index α, isospin i):

$$-ie\big(\frac{1}{2}(1+\tau^3)\delta^{ij} - i\epsilon^{ij3}\big)\big(\not\varepsilon g^{\alpha\beta} + \frac{d-2}{4}\gamma^\alpha \not\varepsilon \gamma^\beta\big) \tag{B.8}$$

From $\mathcal{L}^{(2)}_{\gamma N\Delta}$ of eq.(3.62)

◇ photon, Δ (isospin i), N

$$ie\frac{b_1}{2M_0}(\not q\varepsilon^\mu - q^\mu \not\varepsilon)\gamma_5\delta^{3i} \tag{B.9}$$

B.2 Loop integrals

In this section we define the pertinent loop functions appearing in the amplitudes given in section 3.5 and B.3.
We make use of the general expression of a loop diagram with generic dimension d and arbitrary number of propagators:

$$\begin{aligned}&\text{II}[d+n][\{\{p_1,m_1\},k_1\},\ldots\{\{p_N,m_N\},k_N\}] = \\ &\quad\int \frac{d^{d+n}l}{(2\pi)^{d+n}} \frac{1}{[(l+p_1)^2-m_1^2+i\varepsilon]^{k_1}\ldots[(l+p_N)^2-m_N^2+i\varepsilon]^{k_N}}\end{aligned} \tag{B.10}$$

As shown in reference [Dav91], each integral of the kind of eq.(B.10) can be written in terms of scalar loop functions. Following references [BKM92b][BHM03b], we define the following two- and three-point scalar loop functions:

$$\begin{aligned}\Delta_\pi &= \frac{1}{i}\int\frac{d^dl}{(2\pi)^d}\,\frac{1}{[m_\pi^2-l^2-i\epsilon]}\,,\\ J_0[s] &= \frac{1}{i}\int\frac{d^dl}{(2\pi)^d}\,\frac{1}{[m_\pi^2-l^2-i\epsilon][m_\Delta^2-(p+q-l)^2-i\epsilon]}\,,\\ J_0^{\pi\pi}[Q^2] &= \frac{1}{i}\int\frac{d^dl}{(2\pi)^d}\,\frac{1}{[m_\pi^2-l^2-i\epsilon][m_\pi^2-(l+q)^2-i\epsilon]}\,,\\ \gamma_0[s] &= \frac{1}{i}\int\frac{d^dl}{(2\pi)^d}\,\frac{1}{[m_\pi^2-l^2-i\epsilon][m_\pi^2-(l+q)^2-i\epsilon][m_\Delta^2-(p-l)^2-i\epsilon]}\,,\\ \Gamma_0[s] &= \frac{1}{i}\int\frac{d^dl}{(2\pi)^d}\,\frac{1}{[m_\pi^2-l^2-i\epsilon][m_\Delta^2-(p+q-l)^2-i\epsilon][m_\Delta^2-(p-l)^2-i\epsilon]}\,,\end{aligned} \tag{B.11}$$

with $s=(p+q)^2$, $p^2=M_0^2$ and $Q^2=-q^2$.
According to **diagrams e)+d)** of fig.3.4 we define further loop functions, which by means of Passarino-Feldmann reduction can also be written in terms of the only scalar loop functions defined above:

$$\begin{aligned}&\frac{1}{i}\int\frac{d^dl}{(2\pi)^d}\,\frac{l_\mu}{[m_\pi^2-l^2-i\epsilon][m_\pi^2-(l+q)^2-i\epsilon][m_\Delta^2-(p-l)^2-i\epsilon]} = \\ &\qquad\{q_\mu\gamma_1(s)+p_\mu\gamma_2(s)\},\\ &\frac{1}{i}\int\frac{d^dl}{(2\pi)^d}\,\frac{l_\mu l_\nu}{[m_\pi^2-l^2-i\epsilon][m_\pi^2-(l+q)^2-i\epsilon][m_\Delta^2-(p-l)^2-i\epsilon]} = \\ &\qquad g_{\mu\nu}\gamma_3(s)+q_\mu q_\nu\gamma_4(s)+(p_\mu q_\nu+p_\nu q_\mu)\gamma_5(s)+p_\mu p_\nu\gamma_6(s).\end{aligned} \tag{B.12}$$

In what follows we provide the basic correspondence between the general basis defined by eq.(B.10) and the scalar basis of eq.(B.11):

$$\begin{aligned}-i\,\Delta_\pi &= \text{II}[d][\{\{0,m_\pi\},1\}]\,,\\ i\,J_0[s] &= \text{II}[d][\{\{0,m_\pi\},1\},\{\{q+p,m_\Delta\},1\}]\,,\\ i\,J_0^{\pi\pi}[Q^2] &= \text{II}[d][\{\{0,m_\pi\},1\},\{\{q,m_\pi\},1\}]\,,\\ -i\,\Gamma_0[s] &= \text{II}[d][\{\{0,m_\pi\},1\},\{\{q+p,m_\Delta\},1\},\{\{p,m_\Delta\},1\}]\,,\\ -i\,\gamma_0[s] &= \text{II}[d][\{\{0,m_\pi\},1\},\{\{q,m_\pi\},1\},\{\{q+p,m_\Delta\},1\}]\,.\end{aligned} \tag{B.13}$$

B.3 The VVCS amplitudes in covariant SSE at $\mathcal{O}(\epsilon^3)$

In this section we provide the $\mathcal{O}(\epsilon^3)$ amplitudes corresponding to the diagrams **b)+c)**, **f)**, **g)**, **k)+l)** and **m)+n)** depicted in fig.3.4, written in terms of the basic integral of eq.(B.10).
The various parameters and couplings are defined in chapter 1 and in section 3.4.
Note that in what follows the kinematical condition $s = M_0^2$ in the loop function is expressed via the introduction of a new momentum $pr \neq p$ such that the equation $q^2 + 2M_0\nu = 0$ is fulfilled.

Diagram b)+c)

$$\mathbf{A}(\nu, Q^2) = \frac{1}{96(-1+d)^3 F^2 M_0} \left(ie^2 \mathrm{g}\pi\mathrm{N}\Delta^2 (5+\tau[3])\right.$$

$$\left(\frac{1}{ds^2 m_\Delta^4 M_0^2}\left(\left(16d^4 s^2 m_\pi^2 m_\Delta M_0^3 + 16 s m_\pi^2 M_0^2 \left(-\left(Q^2+s\right)\left(Q^2+3s\right) + 2\left(Q^2+9s\right) m_\Delta M_0 + 2sM_0^2 + 2m_\Delta M_0^3 + M_0^4\right) + 4d\left(s^2 M_0^2\left(-Q^2-s+M_0^2\right)\left(-Q^2+s+3M_0^2\right) - \right.\right.\right.$$

$$2sm_\Delta M_0^3\left(2\left(Q^4+5Q^2s+s^2\right)+\left(3Q^2-7s\right)M_0^2+M_0^4\right)+2m_\Delta^5 M_0\left(-4s^2+\left(Q^2+9s\right)M_0^2+M_0^4\right)-sm_\Delta^2 M_0^2\left(\left(Q^2-8s\right)\left(Q^2+s\right)+\left(Q^2+4s\right)M_0^2+4M_0^4\right)-$$

$$2sm_\Delta^3 M_0\left(-2s\left(Q^2+s\right)-\left(Q^2-15s\right)M_0^2+5M_0^4\right)+m_\Delta^4\left(2s^2\left(Q^2+s\right)-\left(Q^4+12Q^2s+21s^2\right)M_0^2+6sM_0^4+M_0^6\right)+$$

$$m_\pi^4\left(2s^2\left(Q^2+s\right)-8s^2 m_\Delta M_0+\left(-Q^4+2Q^2s+s^2\right)M_0^2+2\left(Q^2-s\right)m_\Delta M_0^3-4sM_0^4+2m_\Delta M_0^5+M_0^6\right)+2m_\pi^2\left(-sM_0^2\left(-Q^2-s+M_0^2\right)\left(4Q^2+13s+3M_0^2\right)-\right.$$

$$m_\Delta^2\left(-Q^2-s+M_0^2\right)\left(-2s^2+\left(Q^2+3s\right)M_0^2+M_0^4\right)-2m_\Delta^3 M_0\left(-4s^2+\left(Q^2+4s\right)M_0^2+M_0^4\right)-2sm_\Delta M_0\left(s\left(Q^2+s\right)+\left(6Q^2+40s\right)M_0^2+3M_0^4\right)\Big)\Big)+$$

$$d^3\left(s^2M_0^2\left(-Q^2-s+M_0^2\right)\left(-Q^2+s+3M_0^2\right)+m_\pi^4\left(-Q^2-s+M_0^2\right)\left(-2s^2+\left(Q^2-3s\right)M_0^2+M_0^4\right)-2sm_\Delta M_0^3\left(3Q^4+8Q^2s+s^2+\left(4Q^2-2s\right)M_0^2+M_0^4\right)-\right.$$

$$2sm_\Delta^2 M_0^2\left(\left(Q^2+s\right)\left(2Q^2+s\right)+\left(3Q^2+4s\right)M_0^2+3M_0^4\right)-2m_\Delta^3 M_0\left(-2s^2\left(Q^2+s\right)+\left(Q^4+s^2\right)M_0^2+2Q^2M_0^4+M_0^6\right)+m_\Delta^4\left(2s^2\left(Q^2+s\right)+\left(-Q^4+2Q^2s+9s^2\right)M_0^2+4sM_0^4+M_0^6\right)$$

$$+2m_\pi^2\left(-sM_0^2\left(-Q^2-s+M_0^2\right)\left(2Q^2+7s+M_0^2\right)+m_\Delta M_0\left(-2s^2\left(Q^2+s\right)+\left(Q^4-4Q^2s-75s^2\right)M_0^2+2\left(Q^2-2s\right)M_0^4+M_0^6\right)-m_\Delta^2\left(2s^2\left(Q^2+s\right)-\left(Q^4-7s^2\right)M_0^2-2sM_0^4+M_0^6\right)\right)-$$

$$4d^2\left(s^2M_0^2\left(-Q^2-s+M_0^2\right)\left(-Q^2+s+3M_0^2\right)+m_\Delta^5M_0\left(-4s^2+\left(Q^2+3s\right)M_0^2+M_0^4\right)-sm_\Delta M_0^3\left(5Q^4+18Q^2s+3s^2+\left(7Q^2-9s\right)M_0^2+2M_0^4\right)-\right.$$

$$sm_\Delta^2M_0^2\left(2\left(Q^2-s\right)\left(Q^2+s\right)+3\left(Q^2+3s\right)M_0^2+5M_0^4\right)+m_\Delta^4\left(2s^2\left(Q^2+s\right)-\left(Q^2+s\right)\left(Q^2+5s\right)M_0^2+2sM_0^4+M_0^6\right)-$$

$$m_\Delta^3M_0\left(-4s^2\left(Q^2+s\right)+\left(Q^4+2Q^2s+15s^2\right)M_0^2+2\left(Q^2+4s\right)M_0^4+M_0^6\right)+m_\pi^4\left(2s^2\left(Q^2+s\right)-4s^2m_\Delta M_0+\left(-Q^4+2Q^2s+s^2\right)M_0^2+\right.$$

$$\left(Q^2-s\right)m_\Delta M_0^3-4sM_0^4+m_\Delta M_0^5+M_0^6\Big)+m_\pi^2\left(-sM_0^2\left(-Q^2-s+M_0^2\right)\left(5Q^2+17s+3M_0^2\right)-2m_\Delta^3M_0\left(-4s^2+\left(Q^2+s\right)M_0^2+M_0^4\right)+\right.$$

$$m_\Delta M_0\left(-4s^2\left(Q^2+s\right)+\left(Q^4-14Q^2s-121s^2\right)M_0^2+2\left(Q^2-4s\right)M_0^4+M_0^6\right)-m_\Delta^2\left(4s^2\left(Q^2+s\right)+\left(-2Q^4-3Q^2s+7s^2\right)M_0^2-sM_0^4+2M_0^6\right)\Big)\Big)\Big)\Big)\,\mathrm{II}[d]\left[\left\{\{0,m_\pi\},1\right\}\right]$$

$$-\frac{1}{m_\Delta^4}\left(2\left(\frac{(-2+d)m_\Delta\left(-m_\pi-m_\Delta+M_0\right)\left(m_\pi-m_\Delta+M_0\right)\left(-m_\pi+m_\Delta+M_0\right)\left(m_\pi+m_\Delta+M_0\right)\left(m_\pi^2-m_\Delta^2+M_0^2\right)}{M_0}+\right.\right.$$

$$4(-1+d)m_\Delta\left(-(-9+2d)m_\Delta^4M_0+(-2+d)M_0\left(-2Q^2-s+M_0^2\right)\left(-m_\pi^2+M_0^2\right)+(-1+d)m_\Delta^3\left(-Q^2-s+3M_0^2\right)+\right.$$

$$m_\Delta^2M_0\left((-2+d)s+(-9+2d)m_\pi^2+(13-3d)M_0^2\right)+m_\Delta\left(m_\pi-M_0\right)\left(m_\pi+M_0\right)\left((-1+d)\left(Q^2+s\right)+3(-3+d)M_0^2\right)\Big)-$$

$$\frac{1}{M_0^2}\left(\left(m_\pi-m_\Delta-M_0\right)\left(m_\pi+m_\Delta-M_0\right)\left(m_\pi-m_\Delta+M_0\right)\left(m_\pi+m_\Delta+M_0\right)\left(7(-2+d)m_\Delta^3M_0+(-2+d)^2\left(m_\pi-M_0\right)\left(m_\pi+M_0\right)\left(-Q^2-s+M_0^2\right)+(-2+d)\right.\right.$$

$$m_\Delta M_0\left(-2\left(Q^2+s\right)-7m_\pi^2+11M_0^2\right)-m_\Delta^2\left(-(-2+d)^2\left(Q^2+s\right)+(24+(-8+d)d)M_0^2\right)\Big)\Big)-\frac{1}{M_0}\left(2(-1+d)m_\Delta\left(m_\pi^2-m_\Delta^2+M_0^2\right)\right.$$

$$\left(-2(-1+d)m_\Delta^3M_0+2(-5+2d)m_\Delta\left(m_\pi-M_0\right)M_0\left(m_\pi+M_0\right)+(-2+d)\left(-m_\pi+M_0\right)\left(m_\pi+M_0\right)\left(-Q^2-s+M_0^2\right)+m_\Delta^2\left(2\left(Q^2+s\right)+d\left(-Q^2-s+M_0^2\right)\right)\right)\Big)\Big)\Big)$$

$$\mathrm{II}[d]\left[\left\{\{0,m_\pi\},1\right\},\left\{\{p,m_\Delta\},1\right\}\right]\Big)-\frac{1}{s^2m_\Delta^4}\left(\left(2(-2+d)s^2m_\Delta M_0\left(d\left(Q^2-s\right)^2+12Q^2s-2\left((-3+2d)Q^2+s\right)M_0^2-(-2+d)M_0^4\right)-\right.\right.$$

$$m_\Delta^6\left(-\left(Q^2+s\right)\left((-2+d)^2Q^2+(44-d(20+3d))s\right)+4(6+(-2+d)d)sM_0^2+(-2+d)^2M_0^4\right)+$$

$$s^2m_\Delta^2\left(\left(Q^2+s\right)\left(44s+d\left((-4+3d)Q^2+7(-4+d)s\right)\right)+4\left((11+5(-3+d)d)Q^2+(-6+d)s\right)M_0^2-(-2+d)(-10+7d)M_0^4\right)+$$

$$sm_\Delta^4\left(\left(Q^2+s\right)\left(-100s+d\left((-4+3d)Q^2+(52+5d)s\right)\right)+2\left((2+3(-2+d)d)Q^2+80s+6(-11+d)ds\right)M_0^2+(-2+d)(-10+7d)M_0^4\right)+$$

$$(-2+d)^2s^3\left(-s^2+\left(-Q^2+M_0^2\right)^2\right)+4m_\Delta^7M_0\left(3(-6+d)s+(-2+d)\left(Q^2+M_0^2\right)\right)+(-2+d)m_\pi^6\left((-2+d)\left(Q^2-3s+M_0^2\right)\left(-Q^2-s+M_0^2\right)-4m_\Delta M_0\left(Q^2-s+M_0^2\right)\right)+$$

$$2m_\Delta^5M_0\left((128+d(-76+7d))s^2+(-2+d)d\left(Q^2+M_0^2\right)^2-6s\left(dQ^2+(-4+3d)M_0^2\right)\right)+$$

$$4sm_\Delta^3M_0\left(2(-3+2(-3+d)d)s^2+(-2+d)\left((-2+d)Q^2-M_0^2\right)\left(Q^2+M_0^2\right)+3s\left((14+d(-14+3d))Q^2+(-4+3d)M_0^2\right)\right)+$$

$$m_\pi^4\left(-(-2+d)^2s\left(3Q^2-7s+M_0^2\right)\left(-Q^2-s+M_0^2\right)+4m_\Delta^3M_0\left((-14+d)s+3(-2+d)\left(Q^2+M_0^2\right)\right)-\right.$$

$$m_\Delta^2\left(-Q^2-s+M_0^2\right)\left((12+(8-5d)d)s+3(-2+d)^2\left(Q^2+M_0^2\right)\right)+2(-2+d)m_\Delta M_0\left(d\left(Q^2-3s+M_0^2\right)\left(Q^2-s+M_0^2\right)+6s\left(3Q^2-s+M_0^2\right)\right)\Big)+$$

$$m_\pi^2\left(-(-2+d)^2s^2\left(-Q^2-s+M_0^2\right)\left(-3Q^2+5s+M_0^2\right)-4(-2+d)sm_\Delta M_0\left(dQ^4-(-16+5d)Q^2s+2(-1+d)s^2+\left(Q^2+dQ^2+s-2ds\right)M_0^2+M_0^4\right)+\right.$$
$$m_\Delta^4\left(-(-2+d)\left(Q^2+s\right)\left(3(-2+d)Q^2-(34+5d)s\right)-4(-10+d)sM_0^2+3(-2+d)^2M_0^4\right)-$$
$$2sm_\Delta^2\left(\left(Q^2+s\right)\left((6+d(-8+3d))Q^2+(52+d(-38+9d))s\right)+2\left((-1+(-1+d)d)Q^2-6(5+(-4+d)d)s\right)M_0^2+(-2+d)(-4+3d)M_0^4\right)-$$
$$4m_\Delta^5M_0\left((-34+5d)s+3(-2+d)\left(Q^2+M_0^2\right)\right)-4m_\Delta^3M_0\left((-42+d(4+5d))s^2+(-2+d)d\left(Q^2+M_0^2\right)^2-\right.$$
$$\left.\left.2s\left((9+(-5+d)d)Q^2+\left(-3+d+d^2\right)M_0^2\right)\right)\right)\mathrm{II}[d]\left[\{\{0,m_\pi\},1\},\{\{p+q,m_\Delta\},1\}\right]-$$
$$\frac{1}{m_\Delta^3}\left(8(-1+d)\left((-2+d)Q^4\left(m_\pi-M_0\right)M_0\left(m_\pi+M_0\right)-2(-5+d)m_\Delta^4M_0\left(-Q^2-2s+2M_0^2\right)+Q^2m_\Delta^2M_0\left((-2+d)\left(Q^2+2s\right)+2(-5+d)m_\pi^2+(14-4d)M_0^2\right)+\right.\right.$$
$$m_\Delta^5\left(-(-4+d)\left(Q^2+s\right)+(-8+d)M_0^2\right)+Q^2m_\Delta\left(m_\pi-M_0\right)\left(m_\pi+M_0\right)\left((-1+d)\left(Q^2+s\right)+(-5+3d)M_0^2\right)+$$
$$\left.m_\Delta^3\left(\left(Q^2+s\right)\left((-1+d)\left(Q^2+2s\right)-(-4+d)m_\pi^2\right)+\left((-7+2d)Q^2-(8+d)s+(-8+d)m_\pi^2\right)M_0^2-(-14+d)M_0^4\right)\right)\mathrm{II}[d]\left[\{\{0,m_\pi\},1\},\{\{p+q,m_\Delta\},1\},\{\{p,m_\Delta\},1\}\right]$$
$$+\frac{1}{m_\Delta^2}(32(-1+d)\pi\left(\left(Q^2-(-6+d)m_\Delta^2\right)\left(\left(Q^2+s\right)^2+2\left(Q^2-s\right)M_0^2+M_0^4\right)-\left(Q^2+s+M_0^2\right)\left(2m_\Delta^4+(-3+2d)Q^2\left(m_\pi-M_0\right)\left(m_\pi+M_0\right)-m_\Delta^2\left(5Q^2+4s-ds+2m_\pi^2+(-6+d)M_0^2\right)\right)\right)$$
$$\mathrm{II}[2+d]\left[\{\{0,m_\pi\},1\},\{\{p,m_\Delta\},1\},\{\{p+q,m_\Delta\},2\}\right])+$$
$$\frac{1}{m_\Delta^4}\left(16(-1+d)\pi\left(-8(-1+d)m_\Delta^5M_0-2(-2+d)Q^2m_\Delta M_0\left(Q^2+s+2m_\pi^2-3M_0^2\right)+(-2+d)^2Q^2\left(m_\pi-M_0\right)\left(m_\pi+M_0\right)\left(-Q^2-s+M_0^2\right)-\right.\right.$$
$$8m_\Delta^4\left((-3+d)\left(Q^2+s\right)+2M_0^2\right)+4m_\Delta^3M_0\left(9Q^2-2dQ^2+11s-3ds+2(-1+d)m_\pi^2+(-9+d)M_0^2\right)-$$
$$\left.m_\Delta^2\left((-2+d)\left(Q^2+s\right)\left(dQ^2+2(-1+d)s\right)+\left(-(-8+d)(-2+d)Q^2-2(12+(-5+d)d)s\right)M_0^2-4(-5+d)M_0^4+2(-8+(-1+d)d)m_\pi^2\left(-Q^2-s+M_0^2\right)\right)\right)$$
$$\mathrm{II}[2+d]\left[\{\{0,m_\pi\},1\},\{\{p+q,m_\Delta\},1\},\{\{p,m_\Delta\},1\}\right])+\frac{1}{m_\Delta^2}\left(64(-1+d)\pi M_0^2\left(-2m_\Delta^4+(-3+2d)Q^2\left(-m_\pi+M_0\right)\left(m_\pi+M_0\right)+m_\Delta^2\left(5Q^2+4s-ds+2m_\pi^2+(-6+d)M_0^2\right)\right)\right.$$
$$\mathrm{II}[2+d]\left[\{\{0,m_\pi\},1\},\{\{p+q,m_\Delta\},1\},\{\{p,m_\Delta\},2\}\right])+\frac{256(-1+d)\pi^2\left(Q^2-(-6+d)m_\Delta^2\right)\left(\left(Q^2+s\right)^2+2\left(Q^2-s\right)M_0^2+M_0^4\right)\mathrm{II}[4+d][\{\{0,m_\pi\},1\},\{\{p,m_\Delta\},1\},\{\{p+q,m_\Delta\},3\}]}{m_\Delta^2}-$$
$$\left.\left.128(-4+d)(-1+d)\pi^2\left(\left(Q^2+s\right)^2+2\left(Q^2-s\right)M_0^2+M_0^4\right)\mathrm{II}[4+d]\left[\{\{0,m_\pi\},1\},\{\{p+q,m_\Delta\},2\},\{\{p,m_\Delta\},2\}\right]\right)\right)$$

$$\mathbf{B}(\nu,Q^2)=\frac{1}{96(-1+d)^3F^2m_\Delta^4M_0}\left(ie^2\mathrm{g}\pi\mathrm{N}\Delta^2(5+\tau[3])\left(\left(\left(\frac{2(-2+d)(-1+d)\left(-\frac{4sm_\pi^2}{d}+\frac{\left(s+m_\pi^2-m_\Delta^2\right)^2}{-1+d}\right)M_0\left(-8m_\Delta+(-2+d)M_0\right)}{s^2}-\frac{4(-2+d)\left(s+m_\pi^2-m_\Delta^2\right)M_0\left((-3+d)m_\Delta+(-2+d)M_0\right)}{s}+\right.\right.\right.\right.$$
$$16(-2+d)(-1+d)M_0\left((-1+d)m_\Delta+(-2+d)M_0\right)-4(-2+d)^2(-1+d)\left(-m_\pi^2+m_\Delta^2+M_0^2\right)+$$
$$\frac{(-2+d)\left(8sm_\pi^2+d^2\left(s+m_\pi^2-m_\Delta^2\right)^2+2d\left(m_\pi^4+\left(s-m_\Delta^2\right)^2-2m_\pi^2\left(s+m_\Delta^2\right)\right)\right)M_0\left(-2m_\Delta\left(Q^2-3s+M_0^2\right)+(-2+d)M_0\left(Q^2-s+M_0^2\right)\right)}{ds^3}-\frac{1}{s^2}\left(2d\left(s+m_\pi^2-m_\Delta^2\right)M_0\right.$$
$$\left.\left(-6(-4+d)m_\Delta^3+(20+(-10+d)d)m_\Delta^2M_0+(-2+d)^2M_0\left(3Q^2-3s+m_\pi^2+2M_0^2\right)+(-2+d)m_\Delta\left((-6+d)Q^2-(-12+d)s+2m_\pi^2+(-2+d)M_0^2\right)\right)\right)+$$
$$\frac{1}{s}\left(4(-1+d)M_0\left((32+(-15+d)d)m_\Delta^3+4(10+(-5+d)d)m_\Delta^2M_0+(-2+d)^2M_0\left(3Q^2-3s+2m_\pi^2+M_0^2\right)+\right.\right.$$
$$\left.\left.\left.(-2+d)m_\Delta\left(-5Q^2+7s+(5+d)m_\pi^2+d\left(2Q^2-2s+M_0^2\right)\right)\right)\right)\right)\mathrm{II}[d]\left[\{\{0,m_\pi\},1\}\right]+$$
$$4(-2+d)(-1+d)\left(m_\pi-m_\Delta-M_0\right)\left(m_\pi+m_\Delta+M_0\right)\left((-2+d)m_\Delta^2+2m_\Delta M_0+(-2+d)\left(-m_\pi+M_0\right)\left(m_\pi+M_0\right)\right)\mathrm{II}[d]\left[\{\{0,m_\pi\},1\},\{\{p,m_\Delta\},1\}\right]+$$
$$M_0\left(-\frac{2(-2+d)\left(s+m_\pi^2-m_\Delta^2\right)\left(m_\pi^4+\left(s-m_\Delta^2\right)^2-2m_\pi^2\left(s+m_\Delta^2\right)\right)\left(-8m_\Delta+(-2+d)M_0\right)}{s^2}+\frac{4(-2+d)\left(m_\pi^4+\left(s-m_\Delta^2\right)^2-2m_\pi^2\left(s+m_\Delta^2\right)\right)\left((-3+d)m_\Delta+(-2+d)M_0\right)}{s}+\right.$$
$$8(-1+d)\left((-2+d)\left((-1+d)\left(Q^2-s\right)+(3+d)m_\pi^2\right)m_\Delta+3(-2+d)(-1+d)m_\Delta^3+(-2+d)^2\left(Q^2-s+m_\pi^2\right)M_0+(20+d(-10+3d))m_\Delta^2M_0\right)+$$
$$\frac{(-2+d)\left(s+m_\pi^2-m_\Delta^2\right)\left((2+d)m_\pi^4+(2+d)\left(s-m_\Delta^2\right)^2+2m_\pi^2\left((-4+d)s-(2+d)m_\Delta^2\right)\right)\left(2m_\Delta\left(Q^2-3s+M_0^2\right)-(-2+d)M_0\left(Q^2-s+M_0^2\right)\right)}{s^3}+$$

$$\frac{1}{s^2}\left(2\left(-4sm_\pi^2+d\left(s+m_\pi^2-m_\Delta^2\right)^2\right)\left(-6(-4+d)m_\Delta^3+(20+(-10+d)d)m_\Delta^2M_0+(-2+d)^2M_0\left(3Q^2-3s+m_\pi^2+2M_0^2\right)+\right.\right.$$
$$(-2+d)m_\Delta\left((-6+d)Q^2-(-12+d)s+2m_\pi^2+(-2+d)M_0^2\right)\Big)\Big)-\frac{1}{s}\left(4(-1+d)\left(s+m_\pi^2-m_\Delta^2\right)\right.$$
$$\left((32+(-15+d)d)m_\Delta^3+4(10+(-5+d)d)m_\Delta^2M_0+(-2+d)^2M_0\left(3Q^2-3s+2m_\pi^2+M_0^2\right)+(-2+d)m_\Delta\left(-5Q^2+7s+(5+d)m_\pi^2+d\left(2Q^2-2s+M_0^2\right)\right)\right)\Big)\Big)$$
$$\mathrm{II}[d]\left[\{\{0,m_\pi\},1\},\{\{p+q,m_\Delta\},1\}\right]+8(-1+d)M_0\left(m_\pi^2\left((-2+d)(-1+d)Q^2m_\Delta+(-2+d)^2Q^2M_0+12m_\Delta^2M_0\right)-\right.$$
$$(m_\Delta+M_0)\left(-(-2+d)(-1+d)\left(Q^2+2s\right)m_\Delta^2+(-2+d)Q^2m_\Delta M_0-12m_\Delta^3M_0+\left((-2+d)^2Q^2+2(8+(-3+d)d)m_\Delta^2\right)M_0^2\right)\Big)$$
$$\mathrm{II}[d]\left[\{\{0,m_\pi\},1\},\{\{p+q,m_\Delta\},1\},\{\{p,m_\Delta\},1\}\right]-32(-1+d)\pi M_0\left(-2(-1+(-4+d)d)m_\Delta^5-2(-7+d)dm_\Delta^4M_0+2(-2+d)^2Q^2M_0\left(-m_\pi^2+M_0^2\right)+\right.$$
$$(-2+d)Q^2m_\Delta\left(Q^2+s-(-3+d)m_\pi^2+(-4+d)M_0^2\right)-m_\Delta^3\left((-3+d)\left((-6+d)Q^2+2(-4+d)s\right)+(2-2(-4+d)d)m_\pi^2+(-26+6d)M_0^2\right)+$$
$$2m_\Delta^2M_0\left(-(2+(-5+d)d)Q^2-2(-2+d)(-1+d)s+(-14+(-1+d)d)m_\pi^2+(18+(-5+d)d)M_0^2\right)\Big)$$
$$\mathrm{II}[2+d]\left[\{\{0,m_\pi\},1\},\{\{p,m_\Delta\},1\},\{\{p+q,m_\Delta\},2\}\right]-64(-1+d)\pi m_\Delta M_0\left((-2+d)m_\Delta^2-2m_\Delta M_0+(-2+d)\left(-m_\pi+M_0\right)\left(m_\pi+M_0\right)\right)$$
$$\mathrm{II}[2+d]\left[\{\{0,m_\pi\},1\},\{\{p+q,m_\Delta\},1\},\{\{p,m_\Delta\},1\}\right]-$$
$$32(-1+d)\pi M_0^2\left(12m_\Delta^4+(-2+d)^2Q^2\left(-m_\pi+M_0\right)\left(m_\pi+M_0\right)-m_\Delta^2\left((-2+d)\left(dQ^2+2(-1+d)s\right)+12m_\pi^2-2(8+(-3+d)d)M_0^2\right)\right)$$
$$\mathrm{II}[2+d]\left[\{\{0,m_\pi\},1\},\{\{p+q,m_\Delta\},1\},\{\{p,m_\Delta\},2\}\right]-$$
$$256(-1+d)\pi^2M_0\left(4(-1+d)m_\Delta^5-2(-4+(-5+d)d)m_\Delta^4M_0+(-2+d)Q^2m_\Delta\left(Q^2+s+2m_\pi^2-3M_0^2\right)+(-2+d)^2Q^2M_0\left(-m_\pi^2+M_0^2\right)-\right.$$
$$2m_\Delta^3\left(9Q^2-2dQ^2+11s-3ds+2(-1+d)m_\pi^2+(-9+d)M_0^2\right)-m_\Delta^2M_0\left((-6+d)dQ^2+2(-2+d)(-1+d)s-2(-8+(-1+d)d)m_\pi^2+4(-5+d)M_0^2\right)\Big)$$
$$\mathrm{II}[4+d]\left[\{\{0,m_\pi\},1\},\{\{p,m_\Delta\},1\},\{\{p+q,m_\Delta\},3\}\right]-128(-1+d)\pi^2M_0^2$$
$$\left(-2(-8+(-1+d)d)m_\Delta^4+(-2+d)^2Q^2\left(-m_\pi+M_0\right)\left(m_\pi+M_0\right)+m_\Delta^2\left(-(-2+d)\left(dQ^2+2(-1+d)s\right)+2(-8+(-1+d)d)m_\pi^2-4(-5+d)M_0^2\right)\right)$$
$$\mathrm{II}[4+d]\left[\{\{0,m_\pi\},1\},\{\{p+q,m_\Delta\},2\},\{\{p,m_\Delta\},2\}\right]\Big)\Big)$$

Diagram f)

$$\mathbf{A}(\nu,Q^2)=-\frac{1}{96(-1+d)^3F^2m_\Delta^4M_0}$$
$$\left(ie^2\mathrm{g}\pi\mathrm{N}\Delta^2\left(-4(-2+d)^2Q^2\left(Q^2+s-M_0^2\right)+\frac{48(-2+d)^2(-1+d)m_\pi^2\left(-Q^2-s+M_0^2\right)}{d}-8(-2+d)\left(-Q^2-s+M_0^2\right)\left(-3(-2+d)m_\pi^2-2(1+d)m_\Delta^2+(-10+d)m_\Delta M_0+\right.\right.\right.$$
$$(-2+d)M_0^2\Big)-\frac{4(-2+d)\left(-Q^2-s+M_0^2\right)\left(m_\pi^2-m_\Delta^2+M_0^2\right)\left((-3+2d)m_\Delta^2-m_\Delta M_0-(-2+d)\left(m_\pi-M_0\right)\left(m_\pi+M_0\right)\right)}{M_0^2}+$$
$$\frac{1}{ds^2}\left((-2+d)\left(-4sm_\pi^2+d\left(s^2-m_\pi^4-m_\Delta^4+2m_\pi^2\left(2s+m_\Delta^2\right)\right)\right)\left(-(-2+d)Q^2\left(Q^2+s-M_0^2\right)-2m_\Delta M_0\left(Q^2-s+M_0^2\right)\right)\right)-$$
$$\frac{1}{ds^2}\left((-2+d)\left(4sm_\pi^2+d\left(m_\pi^4+\left(s-m_\Delta^2\right)^2-2m_\pi^2\left(s+m_\Delta^2\right)\right)\right)\left((-2+d)\left(-s+M_0^2\right)\left(-Q^2-s+M_0^2\right)+m_\Delta M_0\left(Q^2+7s+M_0^2\right)\right)\right)+$$
$$\frac{2(-2+d)(-1+d)m_\Delta\left(-Q^2-s+M_0^2\right)\left(-\frac{4m_\pi^2M_0^2}{d}+\frac{\left(m_\pi^2-m_\Delta^2+M_0^2\right)^2}{-1+d}\right)}{M_0^3}-$$
$$\frac{1}{-2Q^2-s+2M_0^2}\left(2\left(-2Q^2-s+m_\pi^2-m_\Delta^2+2M_0^2\right)\left(-2(-21+5d)m_\Delta^3M_0+(-2+d)m_\Delta M_0\left(-7Q^2-s+M_0^2\right)+(-2+d)^2\left(-Q^2-s+M_0^2\right)\left(-2Q^2-s+2M_0^2\right)+\right.\right.$$
$$(-2+d)m_\pi^2\left((-2+d)\left(Q^2+s\right)+8m_\Delta M_0-(-2+d)M_0^2\right)-m_\Delta^2\left(-(2+d)\left(Q^2+s\right)+9(-2+d)M_0^2\right)\Big)\Big)-$$
$$\frac{1}{s}\left(2\left(s+m_\pi^2-m_\Delta^2\right)\left(2(-21+5d)m_\Delta^3M_0+(-2+d)^2\left(-s+m_\pi^2\right)\left(Q^2+s-M_0^2\right)+(-2+d)m_\Delta M_0\left(5Q^2-s-8m_\pi^2+M_0^2\right)+m_\Delta^2\left((2+d)\left(Q^2+s\right)+(-22+7d)M_0^2\right)\right)\right)+$$
$$\frac{1}{d\left(2Q^2+s-2M_0^2\right)^2}\left((-2+d)\left((-2+d)\left(-2Q^2-s+M_0^2\right)\left(-Q^2-s+M_0^2\right)+m_\Delta M_0\left(-13Q^2-7s+15M_0^2\right)\right)\right.$$
$$\left(4m_\pi^2\left(-2Q^2-s+2M_0^2\right)+d\left(m_\pi^4+\left(2Q^2+s+m_\Delta^2-2M_0^2\right)^2-2m_\pi^2\left(-2Q^2-s+m_\Delta^2+2M_0^2\right)\right)\right)\Big)+\frac{1}{d\left(2Q^2+s-2M_0^2\right)^2}$$

$$\left((-2+d)\left(-(-2+d)Q^2\left(Q^2+s-M_0^2\right)-2m_\Delta M_0\left(-3Q^2-s+M_0^2\right)\right)\left(4m_\pi^2\left(2Q^2+s-2M_0^2\right)-d\left(m_\pi^4+m_\Delta^4-\left(2Q^2+s-2M_0^2\right)^2-2m_\pi^2\left(-4Q^2-2s+m_\Delta^2+4M_0^2\right)\right)\right)\right)\right)$$
$$(5+\tau[3])\mathrm{II}[d]\left[\left\{\left\{0,m_\pi\right\},1\right\}\right]\right)-\frac{1}{48(-1+d)^3F^2m_\Delta^4M_0^4}\left(i(-2+d)e^2\mathrm{g\pi N\Delta}^2\left(m_\pi-m_\Delta-M_0\right)\left(m_\pi+m_\Delta-M_0\right)\left(m_\pi-m_\Delta+M_0\right)\left(m_\pi+m_\Delta+M_0\right)\right.$$
$$\left(-Q^2-s+M_0^2\right)\left(m_\Delta^3+2(-3+2d)m_\Delta^2M_0+2(-2+d)M_0\left(-m_\pi^2+M_0^2\right)-m_\Delta\left(m_\pi^2+3M_0^2\right)\right)(5+\tau[3])\mathrm{II}[d]\left[\left\{\left\{0,m_\pi\right\},1\right\},\left\{\left\{p,m_\Delta\right\},1\right\}\right]+$$
$$\frac{1}{96(-1+d)^3F^2m_\Delta^4M_0}\left(ie^2\mathrm{g\pi N\Delta}^2\left(m_\pi^4+\left(2Q^2+s+m_\Delta^2-2M_0^2\right)^2-2m_\pi^2\left(-2Q^2-s+m_\Delta^2+2M_0^2\right)\right)\left(\frac{(-2+d)\left(2Q^2+s+m_\pi^2-m_\Delta^2-2M_0^2\right)\left((-2+d)Q^2\left(Q^2+s-M_0^2\right)+2m_\Delta M_0\left(-3Q^2-s+M_0^2\right)\right)}{\left(2Q^2+s-2M_0^2\right)^2}+\right.\right.$$
$$\frac{1}{\left(2Q^2+s-2M_0^2\right)^2}\left((-2+d)\left(-2Q^2-s+m_\pi^2-m_\Delta^2+2M_0^2\right)\left((-2+d)\left(-2Q^2-s+M_0^2\right)\left(-Q^2-s+M_0^2\right)+m_\Delta M_0\left(-13Q^2-7s+15M_0^2\right)\right)\right)+$$
$$\frac{1}{-2Q^2-s+2M_0^2}\left(2\left(2(-21+5d)m_\Delta^3M_0-(-2+d)m_\Delta M_0\left(-7Q^2-s+M_0^2\right)-(-2+d)^2\left(-Q^2-s+M_0^2\right)\left(-2Q^2-s+2M_0^2\right)+m_\Delta^2\right.\right.$$
$$\left.\left.\left.\left(-(2+d)\left(Q^2+s\right)+9(-2+d)M_0^2\right)+(-2+d)m_\pi^2\left(-8m_\Delta M_0+(-2+d)\left(-Q^2-s+M_0^2\right)\right)\right)\right)\right)(5+\tau[3])\mathrm{II}[d]\left[\left\{\left\{0,m_\pi\right\},1\right\},\left\{\left\{p-q,m_\Delta\right\},1\right\}\right]\right)+$$
$$\frac{1}{48(-1+d)^3F^2m_\Delta^4M_0}\left(i(-2+d)e^2\mathrm{g\pi N\Delta}^2\left(Q^2+4m_\pi^2\right)\left(Q^2+s-M_0^2\right)\left(4(1+d)m_\Delta^2-2(-10+d)m_\Delta M_0+(-2+d)\left(Q^2+6m_\pi^2-2M_0^2\right)\right)\right.$$
$$\left.(5+\tau[3])\mathrm{II}[d]\left[\left\{\left\{0,m_\pi\right\},1\right\},\left\{\left\{q,m_\pi\right\},1\right\}\right]\right)-$$
$$\frac{1}{96(-1+d)^3F^2s^2m_\Delta^4M_0}\left(ie^2\mathrm{g\pi N\Delta}^2\left(m_\pi^4+\left(s-m_\Delta^2\right)^2-2m_\pi^2\left(s+m_\Delta^2\right)\right)\left((-2+d)sm_\Delta M_0\left(13Q^2+3s+5M_0^2\right)+(-2+d)^2s\left(-s^2+\left(-Q^2+M_0^2\right)^2\right)+\right.\right.$$
$$m_\Delta^3M_0\left(11(-6+d)s+(-2+d)\left(Q^2+M_0^2\right)\right)+(-2+d)m_\pi^2\left((-2+d)\left(Q^2-3s+M_0^2\right)\left(-Q^2-s+M_0^2\right)-m_\Delta M_0\left(Q^2+7s+M_0^2\right)\right)-$$
$$\left.\left.m_\Delta^2\left((-6+d)ds^2-2s\left((2+d)Q^2+(-3+d)(6+d)M_0^2\right)+(-2+d)^2\left(-Q^4+M_0^4\right)\right)\right)(5+\tau[3])\mathrm{II}[d]\left[\left\{\left\{0,m_\pi\right\},1\right\},\left\{\left\{p+q,m_\Delta\right\},1\right\}\right]\right)+$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^4M_0}\left(ie^2\mathrm{g\pi N\Delta}^2\pi\left(\left(Q^2+s\right)m_\Delta^2\left((-2+d)\left((-4+3d)Q^2+2(-1+d)s\right)-2(2+(-4+d)d)m_\Delta^2\right)-2\left(Q^2+s\right)m_\Delta\left((-2+d)Q^2+2(-5+d)m_\Delta^2\right)M_0+\right.\right.$$
$$\left((-2+d)^2Q^2\left(Q^2+s\right)-\left((20+d(-30+7d))Q^2+4s+6(-3+d)ds\right)m_\Delta^2+2(-2+(-4+d)d)m_\Delta^4\right)M_0^2+$$
$$\left.2m_\Delta\left((-2+d)Q^2+2(-5+d)m_\Delta^2\right)M_0^3-\left((-2+d)^2Q^2-4(-3+d)dm_\Delta^2\right)M_0^4+m_\pi^2\left((-2+d)^2Q^2-2(-2+(-3+d)d)m_\Delta^2\right)\left(-Q^2-s+M_0^2\right)\right)$$
$$\left.(5+\tau[3])\mathrm{II}[2+d]\left[\left\{\left\{0,m_\pi\right\},1\right\},\left\{\left\{p-q,m_\Delta\right\},1\right\},\left\{\left\{p,m_\Delta\right\},1\right\}\right]\right)+\frac{1}{6(-1+d)^2F^2m_\Delta^4M_0}$$
$$\left(ie^2\mathrm{g\pi N\Delta}^2\pi\left(-2(-23+6d)m_\Delta^5M_0+(-2+d)^2M_0^2\left(-Q^2-s+M_0^2\right)\left(-2Q^2-s+2M_0^2\right)+2m_\Delta^3M_0\left(-5(-3+d)Q^2+11s-3ds+(6+d)M_0^2\right)+\right.\right.$$
$$m_\Delta^4\left(-(-4+d)(2+d)\left(Q^2+s\right)+(26+(-10+d)d)M_0^2\right)-(-2+d)m_\Delta M_0\left(3\left(Q^2+s\right)\left(2Q^2+s\right)-\left(11Q^2+10s\right)M_0^2+7M_0^4\right)-$$
$$m_\Delta^2\left((-2+d)\left(Q^2+s\right)\left(2Q^2+(-1+2d)s\right)+\left((-16+(19-4d)d)Q^2-6(-2+d)(-1+d)s\right)M_0^2+(4+d(-13+4d))M_0^4\right)+$$
$$(-2+d)m_\pi^4\left(-8m_\Delta M_0+(-2+d)\left(-Q^2-s+M_0^2\right)\right)+m_\pi^2\left(2(-31+10d)m_\Delta^3M_0-(-2+d)^2\left(-Q^2-s+M_0^2\right)\left(-2Q^2-s+3M_0^2\right)+2(-2+d)m_\Delta M_0\left(3Q^2+4M_0^2\right)-\right.$$
$$\left.\left.\left.2m_\Delta^2\left(-(2+(-4+d)d)\left(Q^2+s\right)+(23+(-12+d)d)M_0^2\right)\right)\right)(5+\tau[3])\mathrm{II}[2+d]\left[\left\{\left\{0,m_\pi\right\},1\right\},\left\{\left\{q,m_\pi\right\},1\right\},\left\{\left\{p,m_\Delta\right\},1\right\}\right]\right)+$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^4M_0}\left(ie^2\mathrm{g\pi N\Delta}^2\pi\left(2(-23+6d)m_\Delta^5M_0+(-2+d)^2sM_0^2\left(-Q^2-s+M_0^2\right)+2m_\Delta^3M_0\left(7Q^2+11s-d\left(Q^2+3s\right)+(-28+5d)M_0^2\right)+\right.\right.$$
$$m_\Delta^4\left(-(-4+d)(2+d)\left(Q^2+s\right)+(-42+d(6+d))M_0^2\right)-(-2+d)m_\Delta M_0\left(-3s\left(Q^2+s\right)+\left(-3Q^2+2s\right)M_0^2+M_0^4\right)+$$
$$m_\Delta^2\left((-2+d)\left(Q^2+s\right)\left(4(-1+d)Q^2+(-1+2d)s\right)+\left((-12+(17-4d)d)Q^2+2\left(2+d-d^2\right)s\right)M_0^2+3(-4+d)M_0^4\right)+$$
$$(-2+d)m_\pi^4\left(8m_\Delta M_0+(-2+d)\left(-Q^2-s+M_0^2\right)\right)+$$
$$\left.m_\pi^2\left(-2(-31+10d)m_\Delta^3M_0-(-2+d)^2\left(-Q^2-s+M_0^2\right)\left(s+M_0^2\right)-2(-2+d)m_\Delta M_0\left(3Q^2+4M_0^2\right)-2m_\Delta^2\left(-(2+(-4+d)d)\left(Q^2+s\right)+(-19+d(4+d))M_0^2\right)\right)\right)$$
$$\left.(5+\tau[3])\mathrm{II}[2+d]\left[\left\{\left\{0,m_\pi\right\},1\right\},\left\{\left\{q,m_\pi\right\},1\right\},\left\{\left\{p+q,m_\Delta\right\},1\right\}\right]\right)+$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^4M_0}\left(ie^2\mathrm{g\pi N\Delta}^2\pi\left(2(-2+d)Q^2m_\Delta M_0\left(-Q^2-s+M_0^2\right)+4(-5+d)m_\Delta^3M_0\left(-Q^2-s+M_0^2\right)+\right.\right.$$
$$(-2+d)^2Q^2\left(m_\pi-M_0\right)\left(m_\pi+M_0\right)\left(-Q^2-s+M_0^2\right)+2m_\Delta^4\left(-(2+(-4+d)d)\left(Q^2+s\right)+(6+(-4+d)d)M_0^2\right)-$$
$$\left.m_\Delta^2\left((-2+d)\left(Q^2+s\right)\left(dQ^2+2(-1+d)s\right)+\left(-(20+(-10+d)d)Q^2-2(6+(-3+d)d)s\right)M_0^2+8M_0^4+2(-2+(-3+d)d)m_\pi^2\left(-Q^2-s+M_0^2\right)\right)\right)$$
$$\left.(5+\tau[3])\mathrm{II}[2+d]\left[\left\{\left\{0,m_\pi\right\},1\right\},\left\{\left\{p+q,m_\Delta\right\},1\right\},\left\{\left\{p,m_\Delta\right\},1\right\}\right]\right)+$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^4M_0}\left(ie^2\mathrm{g\pi N\Delta}^2\pi\left(-Q^2-s+M_0^2\right)\left(-2(-3+d)(-2+d)m_\Delta^6-8(-5+d)m_\Delta^5M_0+4(-2+d)Q^2m_\Delta\left(m_\pi-M_0\right)M_0\left(m_\pi+M_0\right)+\right.\right.$$

$$(-2+d)^2Q^2\left(m_\pi^2-M_0^2\right)^2+m_\Delta^4\left((-2+d)^2Q^2+4(2+(-4+d)d)m_\pi^2+4(12+(-6+d)d)M_0^2\right)-$$
$$4m_\Delta^3M_0\left((-2+d)Q^2-2(-5+d)\left(m_\pi-M_0\right)\left(m_\pi+M_0\right)\right)-2m_\Delta^2\left(m_\pi-M_0\right)\left(m_\pi+M_0\right)\left((-2+d)^2Q^2+(-2+(-3+d)d)\left(m_\pi-M_0\right)\left(m_\pi+M_0\right)\right)\Big)$$
$$(5+\tau[3])\mathrm{II}[2+d]\left[\left\{\left\{0,m_\pi\right\},1\right\},\left\{\left\{q,m_\pi\right\},1\right\},\left\{\left\{p+q,m_\Delta\right\},1\right\},\left\{\left\{p,m_\Delta\right\},1\right\}\right]\Big)+$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^3M_0}\left(8i(-2+d)e^2\mathrm{g\pi N\Delta}^2\pi^2M_0\left(-2Q^2-s+M_0^2\right)(5+\tau[3])\mathrm{II}[4+d]\left[\left\{\left\{0,m_\pi\right\},1\right\},\left\{\left\{q,m_\pi\right\},1\right\},\left\{\left\{p,m_\Delta\right\},1\right\}\right]\right)-$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^4M_0}\left(4ie^2\mathrm{g\pi N\Delta}^2\pi^2\left((-2+d)^2\left(m_\pi-M_0\right)\left(m_\pi+M_0\right)\left(-2Q^2-s+M_0^2\right)\left(-Q^2-s+M_0^2\right)-2m_\Delta^3M_0\left((21-9d)Q^2+3(5-2d)s+(-37+14d)M_0^2\right)+\right.\right.$$
$$m_\Delta^2\left((-2+d)\left(Q^2+s\right)\left(2Q^2+ds\right)-\left((38+d(-32+5d))Q^2+2(-3+d)(-5+3d)s\right)M_0^2+(66+d(-42+5d))M_0^4\right)-$$
$$(-2+d)m_\Delta M_0\left(-5\left(Q^2+s\right)\left(2Q^2+s\right)+\left(17Q^2+10s\right)M_0^2+3M_0^4+m_\pi^2\left(14Q^2+8s-16M_0^2\right)\right)\Big)$$
$$(5+\tau[3])\mathrm{II}[4+d]\left[\left\{\left\{0,m_\pi\right\},1\right\},\left\{\left\{q,m_\pi\right\},1\right\},\left\{\left\{p,m_\Delta\right\},2\right\}\right]\Big)+$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^3M_0}\left(8i(-2+d)e^2\mathrm{g\pi N\Delta}^2\pi^2M_0\left(-s+M_0^2\right)(5+\tau[3])\mathrm{II}[4+d]\left[\left\{\left\{0,m_\pi\right\},1\right\},\left\{\left\{q,m_\pi\right\},1\right\},\left\{\left\{p+q,m_\Delta\right\},1\right\}\right]\right)+$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^4M_0}\left(4ie^2\mathrm{g\pi N\Delta}^2\pi^2\left((-2+d)^2\left(m_\pi-M_0\right)\left(m_\pi+M_0\right)\left(-s+M_0^2\right)\left(-Q^2-s+M_0^2\right)-\right.\right.$$
$$2m_\Delta^3M_0\left(3(-3+d)Q^2+3(-5+2d)s+(-7+2d)M_0^2\right)-(-2+d)m_\Delta M_0\left(-5s\left(Q^2+s\right)-2\left(Q^2+4s\right)m_\pi^2+\left(7Q^2+10s\right)M_0^2+3M_0^4\right)-$$
$$m_\Delta^2\left(-(-2+d)\left(Q^2+s\right)\left(2(-1+d)Q^2+ds\right)+\left(-(14+(-8+d)d)Q^2-2(15+(-10+d)d)s\right)M_0^2+3(-2+(-2+d)d)M_0^4\right)\Big)$$
$$(5+\tau[3])\mathrm{II}[4+d]\left[\left\{\left\{0,m_\pi\right\},1\right\},\left\{\left\{q,m_\pi\right\},1\right\},\left\{\left\{p+q,m_\Delta\right\},2\right\}\right]\Big)+$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^4M_0}\left(4ie^2\mathrm{g\pi N\Delta}^2\pi^2\left(-(-2+d)^2Q^2\left(m_\pi-M_0\right)\left(m_\pi+M_0\right)\left(-Q^2-s+M_0^2\right)-2(-2+d)m_\Delta^3M_0\left(3Q^2-2s+2M_0^2\right)-\right.\right.$$
$$m_\Delta^2\left(-(-2+d)\left(Q^2+s\right)\left(Q^2+s-ds\right)+\left(-(10+(-8+d)d)Q^2+2\left(3+d-d^2\right)s\right)M_0^2+\left(-8+d+d^2\right)M_0^4\right)+$$
$$(-2+d)m_\Delta M_0\left(5Q^4+2Q^2s-s^2-2\left(Q^2-2s\right)M_0^2-3M_0^4+2m_\pi^2\left(2Q^2-s+M_0^2\right)\right)\Big)(5+\tau[3])\mathrm{II}[4+d]\left[\left\{\left\{0,m_\pi\right\},1\right\},\left\{\left\{q,m_\pi\right\},2\right\},\left\{\left\{-p,m_\Delta\right\},1\right\}\right]\Big)+$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^4M_0}\left(4ie^2\mathrm{g\pi N\Delta}^2\pi^2\left(-(-2+d)^2Q^2\left(m_\pi-M_0\right)\left(m_\pi+M_0\right)\left(-Q^2-s+M_0^2\right)-2(-2+d)m_\Delta^3M_0\left(-7Q^2-2s+2M_0^2\right)+\right.\right.$$
$$m_\Delta^2\left((-2+d)\left(Q^2+s\right)\left((-1+2d)Q^2+(-1+d)s\right)+\left((-30+(20-3d)d)Q^2-2(7+(-5+d)d)s\right)M_0^2+(-4+d)(-3+d)M_0^4\right)+$$
$$(-2+d)m_\Delta M_0\left(3Q^4+6Q^2s+s^2-2Q^2M_0^2-M_0^4+2m_\pi^2\left(-4Q^2-s+M_0^2\right)\right)\Big)(5+\tau[3])\mathrm{II}[4+d]\left[\left\{\left\{0,m_\pi\right\},1\right\},\left\{\left\{q,m_\pi\right\},2\right\},\left\{\left\{p,m_\Delta\right\},1\right\}\right]\Big)-$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^4M_0}\left(4ie^2\mathrm{g\pi N\Delta}^2\pi^2\left(-(-2+d)^2Q^2\left(m_\pi-M_0\right)\left(m_\pi+M_0\right)\left(-Q^2-s+M_0^2\right)-2m_\Delta^3M_0\left((13-5d)Q^2+(7-2d)s+(-7+2d)M_0^2\right)+\right.\right.$$
$$m_\Delta^2\left((-2+d)\left(Q^2+s\right)\left(Q^2+(-1+d)s\right)+\left(-(20+(-14+d)d)Q^2-2(-3+d)(-2+d)s\right)M_0^2+(-5+d)(-2+d)M_0^4\right)+$$
$$(-2+d)m_\Delta M_0\left(\left(Q^2+s\right)\left(7Q^2+s\right)-4Q^2M_0^2-M_0^4+2m_\pi^2\left(-4Q^2-s+M_0^2\right)\right)\Big)(5+\tau[3])\mathrm{II}[4+d]\left[\left\{\left\{0,m_\pi\right\},1\right\},\left\{\left\{q,m_\pi\right\},2\right\},\left\{\left\{-p+q,m_\Delta\right\},1\right\}\right]\Big)+$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^4M_0}\left(4ie^2\mathrm{g\pi N\Delta}^2\pi^2\left((-2+d)^2Q^2\left(m_\pi-M_0\right)\left(m_\pi+M_0\right)\left(-Q^2-s+M_0^2\right)+2m_\Delta^3M_0\left(Q^2+dQ^2+7s-2ds+(-7+2d)M_0^2\right)+\right.\right.$$
$$m_\Delta^2\left((-2+d)\left(Q^2+s\right)\left((-3+2d)Q^2+(-1+d)s\right)+\left((-12+(14-3d)d)Q^2+2\left(2+d-d^2\right)s\right)M_0^2+\left(-6+d+d^2\right)M_0^4\right)+$$
$$(-2+d)m_\Delta M_0\left(-5Q^4+s^2+4Q^2M_0^2+3M_0^4-4s\left(Q^2+M_0^2\right)-2m_\pi^2\left(2Q^2-s+M_0^2\right)\right)\Big)$$
$$(5+\tau[3])\mathrm{II}[4+d]\left[\left\{\left\{0,m_\pi\right\},1\right\},\left\{\left\{q,m_\pi\right\},2\right\},\left\{\left\{p+q,m_\Delta\right\},1\right\}\right]\Big)+\frac{1}{6(-1+d)^2F^2M_0}$$
$$\left(8i(-4+d)e^2\mathrm{g\pi N\Delta}^2\pi^2\left(\left(Q^2+s\right)^2+2\left(Q^2-s\right)M_0^2+M_0^4\right)(5+\tau[3])\mathrm{II}[4+d]\left[\left\{\left\{0,m_\pi\right\},1\right\},\left\{\left\{q,m_\pi\right\},1\right\},\left\{\left\{p,m_\Delta\right\},1\right\},\left\{\left\{p+q,m_\Delta\right\},2\right\}\right]\right)-$$
$$\frac{1}{6(-1+d)^2F^2M_0}\left(8i(-4+d)e^2\mathrm{g\pi N\Delta}^2\pi^2\left(\left(Q^2+s\right)^2+2\left(Q^2-s\right)M_0^2+M_0^4\right)(5+\tau[3])\mathrm{II}[4+d]\left[\left\{\left\{0,m_\pi\right\},1\right\},\left\{\left\{q,m_\pi\right\},1\right\},\left\{\left\{p+q,m_\Delta\right\},1\right\},\left\{\left\{p,m_\Delta\right\},2\right\}\right]\right)-$$
$$\frac{1}{6(-1+d)^2F^2M_0}\left(8i(-4+d)e^2\mathrm{g\pi N\Delta}^2\pi^2\left(\left(Q^2+s\right)^2+2\left(Q^2-s\right)M_0^2+M_0^4\right)(5+\tau[3])\mathrm{II}[4+d]\left[\left\{\left\{0,m_\pi\right\},1\right\},\left\{\left\{q,m_\pi\right\},2\right\},\left\{\left\{-p,m_\Delta\right\},1\right\},\left\{\left\{-p+q,m_\Delta\right\},1\right\}\right]\right)+\frac{1}{6(-1+d)^2F^2M_0}$$
$$\left(8i(-4+d)e^2\mathrm{g\pi N\Delta}^2\pi^2\left(\left(Q^2+s\right)^2+2\left(Q^2-s\right)M_0^2+M_0^4\right)(5+\tau[3])\mathrm{II}[4+d]\left[\left\{\left\{0,m_\pi\right\},1\right\},\left\{\left\{q,m_\pi\right\},2\right\},\left\{\left\{p+q,m_\Delta\right\},1\right\},\left\{\left\{p,m_\Delta\right\},1\right\}\right]\right)-$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^3M_0}\left(64i(-2+d)e^2\mathrm{g\pi N\Delta}^2\pi^3M_0\left(-2Q^2-s+M_0^2\right)\left(-Q^2-s+3M_0^2\right)(5+\tau[3])\mathrm{II}[6+d]\left[\left\{\left\{0,m_\pi\right\},1\right\},\left\{\left\{q,m_\pi\right\},1\right\},\left\{\left\{p,m_\Delta\right\},3\right\}\right]\right)-$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^3M_0}\left(64i(-2+d)e^2\mathrm{g\pi N\Delta}^2\pi^3M_0\left(-s+M_0^2\right)\left(Q^2+s+M_0^2\right)(5+\tau[3])\mathrm{II}[6+d]\left[\left\{\left\{0,m_\pi\right\},1\right\},\left\{\left\{q,m_\pi\right\},1\right\},\left\{\left\{p+q,m_\Delta\right\},3\right\}\right]\right)-$$

$$\frac{1}{6(-1+d)^2F^2m_\Delta^3M_0}\left(16i(-2+d)e^2\mathrm{g\pi N\Delta}^2\pi^3M_0\left(-\left(3Q^2-s\right)\left(Q^2+s\right)-2\left(Q^2+s\right)M_0^2+M_0^4\right)(5+\tau[3])\mathrm{II}[6+d]\left[\left\{\left\{0,m_\pi\right\},1\right\},\left\{\left\{q,m_\pi\right\},2\right\},\left\{\left\{-p,m_\Delta\right\},2\right\}\right]\right)+\frac{1}{6(-1+d)^2F^2m_\Delta^3M_0}$$
$$\left(16i(-2+d)e^2\mathrm{g\pi N\Delta}^2\pi^3M_0\left(\left(Q^2+s\right)\left(5Q^2+s\right)-2\left(5Q^2+s\right)M_0^2+M_0^4\right)(5+\tau[3])\mathrm{II}[6+d]\left[\left\{\left\{0,m_\pi\right\},1\right\},\left\{\left\{q,m_\pi\right\},2\right\},\left\{\left\{p,m_\Delta\right\},2\right\}\right]\right)-$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^3M_0}\left(16i(-2+d)e^2\mathrm{g\pi N\Delta}^2\pi^3M_0\left(\left(Q^2+s\right)\left(5Q^2+s\right)-2\left(5Q^2+s\right)M_0^2+M_0^4\right)(5+\tau[3])\mathrm{II}[6+d]\left[\left\{\left\{0,m_\pi\right\},1\right\},\left\{\left\{q,m_\pi\right\},2\right\},\left\{\left\{-p+q,m_\Delta\right\},2\right\}\right]\right)+\frac{1}{6(-1+d)^2F^2m_\Delta^3M_0}$$
$$\left(16i(-2+d)e^2\mathrm{g\pi N\Delta}^2\pi^3M_0\left(-\left(3Q^2-s\right)\left(Q^2+s\right)-2\left(Q^2+s\right)M_0^2+M_0^4\right)(5+\tau[3])\mathrm{II}[6+d]\left[\left\{\left\{0,m_\pi\right\},1\right\},\left\{\left\{q,m_\pi\right\},2\right\},\left\{\left\{p+q,m_\Delta\right\},2\right\}\right]\right)+$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^3M_0}\left(64i(-2+d)e^2\mathrm{g\pi N\Delta}^2\pi^3Q^2M_0\left(Q^2-s+M_0^2\right)(5+\tau[3])\mathrm{II}[6+d]\left[\left\{\left\{0,m_\pi\right\},1\right\},\left\{\left\{q,m_\pi\right\},3\right\},\left\{\left\{-p,m_\Delta\right\},1\right\}\right]\right)+$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^3M_0}\left(64i(-2+d)e^2\mathrm{g\pi N\Delta}^2\pi^3Q^2M_0\left(-3Q^2-s+M_0^2\right)(5+\tau[3])\mathrm{II}[6+d]\left[\left\{\left\{0,m_\pi\right\},1\right\},\left\{\left\{q,m_\pi\right\},3\right\},\left\{\left\{p,m_\Delta\right\},1\right\}\right]\right)+$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^3M_0}\left(64i(-2+d)e^2\mathrm{g\pi N\Delta}^2\pi^3Q^2M_0\left(-Q^2-s+M_0^2\right)(5+\tau[3])\mathrm{II}[6+d]\left[\left\{\left\{0,m_\pi\right\},1\right\},\left\{\left\{q,m_\pi\right\},3\right\},\left\{\left\{-p+q,m_\Delta\right\},1\right\}\right]\right)+$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^3M_0}\left(64i(-2+d)e^2\mathrm{g\pi N\Delta}^2\pi^3Q^2M_0\left(-Q^2-s+M_0^2\right)(5+\tau[3])\mathrm{II}[6+d]\left[\left\{\left\{0,m_\pi\right\},1\right\},\left\{\left\{q,m_\pi\right\},3\right\},\left\{\left\{p+q,m_\Delta\right\},1\right\}\right]\right)$$

$$\mathbf{B}(\nu,Q^2)=\frac{1}{192(-1+d)^3F^2m_\Delta^4M_0^5}\left(ie^2\mathrm{g\pi N\Delta}^2\left(-24(-2+d)^2m_\Delta M_0^4+8(-2+d)M_0^4\left(-6m_\Delta+(-2+d)M_0\right)+\right.\right.$$
$$\frac{2(-2+d)(-1+d)\left(-\frac{4sm_\pi^2}{d}+\frac{\left(s+m_\pi^2-m_\Delta^2\right)^2}{-1+d}\right)M_0^4\left(-4m_\Delta+(-2+d)M_0\right)}{s^2}-\frac{4(-2+d)\left(s+m_\pi^2-m_\Delta^2\right)M_0^4\left((-3+d)m_\Delta+(-2+d)M_0\right)}{s}+$$
$$8(-2+d)(-1+d)M_0^4\left((2+d)m_\Delta+(-2+d)M_0\right)-8(-2+d)M_0^4\left((-4+d)m_\Delta+2(-2+d)M_0\right)+\frac{(-2+d)^2d\left(s+m_\pi^2-m_\Delta^2\right)M_0^5\left(-Q^2-s+M_0^2\right)}{s^2}+$$
$$\frac{1}{ds^3}\left(\left(-2+d\right)^2\left(8sm_\pi^2+d^2\left(-s^2+\left(m_\pi-m_\Delta\right)^2\left(m_\pi+m_\Delta\right)^2\right)+2d\left(m_\pi^4+\left(s-m_\Delta^2\right)^2-2m_\pi^2\left(s+m_\Delta^2\right)\right)\right)M_0^5\left(-Q^2-s+M_0^2\right)\right)-$$
$$\frac{1}{ds^3}\left(2(-2+d)m_\Delta\left(8sm_\pi^2+2d\left(-5s^2+m_\pi^4-2sm_\Delta^2+m_\Delta^4-2m_\pi^2\left(s+m_\Delta^2\right)\right)+d^2\left(7s^2+m_\pi^4+4sm_\Delta^2+m_\Delta^4-2m_\pi^2\left(2s+m_\Delta^2\right)\right)\right)M_0^4\left(Q^2-s+M_0^2\right)\right)-$$
$$2(-2+d)m_\Delta M_0^2\left(m_\pi^2-m_\Delta^2+M_0^2\right)-\frac{2(-2+d)M_0^4\left((-1+2d)m_\Delta+2(-2+d)M_0\right)\left(2Q^2+s-m_\pi^2+m_\Delta^2-2M_0^2\right)}{-2Q^2-s+2M_0^2}+$$
$$\frac{1}{ds^3}\left(2(-2+d)\left(8sm_\pi^2+d^2\left(s-m_\pi^2+m_\Delta^2\right)^2+2d\left(-s^2+m_\pi^4-2sm_\Delta^2+m_\Delta^4-2m_\pi^2\left(s+m_\Delta^2\right)\right)\right)M_0^4\left((-2+d)Q^2M_0+m_\Delta\left(Q^2+s+M_0^2\right)\right)\right)+$$
$$2\left(2M_0^2+d\left(m_\pi^2-m_\Delta^2-M_0^2\right)\right)\left(8(-2+d)m_\Delta^3+(-16+d(4+d))m_\Delta^2M_0-2(-2+d)m_\Delta\left(Q^2+s+4m_\pi^2-5M_0^2\right)+(-2+d)^2M_0\left(-m_\pi^2+M_0^2\right)\right)-$$
$$2d\left(m_\pi^2-m_\Delta^2+M_0^2\right)\left(4(-2+d)m_\Delta^3+\left(-6+d^2\right)m_\Delta^2M_0-(-2+d)m_\Delta\left(Q^2+s+4m_\pi^2-5M_0^2\right)+(-2+d)^2M_0\left(-m_\pi^2+M_0^2\right)\right)+$$
$$\frac{1}{s}\left(2(-1+d)M_0^4\left(-(4+(-2+d)d)m_\Delta^2M_0-(-2+d)dm_\Delta\left(-Q^2-s+M_0^2\right)-(-2+d)^2M_0\left(-2\left(Q^2+s\right)+m_\pi^2+M_0^2\right)\right)\right)+$$
$$\frac{1}{s^2}\left(4\left(-4s+d\left(3s-m_\pi^2+m_\Delta^2\right)\right)M_0^4\left(-2(-4+d)m_\Delta^3+(-2+d)^2Q^2M_0-(-5+2d)m_\Delta^2M_0+(-2+d)m_\Delta\left(dQ^2-s+2m_\pi^2+M_0^2\right)\right)\right)+\frac{1}{s}\left(2(-1+d)M_0^4\right.$$
$$\left.\left(2(-4+d)(4+d)m_\Delta^3+2(-2+d)^2\left(-2Q^2-s+m_\pi^2\right)M_0+2(-1+d)(3+d)m_\Delta^2M_0+(-2+d)m_\Delta\left(3\left(Q^2+s\right)-2d\left(2Q^2+s\right)+2(-4+d)m_\pi^2+M_0^2\right)\right)\right)+$$
$$(-2+d)(-1+d)M_0^4\left(m_\Delta\left(11Q^2+5s-9M_0^2\right)+(-2+d)M_0\left(-3Q^2-s+M_0^2\right)\right)\left(-\frac{8m_\pi^2}{d\left(2Q^2+s-2M_0^2\right)^2}+\frac{\left(2Q^2+s-m_\pi^2+m_\Delta^2-2M_0^2\right)^2}{\left(-2Q^2-s+2M_0^2\right)^3}+\right.$$
$$\left.\frac{3\left(2Q^2+s-m_\pi^2+m_\Delta^2-2M_0^2\right)^2}{(-1+d)\left(-2Q^2-s+2M_0^2\right)^3}\right)-2m_\Delta\left(-4(-2+d)m_\Delta^2-2(-5+2d)m_\Delta M_0+(-2+d)\left(Q^2+s+4m_\pi^2-5M_0^2\right)\right)\left(-4M_0^2+d\left(-m_\pi^2+m_\Delta^2+3M_0^2\right)\right)+$$
$$\frac{1}{-2Q^2-s+2M_0^2}\left(2(-1+d)M_0^4\left(2(-27+d(5+d))m_\Delta^3+(-26+d(-4+3d))m_\Delta^2M_0+(-2+d)^2M_0\left(-4\left(3Q^2+s\right)-3m_\pi^2+7M_0^2\right)-\right.\right.$$
$$\left.\left.(-2+d)m_\Delta\left(-37Q^2+9dQ^2-13s+3ds+2(4+d)m_\pi^2+(21-5d)M_0^2\right)\right)\right)+$$
$$\frac{1}{Q^2}\left(8(-2+d)M_0^4\left(-3m_\Delta^3+m_\Delta^2M_0+(-2+d)^2M_0\left(-2Q^2-s+M_0^2\right)+(-2+d)m_\Delta\left(4Q^2+3s-d\left(2Q^2+s\right)+(-3+d)M_0^2\right)\right)\right)+$$
$$\frac{1}{s^2}\left(2\left(-2s+d\left(s-m_\pi^2+m_\Delta^2\right)\right)M_0^4\left((4+(-2+d)d)m_\Delta^2M_0+(-2+d)^2M_0\left(-3Q^2-2s+m_\pi^2+M_0^2\right)+(-2+d)m_\Delta\left(-(1+d)\left(Q^2+s\right)+(-1+d)M_0^2\right)\right)\right)-$$

$$\frac{1}{(2Q^2+s-2M_0^2)^2}\,(dM_0^4\,(-2Q^2-s+m_\pi^2-m_\Delta^2+2M_0^2)$$
$$(4(-13+3d)m_\Delta^3+2\,(-10+d^2)\,m_\Delta^2M_0+(-2+d)^2M_0\,(-5\,(3Q^2+s)-2m_\pi^2+7M_0^2)+(-2+d)m_\Delta\,(49Q^2+19s-2d\,(3Q^2+s)-8m_\pi^2+(-31+2d)M_0^2)))-$$
$$2(-1+d)M_0^2\,(2(-2+d)(4+d)m_\Delta^3+2(-15+d(4+d))m_\Delta^2M_0+2(-2+d)^2M_0\,(-m_\pi^2+M_0^2)-(-2+d)m_\Delta\,(3\,(Q^2+s)+2(4+d)m_\pi^2-(11+2d)M_0^2))-$$
$$2(-1+d)M_0^2\,(-2(-2+d)(4+d)m_\Delta^3-6(-3+(-1+d)d)m_\Delta^2M_0+4(-2+d)^2\,(m_\pi-M_0)\,M_0\,(m_\pi+M_0)+(-2+d)m_\Delta\,(3\,(Q^2+s)+2(4+d)m_\pi^2-(11+2d)M_0^2))-$$
$$\frac{1}{d(2Q^2+s-2M_0^2)^2}\Big(2(-2+d)M_0^4\,(-2m_\Delta+(-2+d)M_0)\Big(4m_\pi^2\,(-2Q^2-s+2M_0^2)+d\Big(m_\pi^4+(2Q^2+s+m_\Delta^2-2M_0^2)^2-2m_\pi^2\,(-2Q^2-s+m_\Delta^2+2M_0^2)\Big)\Big)\Big)-$$
$$\frac{1}{d(2Q^2+s-2M_0^2)^2}\Big(8(-2+d)m_\Delta M_0^4\Big(4m_\pi^2\,(2Q^2+s-2M_0^2)-d\Big(m_\pi^4+m_\Delta^4-(2Q^2+s-2M_0^2)^2-2m_\pi^2\,(-4Q^2-2s+m_\Delta^2+4M_0^2)\Big)\Big)\Big)\Big)$$
$$(5+\tau[3])\text{II}[d]\,[\{\{0,m_\pi\},1\}])+\frac{1}{48(-1+d)^3F^2m_\Delta^3M_0^2}\Big(i(-2+d)e^2\text{g}\pi\text{N}\Delta^2\,(-m_\pi^2+m_\Delta^2+2(-2+d)dm_\Delta M_0+M_0^2)\Big(-m_\pi^2+(m_\Delta+M_0)^2\Big)(5+\tau[3])$$
$$\text{II}[d]\,[\{\{0,m_\pi\},1\},\{\{p,m_\Delta\},1\}])-$$
$$\frac{1}{96(-1+d)^3F^2m_\Delta^4M_0}\Big(ie^2\text{g}\pi\text{N}\Delta^2M_0\Big(\frac{1}{(2Q^2+s-2M_0^2)^2}\Big(8(-2+d)m_\Delta\,(2Q^2+s+m_\pi^2-m_\Delta^2-2M_0^2)\Big(m_\pi^4+(2Q^2+s+m_\Delta^2-2M_0^2)^2-2m_\pi^2\,(-2Q^2-s+m_\Delta^2+2M_0^2)\Big)\Big)-$$
$$\frac{1}{(2Q^2+s-2M_0^2)^2}\Big(2(-2+d)\,(-2m_\Delta+(-2+d)M_0)\,(-2Q^2-s+m_\pi^2-m_\Delta^2+2M_0^2)\Big(m_\pi^4+(2Q^2+s+m_\Delta^2-2M_0^2)^2-2m_\pi^2\,(-2Q^2-s+m_\Delta^2+2M_0^2)\Big)\Big)+$$
$$\frac{1}{-2(2Q^2+s)+4M_0^2}\Big(4(-2+d)\,((-1+2d)m_\Delta+2(-2+d)M_0)\Big(m_\pi^4+(2Q^2+s+m_\Delta^2-2M_0^2)^2-2m_\pi^2\,(-2Q^2-s+m_\Delta^2+2M_0^2)\Big)\Big)-$$
$$\frac{1}{-2Q^2-s+2M_0^2}\,(2(-1+d)\,(-2Q^2-s+m_\pi^2-m_\Delta^2+2M_0^2)\,(-2(-27+d(5+d))m_\Delta^3-(-26+d(-4+3d))m_\Delta^2M_0+$$
$$(-2+d)^2M_0\,(4\,(3Q^2+s)+3m_\pi^2-7M_0^2)+(-2+d)m_\Delta\,(-37Q^2+9dQ^2-13s+3ds+2(4+d)m_\pi^2+(21-5d)M_0^2)))+$$
$$4(-1+d)\,(-(-14+d(2+d))m_\Delta^3-(-4+d)(2+d)m_\Delta^2M_0+(-2+d)^2M_0\,(3Q^2+s+m_\pi^2-2M_0^2)+(-2+d)m_\Delta\,((-3+d)\,(3Q^2+s)+(2+d)m_\pi^2+(5-2d)M_0^2))+$$
$$\frac{1}{(2Q^2+s-2M_0^2)^2}\Big(\Big(4m_\pi^2\,(2Q^2+s-2M_0^2)+d\,(2Q^2+s-m_\pi^2+m_\Delta^2-2M_0^2)^2\Big)$$
$$(-4(-13+3d)m_\Delta^3-2\,(-10+d^2)\,m_\Delta^2M_0+(-2+d)^2M_0\,(5\,(3Q^2+s)+2m_\pi^2-7M_0^2)+(-2+d)m_\Delta\,(-49Q^2+6dQ^2-19s+2ds+8m_\pi^2+(31-2d)M_0^2)))+$$
$$\frac{1}{(-2Q^2-s+2M_0^2)^3}\,((-2+d)\,(-2Q^2-s+m_\pi^2-m_\Delta^2+2M_0^2)\,(m_\Delta\,(11Q^2+5s-9M_0^2)+(-2+d)M_0\,(-3Q^2-s+M_0^2))$$
$$\Big((2+d)m_\pi^4+(2+d)\,(2Q^2+s+m_\Delta^2-2M_0^2)^2+2m_\pi^2\,(-(2+d)m_\Delta^2+(-4+d)\,(-2Q^2-s+2M_0^2))\Big)\Big)\Big)$$
$$(5+\tau[3])\text{II}[d]\,[\{\{0,m_\pi\},1\},\{\{p-q,m_\Delta\},1\}])-\frac{ie^2\text{g}\pi\text{N}\Delta^2}{24(-1+d)^3F^2Q^2m_\Delta^4M_0}\,((Q^2+4m_\pi^2)\,M_0\,(-3m_\Delta^3+m_\Delta^2M_0+(-3+d)(-2+d)m_\Delta\,(-Q^2-s+M_0^2)+(-2+d)^2M_0\,(-Q^2-s+M_0^2))$$
$$(5+\tau[3])\text{II}[d]\,[\{\{0,m_\pi\},1\},\{\{q,m_\pi\},1\}])+$$
$$\frac{1}{96(-1+d)^3F^2s^3m_\Delta^4M_0}\,(ie^2\text{g}\pi\text{N}\Delta^2M_0\,(2sm_\Delta\,((-2+d)s^2\,((-3+d)Q^2+(-1+d)s)+s\,((-2+d)(7+(-4+d)d)Q^2+10s-(-1+d)^2ds)\,m_\Delta^2+$$
$$((-3+d)(-2+d)dQ^2+(-5+d)\,(4+d^2)\,s)\,m_\Delta^4-2(-6+d)dm_\Delta^6)+(sm_\Delta^2\,(-(-28+d(16+(-6+d)d))s^2+2(-30+d(12+d))sm_\Delta^2+d(24+(-8+d)d)m_\Delta^4)+$$
$$(-2+d)^2Q^2\,(2s^3+(-2+d)s^2m_\Delta^2+2(-3+d)sm_\Delta^4+(2+d)m_\Delta^6))\,M_0+2(-2+d)(-1+d)sm_\Delta\,(s-m_\Delta^2)\,(s-dm_\Delta^2)\,M_0^2+$$
$$(-2+d)^2m_\Delta^2\,(-s+m_\Delta^2)\,((-4+d)s+(2+d)m_\Delta^2)\,M_0^3-(-2+d)m_\pi^6\,(-4dsm_\Delta-3(-2+d)dsM_0+(-4+d^2)\,M_0\,(Q^2+M_0^2))+$$
$$m_\pi^2\,(4d(-14+3d)sm_\Delta^5+4s\,(-(-2+d)^2dQ^2+(-19+(-4+d)^2d)\,s)\,m_\Delta^2M_0+(-2+d)^2s^2M_0\,(-(6+d)Q^2+(2+d)s+(-4+d)M_0^2)+$$
$$2(-2+d)s^2m_\Delta\,(5\,(Q^2+s)+d\,(-dQ^2+(-4+d)s)+(-3+d)(-1+d)M_0^2)-m_\Delta^4M_0\,(-d(-36+d(4+d))s+3(-2+d)^2(2+d)\,(Q^2+M_0^2))+$$
$$4sm_\Delta^3\,((-12+d(6+(-3+d)d))s-(-2+d)d\,((-3+d)Q^2+(-1+d)M_0^2)))+$$
$$m_\pi^4\,(-4d(-10+3d)sm_\Delta^3-2(-2+d)^2sM_0\,(-(3+d)Q^2+s+2ds-3M_0^2)+m_\Delta^2M_0\,(-d^2(-16+5d)s+3(-2+d)^2(2+d)\,(Q^2+M_0^2))-$$
$$2(-2+d)sm_\Delta\,((6+d(-5+3d))s+d\,(3Q^2+M_0^2-d\,(Q^2+M_0^2)))))\,(5+\tau[3])\text{II}[d]\,[\{\{0,m_\pi\},1\},\{\{p+q,m_\Delta\},1\}])+$$
$$\frac{1}{24(-1+d)^2F^2m_\Delta^4M_0}\,(ie^2\text{g}\pi\text{N}\Delta^2M_0\,(-2(-2+(-3+d)d)m_\Delta^5-2(8+(-5+d)d)m_\Delta^4M_0+(-2+d)^2Q^2m_\Delta M_0^2+(-2+d)^2Q^2M_0^3-$$

$$m_\pi^2\left((-2+d)^2Q^2-2(-2+(-3+d)d)m_\Delta^2\right)(m_\Delta+M_0)-m_\Delta^3\left(-(-4+d)d\left(3Q^2+2s\right)-2\left(5Q^2+3s\right)+2(1+d(-7+2d))M_0^2\right)-$$
$$m_\Delta^2M_0\left(-4\left(Q^2+s\right)+d\left(8Q^2-3dQ^2+6s-2ds+4(-3+d)M_0^2\right)\right)\right)(5+\tau[3])\mathrm{II}[d]\left[\{\{0,m_\pi\},1\},\{\{p-q,m_\Delta\},1\},\{\{p,m_\Delta\},1\}\right]\right)-$$
$$\frac{1}{24(-1+d)^2F^2m_\Delta^4M_0}\left(ie^2\mathrm{g}\pi\mathrm{N}\Delta^2M_0\left((-14+d(2+d))m_\Delta^5+(-6+d(4+d))m_\Delta^4M_0+(-2+d)^2(m_\pi-M_0)M_0(m_\pi+M_0)\left(3Q^2+s+m_\pi^2-2M_0^2\right)-\right.\right.$$
$$m_\Delta^3\left(26Q^2+14s-d\left((8+d)Q^2+(-2+3d)s\right)+2\left(-9+d+d^2\right)m_\pi^2+(-2+d)(2+5d)M_0^2\right)-$$
$$m_\Delta^2M_0\left(10Q^2-2s-d\left((6+d)Q^2+3(-2+d)s\right)+2\left(-9+d^2\right)m_\pi^2+(12+5(-2+d)d)M_0^2\right)+$$
$$(-2+d)m_\Delta\left(6Q^4+8Q^2s+2s^2+(2+d)m_\pi^4-6Q^2M_0^2-4sM_0^2+dM_0^2\left(-3Q^2-s+2M_0^2\right)+m_\pi^2\left((-8+3d)Q^2+(-2+d)s+(2-3d)M_0^2\right)\right)\right)$$
$$(5+\tau[3])\mathrm{II}[d]\left[\{\{0,m_\pi\},1\},\{\{q,m_\pi\},1\},\{\{p,m_\Delta\},1\}\right]\right)+$$
$$\frac{1}{24(-1+d)^2F^2m_\Delta^4M_0}\left(ie^2\mathrm{g}\pi\mathrm{N}\Delta^2M_0\left(-(12+(-4+d)d)m_\Delta^5-(10+(-4+d)d)m_\Delta^4M_0+(-2+d)^2\left(-Q^2-s+m_\pi^2\right)M_0\left(-m_\pi^2+M_0^2\right)+\right.\right.$$
$$(-2+d)m_\Delta\left(-m_\pi^2+M_0^2\right)\left(Q^2+s-d\left(Q^2+s\right)+(-2+d)m_\pi^2+M_0^2\right)+m_\Delta^3\left((-1+d)\left(-4Q^2+3dQ^2+ds\right)+2(8+(-4+d)d)m_\pi^2+(-4+d)(1+d)M_0^2\right)+$$
$$m_\Delta^2M_0\left((-2+d)\left((-4+3d)Q^2+(-2+d)s\right)+2(1+(-4+d)d)m_\pi^2+\left(2+d^2\right)M_0^2\right)\right)(5+\tau[3])\mathrm{II}[d]\left[\{\{0,m_\pi\},1\},\{\{q,m_\pi\},1\},\{\{p+q,m_\Delta\},1\}\right]-$$
$$\frac{1}{24(-1+d)^2F^2m_\Delta^4M_0}\left(ie^2\mathrm{g}\pi\mathrm{N}\Delta^2M_0\left(m_\Delta\left((-2+d)Q^4+(-2+d)Q^2s+(-4+d)(2+d)Q^2m_\Delta^2+2(-2+(-3+d)d)m_\Delta^2\left(s+m_\Delta^2\right)\right)+\right.\right.$$
$$m_\Delta^2\left(\left(-2+d^2\right)Q^2+2(-2+d)\left((-1+d)s+(-3+d)m_\Delta^2\right)\right)M_0-(-2+d)(-1+d)Q^2m_\Delta M_0^2-$$
$$\left((-2+d)^2Q^2+8m_\Delta^2\right)M_0^3+m_\pi^2\left((-2+d)^2Q^2-2(-2+(-3+d)d)m_\Delta^2\right)(m_\Delta+M_0)\right)$$
$$(5+\tau[3])\mathrm{II}[d]\left[\{\{0,m_\pi\},1\},\{\{p+q,m_\Delta\},1\},\{\{p,m_\Delta\},1\}\right]\right)-\frac{1}{24(-1+d)^2F^2m_\Delta^4M_0}$$
$$\left(ie^2\mathrm{g}\pi\mathrm{N}\Delta^2M_0\left(-2(-2+(-3+d)d)m_\Delta^7-2(-3+d)dm_\Delta^6M_0+(-2+d)^2Q^2M_0\left(m_\pi^2-M_0^2\right)^2+(-2+d)Q^2m_\Delta(m_\pi-M_0)(m_\pi+M_0)\left(Q^2+s+(-2+d)m_\pi^2-(-1+d)M_0^2\right)+\right.\right.$$
$$m_\Delta^4M_0\left((18+(-12+d)d)Q^2-4(-2+d)s+4(7+(-5+d)d)m_\pi^2+4(-3+d)(1+d)M_0^2\right)+$$
$$m_\Delta^5\left((14+(-6+d)d)Q^2-2(-5+d)s+4(-2+(-3+d)d)m_\pi^2+2(-9+d(-5+2d))M_0^2\right)-2m_\Delta^2(m_\pi-M_0)M_0(m_\pi+M_0)$$
$$\left((-3+d)(-1+d)Q^2+(-2+(-3+d)d)(m_\pi-M_0)(m_\pi+M_0)\right)-m_\Delta^3\left(\left(Q^2+s\right)\left((-4+3d)Q^2+2(-1+d)s\right)+2(-2+(-3+d)d)m_\pi^4-\right.$$
$$\left.\left((8+d(-5+2d))Q^2+2(3+d)s\right)M_0^2+2(-2+d)(-1+d)M_0^4-2m_\pi^2\left(-(9+(-5+d)d)Q^2+(-5+d)s+(1+d(-7+2d))M_0^2\right)\right)\right)$$
$$(5+\tau[3])\mathrm{II}[d]\left[\{\{0,m_\pi\},1\},\{\{q,m_\pi\},1\},\{\{p+q,m_\Delta\},1\},\{\{p,m_\Delta\},1\}\right]\right)+\frac{1}{6(-1+d)^2F^2m_\Delta^4M_0}$$
$$\left(ie^2\mathrm{g}\pi\mathrm{N}\Delta^2\pi M_0\left(-4(-2+(-3+d)d)m_\Delta^5-2(20+d(-13+3d))m_\Delta^4M_0+3(-2+d)^2Q^2M_0\left(-m_\pi^2+M_0^2\right)-(-2+d)Q^2m_\Delta\left(Q^2+s+2(-2+d)m_\pi^2+(3-2d)M_0^2\right)+\right.\right.$$
$$2m_\Delta^3\left((15+d(-13+3d))Q^2+(11+d(-9+2d))s+2(-2+(-3+d)d)m_\pi^2+(-7+(15-4d)d)M_0^2\right)+$$
$$m_\Delta^2M_0\left(2\left(5Q^2+6s\right)+6(-2+(-3+d)d)m_\pi^2+d\left((-22+9d)Q^2+6(-3+d)s-12(-3+d)M_0^2\right)\right)\right)$$
$$(5+\tau[3])\mathrm{II}[2+d]\left[\{\{0,m_\pi\},1\},\{\{p,m_\Delta\},1\},\{\{p-q,m_\Delta\},2\}\right]\right)+\frac{1}{6(-1+d)^2F^2m_\Delta^4M_0}$$
$$\left(ie^2\mathrm{g}\pi\mathrm{N}\Delta^2\pi M_0^2\left(-2(2+(-5+d)d)m_\Delta^4-(-2+d)^2Q^2(m_\pi-M_0)(m_\pi+M_0)+m_\Delta^2\left(-(-2+d)\left(dQ^2+2(-1+d)s\right)+2(-2+(-3+d)d)m_\pi^2+8M_0^2\right)\right)\right.$$
$$(5+\tau[3])\mathrm{II}[2+d]\left[\{\{0,m_\pi\},1\},\{\{p,m_\Delta\},1\},\{\{p+q,m_\Delta\},2\}\right]\right)+$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^3M_0}\left(4ie^2\mathrm{g}\pi\mathrm{N}\Delta^2\pi M_0\left(2(-2+d)m_\Delta^2+(-5+d)m_\Delta M_0-2(-2+d)(m_\pi-M_0)(m_\pi+M_0)\right)(5+\tau[3])\right.$$
$$\mathrm{II}[2+d]\left[\{\{0,m_\pi\},1\},\{\{p-q,m_\Delta\},1\},\{\{p,m_\Delta\},1\}\right]\right)+\frac{1}{6(-1+d)^2F^2m_\Delta^4M_0}$$
$$\left(ie^2\mathrm{g}\pi\mathrm{N}\Delta^2\pi M_0^2\left(m_\pi^2\left(-(-2+d)^2Q^2+2(-2+(-3+d)d)m_\Delta^2\right)+(-2+d)m_\Delta^2\left((-4+3d)Q^2+2(-1+d)\left(s-m_\Delta^2\right)\right)+\left((-2+d)^2Q^2-4(-3+d)dm_\Delta^2\right)M_0^2\right)\right.$$
$$(5+\tau[3])\mathrm{II}[2+d]\left[\{\{0,m_\pi\},1\},\{\{p-q,m_\Delta\},1\},\{\{p,m_\Delta\},2\}\right]\right)+$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^4M_0}\left(ie^2\mathrm{g}\pi\mathrm{N}\Delta^2\pi M_0\left((-2+d)(-1+2d)m_\Delta^3+2(-7+d)m_\Delta^2M_0+2(-2+d)^2M_0\left(-m_\pi^2+M_0^2\right)+(-2+d)m_\Delta\left(-7\left(Q^2+s\right)+(1-2d)m_\pi^2+2(6+d)M_0^2\right)\right)\right.$$
$$(5+\tau[3])\mathrm{II}[2+d]\left[\{\{0,m_\pi\},1\},\{\{q,m_\pi\},1\},\{\{p,m_\Delta\},1\}\right]\right)-$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^4M_0}\left(ie^2\mathrm{g}\pi\mathrm{N}\Delta^2\pi M_0\left(2(-27+d(5+d))m_\Delta^5+3(-2+d)(6+d)m_\Delta^4M_0+(-2+d)^2(m_\pi-M_0)M_0(m_\pi+M_0)\left(4\left(3Q^2+s\right)+3m_\pi^2-7M_0^2\right)-\right.\right.$$
$$2m_\Delta^2M_0\left(4\left(8Q^2+s\right)-d\left((20+d)Q^2+(-6+5d)s\right)+\left(-26+3d^2\right)m_\pi^2+2(3+d(-7+5d))M_0^2\right)-$$

$$m_\Delta^3 \left(110Q^2 + 62s - d\left((40+d)Q^2 + (4+7d)s\right) + 2(-35+d(7+2d))m_\pi^2 + (-48+d(-14+15d))M_0^2\right) +$$

$$(-2+d)m_\Delta \left(9\left(Q^2+s\right)\left(3Q^2+s\right) + 2(4+d)m_\pi^4 - \left((32+9d)Q^2 + 3(6+d)s\right)M_0^2 + (1+5d)M_0^4 + m_\pi^2\left(-34Q^2 + 9dQ^2 - 10s + 3ds + (10-7d)M_0^2\right)\right)\Big)$$

$$(5+\tau[3])\text{II}[2+d]\left[\{\{0,m_\pi\},1\},\{\{q,m_\pi\},1\},\{\{p,m_\Delta\},2\}\right] - \frac{1}{6(-1+d)^2F^2m_\Delta^4M_0}$$

$$\left(ie^2 g\pi N\Delta^2\pi M_0\left(-(-2+d)(-1+2d)m_\Delta^3 + 2(4+d)m_\Delta^2M_0 + 2(-2+d)^2M_0\left(-m_\pi^2+M_0^2\right) - (-2+d)m_\Delta\left(3Q^2+2s+(-5+2d)m_\pi^2+(5-2d)M_0^2\right)\right)\right.$$

$$(5+\tau[3])\text{II}[2+d]\left[\{\{0,m_\pi\},1\},\{\{q,m_\pi\},1\},\{\{p+q,m_\Delta\},1\}\right] +$$

$$\frac{1}{6(-1+d)^2F^2m_\Delta^4M_0}\left(ie^2 g\pi N\Delta^2\pi M_0\left(-(-8+(-4+d)d)m_\Delta^4M_0 + (-2+d)dm_\Delta\left(-Q^2-s+M_0^2\right)\left(-m_\pi^2+M_0^2\right) + (-2+d)^2M_0\left(-m_\pi^2+M_0^2\right)\left(-2\left(Q^2+s\right)+m_\pi^2+M_0^2\right) +\right.\right.$$

$$(-2+d)m_\Delta^3\left((-2+d)\left(Q^2+s\right)+(-2+3d)M_0^2\right) + 2m_\Delta^2M_0\left((-2+d)(-1+d)\left(2Q^2+s\right)+(2+(-4+d)d)m_\pi^2+(4+d(-5+2d))M_0^2\right)\Big)$$

$$(5+\tau[3])\text{II}[2+d]\left[\{\{0,m_\pi\},1\},\{\{q,m_\pi\},1\},\{\{p+q,m_\Delta\},2\}\right] +$$

$$\frac{1}{6(-1+d)^2F^2m_\Delta^4M_0}\left(ie^2 g\pi N\Delta^2\pi M_0\left(-2(22+(-7+d)d)m_\Delta^5 - 2(19+(-6+d)d)m_\Delta^4M_0 + 2(-2+d)^2\left(-2Q^2-s+m_\pi^2\right)M_0\left(-m_\pi^2+M_0^2\right) +\right.\right.$$

$$m_\Delta^3\left((20+d(-19+6d))Q^2 + 3ds + \left(60-26d+4d^2\right)m_\pi^2 + (-4+d)(7+4d)M_0^2\right) +$$

$$2m_\Delta^2M_0\left((-2+d)(-5+3d)Q^2 + s - ds + (13+2(-6+d)d)m_\pi^2 + (4+d(-3+2d))M_0^2\right) +$$

$$(-2+d)m_\Delta\left(\left(Q^2+s\right)\left(2Q^2+s\right) - 2(-4+d)m_\pi^4 + \left(2Q^2-4dQ^2+3s-2ds\right)M_0^2 + m_\pi^2\left(-3\left(Q^2+s\right)+2d\left(2Q^2+s\right)+(-9+2d)M_0^2\right)\right)\Big)$$

$$(5+\tau[3])\text{II}[2+d]\left[\{\{0,m_\pi\},1\},\{\{q,m_\pi\},2\},\{\{-p,m_\Delta\},1\}\right] - \frac{1}{6(-1+d)^2F^2m_\Delta^4M_0}$$

$$\left(ie^2 g\pi N\Delta^2\pi M_0\left(2(-27+d(5+d))m_\Delta^5 + 2(-14+d(6+d))m_\Delta^4M_0 + 2(-2+d)^2\left(m_\pi-M_0\right)M_0\left(m_\pi+M_0\right)\left(4Q^2+s+m_\pi^2-2M_0^2\right) -\right.\right.$$

$$2m_\Delta^2M_0\left(3\left(9Q^2+s\right) - d\left((17+d)Q^2+(-5+4d)s\right) + 2(-14+d(2+d))m_\pi^2 + \left(9-9d+6d^2\right)M_0^2\right) +$$

$$(-2+d)m_\Delta\left(\left(Q^2+s\right)\left(25Q^2+7s\right) + 2(4+d)m_\pi^4 - \left(23Q^2+8dQ^2+13s+2ds\right)M_0^2 + (2+4d)M_0^4 + m_\pi^2\left(-31Q^2+8dQ^2-7s+2ds+(3-6d)M_0^2\right)\right) -$$

$$m_\Delta^3\left(104Q^2+56s+2(-35+d(7+2d))m_\pi^2 - 22M_0^2 + d\left(-35Q^2+s-2d\left(Q^2+4s\right)+3(-5+4d)M_0^2\right)\right)\Big)$$

$$(5+\tau[3])\text{II}[2+d]\left[\{\{0,m_\pi\},1\},\{\{q,m_\pi\},2\},\{\{-p+q,m_\Delta\},1\}\right] +$$

$$\frac{1}{6(-1+d)^2F^2m_\Delta^3M_0}\left(4ie^2 g\pi N\Delta^2\pi M_0\left(2(-2+d)m_\Delta^2 + (-7+3d)m_\Delta M_0 - 2(-2+d)\left(m_\pi-M_0\right)\left(m_\pi+M_0\right)\right)(5+\tau[3])\right.$$

$$\text{II}[2+d]\left[\{\{0,m_\pi\},1\},\{\{p+q,m_\Delta\},1\},\{\{p,m_\Delta\},1\}\right] + \frac{1}{6(-1+d)^2F^2m_\Delta^4M_0}$$

$$\left(ie^2 g\pi N\Delta^2\pi M_0\left(-4(-2+(-3+d)d)m_\Delta^5 - 6(-4+d)(-1+d)m_\Delta^4M_0 + 3(-2+d)^2Q^2M_0^3 - m_\pi^2\left((-2+d)^2Q^2 - 2(-2+(-3+d)d)m_\Delta^2\right)\left(2m_\Delta+3M_0\right) +\right.\right.$$

$$m_\Delta^2M_0\left((10-d(2+3d))Q^2 - 6(-2+d)(-1+d)s + 24M_0^2\right) + 2m_\Delta^3\left(13Q^2+9s+d\left(Q^2-dQ^2+(5-2d)s\right)+(-5+d)M_0^2\right) +$$

$$(-2+d)Q^2m_\Delta\left(-3\left(Q^2+s\right)+(-1+2d)M_0^2\right)\Big)(5+\tau[3])\text{II}[2+d]\left[\{\{0,m_\pi\},1\},\{\{p+q,m_\Delta\},1\},\{\{p,m_\Delta\},2\}\right] +$$

$$\frac{1}{6(-1+d)^2F^2m_\Delta^4M_0}\left(ie^2 g\pi N\Delta^2\pi M_0\left(-4(-2+(-3+d)d)m_\Delta^5 - 4(5+(-5+d)d)m_\Delta^4M_0 + 2(-2+d)^2Q^2M_0^3 -\right.\right.$$

$$2m_\pi^2\left((-2+d)^2Q^2 - 2(-2+(-3+d)d)m_\Delta^2\right)\left(m_\Delta+M_0\right) + 2m_\Delta^2M_0\left(-(-5+d(2+d))Q^2 - 2(-2+d)(-1+d)s + 8M_0^2\right) +$$

$$2m_\Delta^3\left(13Q^2+9s+d\left(Q^2-dQ^2+(5-2d)s\right)+(-5+d)M_0^2\right) + (-2+d)Q^2m_\Delta\left(-3\left(Q^2+s\right)+(-1+2d)M_0^2\right)\Big)$$

$$(5+\tau[3])\text{II}[2+d]\left[\{\{0,m_\pi\},2\},\{\{p+q,m_\Delta\},1\},\{\{p,m_\Delta\},1\}\right] - \frac{1}{6(-1+d)^2F^2m_\Delta^4M_0}$$

$$\left(ie^2 g\pi N\Delta^2\pi M_0^2\left(-2(-3+d)(-2+d)m_\Delta^6 + (-2+d)^2Q^2\left(m_\pi^2-M_0^2\right)^2 + m_\Delta^4\left((4+(-8+d)d)Q^2 - 4ds + 4(2+(-4+d)d)m_\pi^2 + 4(4+(-3+d)d)M_0^2\right) -\right.\right.$$

$$2m_\Delta^2\left(m_\pi-M_0\right)\left(m_\pi+M_0\right)\left((-2+d)^2Q^2 + (-2+(-3+d)d)\left(m_\pi-M_0\right)\left(m_\pi+M_0\right)\right)\Big)\Big)$$

$$(5+\tau[3])\text{II}[2+d]\left[\{\{0,m_\pi\},1\},\{\{q,m_\pi\},1\},\{\{p,m_\Delta\},1\},\{\{p+q,m_\Delta\},2\}\right] + \frac{1}{6(-1+d)^2F^2m_\Delta^3M_0}$$

$$\left(4ie^2 g\pi N\Delta^2\pi M_0\left(-2(-2+d)m_\Delta^4 + (-4+d)^2m_\Delta^3M_0 + 4(-3+d)m_\Delta\left(m_\pi-M_0\right)M_0\left(m_\pi+M_0\right) - 2(-2+d)\left(m_\pi^2-M_0^2\right)^2 + 4m_\Delta^2\left((-2+d)m_\pi^2-(-4+d)M_0^2\right)\right)\right.$$

$$(5+\tau[3])\text{II}[2+d]\left[\{\{0,m_\pi\},1\},\{\{q,m_\pi\},1\},\{\{p+q,m_\Delta\},1\},\{\{p,m_\Delta\},1\}\right] -$$

$$\frac{1}{6(-1+d)^2F^2m_\Delta^4M_0}\left(ie^2 g\pi N\Delta^2\pi M_0\left(-4(-2+(-3+d)d)m_\Delta^7 - 2(-3+d)(-2+3d)m_\Delta^6M_0 + 3(-2+d)^2Q^2M_0\left(m_\pi^2-M_0^2\right)^2 + 2(-2+d)Q^2m_\Delta\left(m_\pi-M_0\right)\right.\right.$$

$$(m_\pi + M_0)\left(2\left(Q^2+s\right) + (-2+d)m_\pi^2 - dM_0^2\right) + 2m_\Delta^5\left((24+(-8+d)d)Q^2 - 4(-5+d)s + 4(-2+(-3+d)d)m_\pi^2 + 4(-7+(-2+d)d)M_0^2\right) +$$
$$m_\Delta^4 M_0\left((56+d(-40+3d))Q^2 - 16(-2+d)s + 4(-2+d)(-8+3d)m_\pi^2 + 12(-2+(-2+d)d)M_0^2\right) -$$
$$2m_\Delta^2\left(m_\pi - M_0\right)M_0\left(m_\pi + M_0\right)\left((10+3(-4+d)d)Q^2 + 3(-2+(-3+d)d)\left(m_\pi - M_0\right)\left(m_\pi + M_0\right)\right) - 4m_\Delta^3\left(\left(Q^2+s\right)\left((-3+2d)Q^2 + (-1+d)s\right) +\right.$$
$$\left.(-2+(-3+d)d)m_\pi^4 - \left((8+(-3+d)d)Q^2 + 8s\right)M_0^2 + (7+(-4+d)d)M_0^4 + m_\pi^2\left((14+(-6+d)d)Q^2 - 2(-5+d)s - 2(-3+d)(-1+d)M_0^2\right)\right)\Big)$$
$$(5+\tau[3])\mathrm{II}[2+d]\left[\{\{0,m_\pi\},1\},\{\{q,m_\pi\},1\},\{\{p+q,m_\Delta\},1\},\{\{p,m_\Delta\},2\}\right] - \frac{1}{6(-1+d)^2F^2m_\Delta^4M_0}$$
$$\Big(2ie^2\mathrm{g}\pi\mathrm{N}\Delta^2\pi M_0\Big(-2(-2+(-3+d)d)m_\Delta^7 - 2(-3+d)dm_\Delta^6M_0 + (-2+d)^2Q^2M_0\left(m_\pi^2 - M_0^2\right)^2 + (-2+d)Q^2m_\Delta\left(m_\pi - M_0\right)\left(m_\pi + M_0\right)\left(2\left(Q^2+s\right) + (-2+d)m_\pi^2 - dM_0^2\right) +$$
$$m_\Delta^5\left((24+(-8+d)d)Q^2 - 4(-5+d)s + 4(-2+(-3+d)d)m_\pi^2 + 4(-7+(-2+d)d)M_0^2\right) +$$
$$m_\Delta^4M_0\left((26+(-16+d)d)Q^2 + 16s - 6ds + 4(7+(-5+d)d)m_\pi^2 + \left(-20-6d+4d^2\right)M_0^2\right) -$$
$$2m_\Delta^2\left(m_\pi - M_0\right)M_0\left(m_\pi + M_0\right)\left((-3+d)(-1+d)Q^2 + (-2+(-3+d)d)\left(m_\pi - M_0\right)\left(m_\pi + M_0\right)\right) - 2m_\Delta^3\left(\left(Q^2+s\right)\left((-3+2d)Q^2 + (-1+d)s\right) +\right.$$
$$\left.(-2+(-3+d)d)m_\pi^4 - \left((8+(-3+d)d)Q^2 + 8s\right)M_0^2 + (7+(-4+d)d)M_0^4 + m_\pi^2\left((14+(-6+d)d)Q^2 - 2(-5+d)s - 2(-3+d)(-1+d)M_0^2\right)\right)\Big)$$
$$(5+\tau[3])\mathrm{II}[2+d]\left[\{\{0,m_\pi\},1\},\{\{q,m_\pi\},2\},\{\{-p,m_\Delta\},1\},\{\{-p+q,m_\Delta\},1\}\right] +$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^3M_0}\left(32ie^2\mathrm{g}\pi\mathrm{N}\Delta^2\pi^2M_0\left((-2+d)m_\Delta^2 + (-3+d)m_\Delta M_0 - (-2+d)\left(m_\pi - M_0\right)\left(m_\pi + M_0\right)\right)(5+\tau[3])\right.$$
$$\left.\mathrm{II}[4+d]\left[\{\{0,m_\pi\},1\},\{\{p,m_\Delta\},1\},\{\{p-q,m_\Delta\},2\}\right]\right) +$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^4M_0}\left(16ie^2\mathrm{g}\pi\mathrm{N}\Delta^2\pi^2M_0\left(-2(4+(-3+d)d)m_\Delta^4M_0 - (-2+d)Q^2m_\Delta\left(Q^2+s-M_0^2\right) + 2(-5+d)m_\Delta^3\left(-Q^2-s+M_0^2\right) +\right.\right.$$
$$\left.(-2+d)^2Q^2M_0\left(-m_\pi^2+M_0^2\right) + m_\Delta^2M_0\left(2\left(Q^2+2s\right) + 2(-2+(-3+d)d)m_\pi^2 + d\left(3(-2+d)Q^2 + 2(-3+d)s - 4(-3+d)M_0^2\right)\right)\right)$$
$$\left.(5+\tau[3])\mathrm{II}[4+d]\left[\{\{0,m_\pi\},1\},\{\{p,m_\Delta\},1\},\{\{p-q,m_\Delta\},3\}\right]\right) +$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^3M_0}\left(32ie^2\mathrm{g}\pi\mathrm{N}\Delta^2\pi^2M_0\left((-2+d)m_\Delta^2 + (-3+d)m_\Delta M_0 - (-2+d)\left(m_\pi - M_0\right)\left(m_\pi + M_0\right)\right)(5+\tau[3])\right.$$
$$\left.\mathrm{II}[4+d]\left[\{\{0,m_\pi\},1\},\{\{p,m_\Delta\},1\},\{\{p+q,m_\Delta\},2\}\right]\right) +$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^3M_0}\left(32ie^2\mathrm{g}\pi\mathrm{N}\Delta^2\pi^2M_0\left((-2+d)m_\Delta^2 + (-3+d)m_\Delta M_0 - (-2+d)\left(m_\pi - M_0\right)\left(m_\pi + M_0\right)\right)(5+\tau[3])\right.$$
$$\left.\mathrm{II}[4+d]\left[\{\{0,m_\pi\},1\},\{\{p-q,m_\Delta\},1\},\{\{p,m_\Delta\},2\}\right]\right) + \frac{1}{6(-1+d)^2F^2m_\Delta^4M_0}$$
$$\left(8ie^2\mathrm{g}\pi\mathrm{N}\Delta^2\pi^2M_0^2\left(m_\pi^2\left(-(-2+d)^2Q^2 + 2(-2+(-3+d)d)m_\Delta^2\right) + (-2+d)m_\Delta^2\left((-4+3d)Q^2 + 2(-1+d)\left(s-m_\Delta^2\right)\right) + \left((-2+d)^2Q^2 - 4(-3+d)dm_\Delta^2\right)M_0^2\right)\right.$$
$$\left.(5+\tau[3])\mathrm{II}[4+d]\left[\{\{0,m_\pi\},1\},\{\{p-q,m_\Delta\},2\},\{\{p,m_\Delta\},2\}\right]\right) +$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^4M_0}\left(8ie^2\mathrm{g}\pi\mathrm{N}\Delta^2\pi^2M_0\left(-2(-2+d)m_\Delta^3 - d(2+d)m_\Delta^2M_0 + (-2+d)^2M_0\left(-m_\pi^2+M_0^2\right) + (-2+d)m_\Delta\left(-7\left(Q^2+s\right) + 2m_\pi^2 + 12M_0^2\right)\right)\right.$$
$$\left.(5+\tau[3])\mathrm{II}[4+d]\left[\{\{0,m_\pi\},1\},\{\{q,m_\pi\},1\},\{\{p,m_\Delta\},2\}\right]\right) -$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^4M_0}\left(8ie^2\mathrm{g}\pi\mathrm{N}\Delta^2\pi^2M_0\left(4(-13+3d)m_\Delta^5 + 2(-24+d(4+d))m_\Delta^4M_0 + (-2+d)^2\left(m_\pi - M_0\right)M_0\left(m_\pi + M_0\right)\left(5\left(3Q^2+s\right) + 2m_\pi^2 - 7M_0^2\right) -\right.\right.$$
$$m_\Delta^3\left((148+d(-61+2d))Q^2 + 88s - d(23+2d)s + 4(-17+5d)m_\pi^2 + (-128+d(19+10d))M_0^2\right) -$$
$$m_\Delta^2M_0\left((122+(-78+d)d)Q^2 + 38s - 3d(2+3d)s + 4\left(-8+d^2\right)m_\pi^2 + (-46+d(-14+25d))M_0^2\right) +$$
$$\left.(-2+d)m_\Delta\left(14\left(Q^2+s\right)\left(3Q^2+s\right) + 8m_\pi^4 - \left(63Q^2+6dQ^2+29s+2ds\right)M_0^2 + (3+2d)M_0^4 + m_\pi^2\left(-47Q^2+6dQ^2-17s+2ds+(21-2d)M_0^2\right)\right)\right)$$
$$\left.(5+\tau[3])\mathrm{II}[4+d]\left[\{\{0,m_\pi\},1\},\{\{q,m_\pi\},1\},\{\{p,m_\Delta\},3\}\right]\right) +$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^3M_0}\left(16i(-2+d)e^2\mathrm{g}\pi\mathrm{N}\Delta^2\pi^2M_0(5+\tau[3])\mathrm{II}[4+d]\left[\{\{0,m_\pi\},1\},\{\{q,m_\pi\},1\},\{\{p+q,m_\Delta\},1\}\right]\right) -$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^4M_0}$$
$$\left(8ie^2\mathrm{g}\pi\mathrm{N}\Delta^2\pi^2M_0\left(-4(-2+d)m_\Delta^3 - (-12+d(2+d))m_\Delta^2M_0 + (-2+d)^2M_0\left(-m_\pi^2+M_0^2\right) - (-2+d)m_\Delta\left(Q^2+s-4m_\pi^2+4M_0^2\right)\right)\right.$$
$$\left.(5+\tau[3])\mathrm{II}[4+d]\left[\{\{0,m_\pi\},1\},\{\{q,m_\pi\},1\},\{\{p+q,m_\Delta\},2\}\right]\right) +$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^4M_0}\left(8i(-2+d)e^2\mathrm{g}\pi\mathrm{N}\Delta^2\pi^2M_0^2\left(-(-2+d)\left(m_\pi - M_0\right)\left(m_\pi + M_0\right)\left(-Q^2-s+M_0^2\right) + m_\Delta^2\left(d\left(Q^2+s\right) + (-4+3d)M_0^2\right)\right)\right.$$

$$(5+\tau[3])\mathrm{II}[4+d]\left[\{\{0,m_\pi\},1\},\{\{q,m_\pi\},1\},\{\{p+q,m_\Delta\},3\}\right]) -$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^3M_0}\left(8ie^2\mathrm{g\pi N\Delta}^2\pi^2M_0\left(6(-2+d)m_\Delta^2+2(-1+d)dm_\Delta M_0-(-2+d)\left(8Q^2+3s+2m_\pi^2-M_0^2\right)\right)\right.$$
$$(5+\tau[3])\mathrm{II}[4+d]\left[\{\{0,m_\pi\},1\},\{\{q,m_\pi\},2\},\{\{-p,m_\Delta\},1\}\right]) +$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^4M_0}\left(4ie^2\mathrm{g\pi N\Delta}^2\pi^2M_0\left(-2(-8+(-4+d)d)m_\Delta^4M_0+2(-2+d)^2M_0\left(-m_\pi^2+M_0^2\right)\left(-3Q^2-2s+m_\pi^2+M_0^2\right)+\right.\right.$$
$$(-2+d)m_\Delta^3\left((-9+2d)\left(Q^2+s\right)+(-7+6d)M_0^2\right)+2m_\Delta^2M_0\left((-2+d)\left((-5+4d)Q^2+(-3+d)s\right)+2(2+(-4+d)d)m_\pi^2+(16+5(-3+d)d)M_0^2\right)+$$
$$(-2+d)m_\Delta\left(\left(Q^2+s\right)\left(3Q^2+2s\right)-\left(2(2+d)Q^2+(3+2d)s\right)M_0^2+(-3+2d)M_0^4+m_\pi^2\left((3+2d)\left(Q^2+s\right)+(1-2d)M_0^2\right)\right)\Big)$$
$$(5+\tau[3])\mathrm{II}[4+d]\left[\{\{0,m_\pi\},1\},\{\{q,m_\pi\},2\},\{\{-p,m_\Delta\},2\}\right]) + \frac{1}{6(-1+d)^2F^2m_\Delta^3M_0}$$
$$\left(32ie^2\mathrm{g\pi N\Delta}^2\pi^2M_0\left((-2+d)m_\Delta^2+(-3+d)m_\Delta M_0-(-2+d)\left(m_\pi-M_0\right)\left(m_\pi+M_0\right)\right)(5+\tau[3])\mathrm{II}[4+d]\left[\{\{0,m_\pi\},1\},\{\{q,m_\pi\},2\},\{\{p,m_\Delta\},1\}\right]\right) +$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^3M_0}\left(8ie^2\mathrm{g\pi N\Delta}^2\pi^2M_0\left(-2(-2+d)m_\Delta^2-2(2+(-1+d)d)m_\Delta M_0+(-2+d)\left(-7\left(Q^2+s\right)+2m_\pi^2+9M_0^2\right)\right)\right.$$
$$(5+\tau[3])\mathrm{II}[4+d]\left[\{\{0,m_\pi\},1\},\{\{q,m_\pi\},2\},\{\{-p+q,m_\Delta\},1\}\right]) -$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^4M_0}\left(4ie^2\mathrm{g\pi N\Delta}^2\pi^2M_0\left(8(-13+3d)m_\Delta^5+2(-40+d(8+d))m_\Delta^4M_0+2(-2+d)^2\left(m_\pi-M_0\right)M_0\left(m_\pi+M_0\right)\left(9Q^2+2s+m_\pi^2-3M_0^2\right)-\right.\right.$$
$$2m_\Delta^2M_0\left(103Q^2+31s-d\left(65Q^2+(5+7d)s\right)+2(-18+d(4+d))m_\pi^2+\left(-35+d+13d^2\right)M_0^2\right)-$$
$$m_\Delta^3\left((278+d(-109+2d))Q^2+158s-3d(11+2d)s+8(-17+5d)m_\pi^2+(-170+d(25+14d))M_0^2\right)+$$
$$(-2+d)m_\Delta\left(7\left(Q^2+s\right)\left(11Q^2+3s\right)+16m_\pi^4-\left(89Q^2+10dQ^2+37s+2ds\right)M_0^2+2(2+d)M_0^4+m_\pi^2\left(-85Q^2+10dQ^2-25s+2ds+(21-2d)M_0^2\right)\right)\Big)$$
$$(5+\tau[3])\mathrm{II}[4+d]\left[\{\{0,m_\pi\},1\},\{\{q,m_\pi\},2\},\{\{-p+q,m_\Delta\},2\}\right]) +$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^3M_0}\left(32ie^2\mathrm{g\pi N\Delta}^2\pi^2M_0\left((-2+d)m_\Delta^2+(-3+d)m_\Delta M_0-(-2+d)\left(m_\pi-M_0\right)\left(m_\pi+M_0\right)\right)(5+\tau[3])\right.$$
$$\mathrm{II}[4+d]\left[\{\{0,m_\pi\},1\},\{\{q,m_\pi\},2\},\{\{p+q,m_\Delta\},1\}\right]) -$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^4M_0}\left(8ie^2\mathrm{g\pi N\Delta}^2\pi^2M_0\left(-4(-10+3d)m_\Delta^5-4(-9+2d)m_\Delta^4M_0+4(-2+d)^2Q^2M_0\left(-m_\pi^2+M_0^2\right)+\right.\right.$$
$$2m_\Delta^3\left(2(-9+4d)Q^2+(4+d(-7+2d))s+2(-14+5d)m_\pi^2+(16+(5-2d)d)M_0^2\right)+$$
$$2m_\Delta^2M_0\left((-7+4d)Q^2+(11+2(-4+d)d)s+(-22+8d)m_\pi^2+(-9-2(-4+d)d)M_0^2\right)-$$
$$(-2+d)m_\Delta\left(9Q^4+8Q^2s+s^2+8m_\pi^4-2\left(Q^2+2dQ^2-3s\right)M_0^2-7M_0^4+2m_\pi^2\left(Q^2+2dQ^2-2s-2M_0^2\right)\right)\Big)$$
$$(5+\tau[3])\mathrm{II}[4+d]\left[\{\{0,m_\pi\},1\},\{\{q,m_\pi\},3\},\{\{-p,m_\Delta\},1\}\right]) + \frac{1}{6(-1+d)^2F^2m_\Delta^4M_0}\left(8ie^2\mathrm{g\pi N\Delta}^2\pi^2M_0\right.$$
$$\left(-4(-13+3d)m_\Delta^5-8(-4+d)m_\Delta^4M_0+4(-2+d)^2Q^2M_0\left(-m_\pi^2+M_0^2\right)+4m_\Delta^2M_0\left(22Q^2+7s-d\left(14Q^2+(2+d)s\right)+2(-5+2d)m_\pi^2+(-5+d(2+d))M_0^2\right)+\right.$$
$$2m_\Delta^3\left(65Q^2+35s-d\left(24Q^2+(5+2d)s\right)+2(-17+5d)m_\pi^2+(-21+d(3+2d))M_0^2\right)+$$
$$(-2+d)m_\Delta\left(-7\left(Q^2+s\right)\left(5Q^2+s\right)+\left((38-4d)Q^2+8s\right)m_\pi^2-8m_\pi^4+2\left(2(7+d)Q^2+5s\right)M_0^2-3M_0^4\right)\Big)$$
$$(5+\tau[3])\mathrm{II}[4+d]\left[\{\{0,m_\pi\},1\},\{\{q,m_\pi\},3\},\{\{-p+q,m_\Delta\},1\}\right]) +$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^3M_0}\left(32ie^2\mathrm{g\pi N\Delta}^2\pi^2M_0\left((-2+d)m_\Delta^2+(-3+d)m_\Delta M_0-(-2+d)\left(m_\pi-M_0\right)\left(m_\pi+M_0\right)\right)(5+\tau[3])\right.$$
$$\mathrm{II}[4+d]\left[\{\{0,m_\pi\},1\},\{\{p+q,m_\Delta\},1\},\{\{p,m_\Delta\},2\}\right]) +$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^4M_0}\left(16ie^2\mathrm{g\pi N\Delta}^2\pi^2M_0\left(-2(-5+d)dm_\Delta^4M_0-(-2+d)Q^2m_\Delta\left(Q^2+s-M_0^2\right)+2(-5+d)m_\Delta^3\left(-Q^2-s+M_0^2\right)+\right.\right.$$
$$(-2+d)^2Q^2M_0\left(-m_\pi^2+M_0^2\right)+m_\Delta^2M_0\left(-(-6+d(2+d))Q^2-2(-2+d)(-1+d)s+2(-2+(-3+d)d)m_\pi^2+8M_0^2\right)\Big)$$
$$(5+\tau[3])\mathrm{II}[4+d]\left[\{\{0,m_\pi\},1\},\{\{p+q,m_\Delta\},1\},\{\{p,m_\Delta\},3\}\right]) + \frac{1}{6(-1+d)^2F^2m_\Delta^4M_0}$$
$$\left(8ie^2\mathrm{g\pi N\Delta}^2\pi^2M_0^2\left(-2(2+(-5+d)d)m_\Delta^4-(-2+d)^2Q^2\left(m_\pi-M_0\right)\left(m_\pi+M_0\right)+m_\Delta^2\left(-(-2+d)\left(dQ^2+2(-1+d)s\right)+2(-2+(-3+d)d)m_\pi^2+8M_0^2\right)\right)\right.$$
$$(5+\tau[3])\mathrm{II}[4+d]\left[\{\{0,m_\pi\},1\},\{\{p+q,m_\Delta\},2\},\{\{p,m_\Delta\},2\}\right]) + \frac{1}{6(-1+d)^2F^2m_\Delta^4M_0}$$
$$\left(8ie^2\mathrm{g\pi N\Delta}^2\pi^2M_0^2\left(-2(2+(-5+d)d)m_\Delta^4-(-2+d)^2Q^2\left(m_\pi-M_0\right)\left(m_\pi+M_0\right)+m_\Delta^2\left(-(-2+d)\left(dQ^2+2(-1+d)s\right)+2(-2+(-3+d)d)m_\pi^2+8M_0^2\right)\right)\right.$$

$$(5+\tau[3])\text{II}[4+d]\left[\{\{0,m_\pi\},2\},\{\{p,m_\Delta\},1\},\{\{p+q,m_\Delta\},2\}\right])+$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^3M_0}\left(32ie^2\text{g}\pi\text{N}\Delta^2\pi^2M_0\left((-2+d)m_\Delta^2+(-3+d)m_\Delta M_0-(-2+d)\left(m_\pi-M_0\right)\left(m_\pi+M_0\right)\right)(5+\tau[3])\right.$$
$$\text{II}[4+d]\left[\{\{0,m_\pi\},2\},\{\{p-q,m_\Delta\},1\},\{\{p,m_\Delta\},1\}\right])+$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^3M_0}\left(32ie^2\text{g}\pi\text{N}\Delta^2\pi^2M_0\left((-2+d)m_\Delta^2+(-3+d)m_\Delta M_0-(-2+d)\left(m_\pi-M_0\right)\left(m_\pi+M_0\right)\right)(5+\tau[3])\right.$$
$$\text{II}[4+d]\left[\{\{0,m_\pi\},2\},\{\{p+q,m_\Delta\},1\},\{\{p,m_\Delta\},1\}\right])+$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^4M_0}\left(8ie^2\text{g}\pi\text{N}\Delta^2\pi^2M_0\left(-2(-2+(-5+d)d)m_\Delta^4M_0-2(-2+d)Q^2m_\Delta\left(Q^2+s-M_0^2\right)+4(-5+d)m_\Delta^3\left(-Q^2-s+M_0^2\right)+\right.\right.$$
$$(-2+d)^2Q^2M_0\left(-m_\pi^2+M_0^2\right)+m_\Delta^2M_0\left(-(-12+d(6+d))Q^2-2(-2+d)(-1+d)s+2(-2+(-3+d)d)m_\pi^2+8M_0^2\right)\left.\right)$$
$$(5+\tau[3])\text{II}[4+d]\left[\{\{0,m_\pi\},2\},\{\{p+q,m_\Delta\},1\},\{\{p,m_\Delta\},2\}\right])+$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^3M_0}\left(16ie^2\text{g}\pi\text{N}\Delta^2\pi^2M_0\left(-2(-3+2d)Q^2m_\Delta M_0+4m_\Delta^3M_0-(-2+d)Q^2\left(Q^2+s-M_0^2\right)-2(-5+d)m_\Delta^2\left(Q^2+s-M_0^2\right)\right)\right.$$
$$(5+\tau[3])\text{II}[4+d]\left[\{\{0,m_\pi\},3\},\{\{p+q,m_\Delta\},1\},\{\{p,m_\Delta\},1\}\right])-$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^3M_0}\left(32ie^2\text{g}\pi\text{N}\Delta^2\pi^2M_0\left(-m_\pi+m_\Delta+M_0\right)\left(m_\pi+m_\Delta+M_0\right)\left((-2+d)m_\Delta^2-2m_\Delta M_0-(-2+d)\left(m_\pi-M_0\right)\left(m_\pi+M_0\right)\right)\right.$$
$$(5+\tau[3])\text{II}[4+d]\left[\{\{0,m_\pi\},1\},\{\{q,m_\pi\},1\},\{\{p,m_\Delta\},1\},\{\{p+q,m_\Delta\},2\}\right])-$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^3M_0}\left(32ie^2\text{g}\pi\text{N}\Delta^2\pi^2M_0\left((-2+d)m_\Delta^4-(-5+d)(-2+d)m_\Delta^3M_0+(-2+d)\left(m_\pi^2-M_0^2\right)^2+2(-3+d)m_\Delta M_0\left(-m_\pi^2+M_0^2\right)-2m_\Delta^2\left((-2+d)m_\pi^2-(-4+d)M_0^2\right)\right)\right.$$
$$(5+\tau[3])\text{II}[4+d]\left[\{\{0,m_\pi\},1\},\{\{q,m_\pi\},1\},\{\{p+q,m_\Delta\},1\},\{\{p,m_\Delta\},2\}\right])-$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^4M_0}\left(16ie^2\text{g}\pi\text{N}\Delta^2\pi^2M_0\left(-2(-3+d)(-2+d)m_\Delta^6M_0+(-2+d)^2Q^2M_0\left(m_\pi^2-M_0^2\right)^2+4(-5+d)m_\Delta^5\left(-Q^2-s+M_0^2\right)+\right.\right.$$
$$2(-2+d)Q^2m_\Delta\left(-Q^2-s+M_0^2\right)\left(-m_\pi^2+M_0^2\right)+2m_\Delta^3\left(-Q^2-s+M_0^2\right)\left((-2+d)Q^2-2(-5+d)\left(m_\pi-M_0\right)\left(m_\pi+M_0\right)\right)-$$
$$2m_\Delta^2M_0\left(-m_\pi^2+M_0^2\right)\left(-(-2+d)^2Q^2-(-2+(-3+d)d)\left(m_\pi-M_0\right)\left(m_\pi+M_0\right)\right)+m_\Delta^4M_0\left(4\left(5Q^2+4s\right)+4(2+(-4+d)d)m_\pi^2-8d\left(2Q^2+s+M_0^2\right)+d^2\left(Q^2+4M_0^2\right)\right)\left.\right)\left.\right)$$
$$(5+\tau[3])\text{II}[4+d]\left[\{\{0,m_\pi\},1\},\{\{q,m_\pi\},1\},\{\{p+q,m_\Delta\},1\},\{\{p,m_\Delta\},3\}\right])-\frac{1}{6(-1+d)^2F^2m_\Delta^4M_0}$$
$$\left(8ie^2\text{g}\pi\text{N}\Delta^2\pi^2M_0^2\left(-2(-3+d)(-2+d)m_\Delta^6+(-2+d)^2Q^2\left(m_\pi^2-M_0^2\right)^2+m_\Delta^4\left((4+(-8+d)d)Q^2-4ds+4(2+(-4+d)d)m_\pi^2+4(4+(-3+d)d)M_0^2\right)-\right.\right.$$
$$2m_\Delta^2\left(m_\pi-M_0\right)\left(m_\pi+M_0\right)\left((-2+d)^2Q^2+(-2+(-3+d)d)\left(m_\pi-M_0\right)\left(m_\pi+M_0\right)\right)\left.\right)\left.\right)$$
$$(5+\tau[3])\text{II}[4+d]\left[\{\{0,m_\pi\},1\},\{\{q,m_\pi\},1\},\{\{p+q,m_\Delta\},2\},\{\{p,m_\Delta\},2\}\right])-$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^3M_0}\left(32ie^2\text{g}\pi\text{N}\Delta^2\pi^2M_0\left((-2+d)m_\Delta^4-(-5+d)(-2+d)m_\Delta^3M_0+(-2+d)\left(m_\pi^2-M_0^2\right)^2+2(-3+d)m_\Delta M_0\left(-m_\pi^2+M_0^2\right)-2m_\Delta^2\left((-2+d)m_\pi^2-(-4+d)M_0^2\right)\right)\right.$$
$$(5+\tau[3])\text{II}[4+d]\left[\{\{0,m_\pi\},1\},\{\{q,m_\pi\},2\},\{\{-p,m_\Delta\},1\},\{\{-p+q,m_\Delta\},1\}\right])-$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^4M_0}\left(8ie^2\text{g}\pi\text{N}\Delta^2\pi^2M_0\left(-2(-3+d)(-2+d)m_\Delta^6M_0+(-2+d)^2Q^2M_0\left(m_\pi^2-M_0^2\right)^2+8(-5+d)m_\Delta^5\left(-Q^2-s+M_0^2\right)+\right.\right.$$
$$4(-2+d)Q^2m_\Delta\left(-Q^2-s+M_0^2\right)\left(-m_\pi^2+M_0^2\right)+m_\Delta^4M_0\left((36+(-24+d)d)Q^2-4(-8+3d)s+4(2+(-4+d)d)m_\pi^2+4(-4+(-1+d)d)M_0^2\right)+$$
$$4m_\Delta^3\left(-Q^2-s+M_0^2\right)\left((-2+d)Q^2-2(-5+d)\left(m_\pi-M_0\right)\left(m_\pi+M_0\right)\right)-2m_\Delta^2M_0\left(-m_\pi^2+M_0^2\right)\left(-(-2+d)^2Q^2-(-2+(-3+d)d)\left(m_\pi-M_0\right)\left(m_\pi+M_0\right)\right)\left.\right)\left.\right)$$
$$(5+\tau[3])\text{II}[4+d]\left[\{\{0,m_\pi\},1\},\{\{q,m_\pi\},2\},\{\{-p,m_\Delta\},1\},\{\{-p+q,m_\Delta\},2\}\right])-\frac{1}{6(-1+d)^2F^2m_\Delta^4M_0}$$
$$\left(8ie^2\text{g}\pi\text{N}\Delta^2\pi^2M_0^2\left(-2(-3+d)(-2+d)m_\Delta^6+(-2+d)^2Q^2\left(m_\pi^2-M_0^2\right)^2+m_\Delta^4\left((4+(-8+d)d)Q^2-4ds+4(2+(-4+d)d)m_\pi^2+4(4+(-3+d)d)M_0^2\right)-\right.\right.$$
$$2m_\Delta^2\left(m_\pi-M_0\right)\left(m_\pi+M_0\right)\left((-2+d)^2Q^2+(-2+(-3+d)d)\left(m_\pi-M_0\right)\left(m_\pi+M_0\right)\right)\left.\right)\left.\right)$$
$$(5+\tau[3])\text{II}[4+d]\left[\{\{0,m_\pi\},1\},\{\{q,m_\pi\},2\},\{\{-p+q,m_\Delta\},1\},\{\{-p,m_\Delta\},2\}\right])-$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^3M_0}\left(32ie^2\text{g}\pi\text{N}\Delta^2\pi^2M_0\left(-m_\pi+m_\Delta+M_0\right)\left(m_\pi+m_\Delta+M_0\right)\left((-2+d)m_\Delta^2-2m_\Delta M_0-(-2+d)\left(m_\pi-M_0\right)\left(m_\pi+M_0\right)\right)\right.$$
$$(5+\tau[3])\text{II}[4+d]\left[\{\{0,m_\pi\},1\},\{\{q,m_\pi\},2\},\{\{p+q,m_\Delta\},1\},\{\{p,m_\Delta\},1\}\right])+$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^3M_0}\left(32ie^2\text{g}\pi\text{N}\Delta^2\pi^2M_0\left(-2(-5+d)m_\Delta^4\left(-Q^2-s+M_0^2\right)+(-2+d)Q^2\left(m_\pi-M_0\right)\left(m_\pi+M_0\right)\left(-Q^2-s+M_0^2\right)-\right.\right.$$

$$2m_\Delta^3 M_0 \left(4\left(Q^2+s\right) - d\left(2Q^2+s\right) + (-4+d)M_0^2\right) + m_\Delta^2 \left(-Q^2-s+M_0^2\right)\left(-(-2+d)Q^2 + 2(-5+d)\left(m_\pi - M_0\right)\left(m_\pi+M_0\right)\right)\Big)$$
$$(5+\tau[3])\mathrm{II}[4+d]\left[\{\{0,m_\pi\},1\},\{\{q,m_\pi\},3\},\{\{-p,m_\Delta\},1\},\{\{-p+q,m_\Delta\},1\}\right]\Big) +$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^3M_0}\left(64i(-2+d)e^2\mathrm{g\pi N\Delta}^2\pi^3 M_0\left(-3\left(Q^2+s\right)+7M_0^2\right)(5+\tau[3])\mathrm{II}[6+d]\left[\{\{0,m_\pi\},1\},\{\{q,m_\pi\},1\},\{\{p,m_\Delta\},3\}\right]\right) -$$
$$\frac{1}{2(-1+d)^2F^2m_\Delta^4M_0}\Big(32ie^2\mathrm{g\pi N\Delta}^2\pi^3 M_0\Big(2(-2+d)^2M_0\left(-3Q^2-s+M_0^2\right)\left(-m_\pi^2+M_0^2\right) - 2m_\Delta^3\left((32-13d)Q^2+(20-7d)s+3(-16+5d)M_0^2\right) -$$
$$2m_\Delta^2M_0\left((34+(-22+d)d)Q^2+14s-d(6+d)s+(-22+d(2+5d))M_0^2\right) + (-2+d)m_\Delta$$
$$\left(9\left(Q^2+s\right)\left(3Q^2+s\right) - 2\left(28Q^2+11s\right)M_0^2 + 5M_0^4 + 2m_\pi^2\left(-11Q^2-5s+9M_0^2\right)\right)\Big)(5+\tau[3])\mathrm{II}[6+d]\left[\{\{0,m_\pi\},1\},\{\{q,m_\pi\},1\},\{\{p,m_\Delta\},4\}\right]\Big) +$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^3M_0}\left(128i(-2+d)e^2\mathrm{g\pi N\Delta}^2\pi^3\left(Q^2+s\right)M_0(5+\tau[3])\mathrm{II}[6+d]\left[\{\{0,m_\pi\},1\},\{\{q,m_\pi\},2\},\{\{-p,m_\Delta\},2\}\right]\right) +$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^4M_0}$$
$$\Big(32i(-2+d)e^2\mathrm{g\pi N\Delta}^2\pi^3M_0\Big(2(-2+d)M_0\left(-Q^2-s+M_0^2\right)\left(-m_\pi^2+M_0^2\right) + 2m_\Delta^2M_0\left(d\left(Q^2+s\right)+(-4+3d)M_0^2\right) + m_\Delta\left(\left(Q^2+s\right)^2 - M_0^4\right)\Big)$$
$$(5+\tau[3])\mathrm{II}[6+d]\left[\{\{0,m_\pi\},1\},\{\{q,m_\pi\},2\},\{\{-p,m_\Delta\},3\}\right]\Big) +$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^3M_0}\left(64i(-2+d)e^2\mathrm{g\pi N\Delta}^2\pi^3M_0\left(-3\left(Q^2+s\right)+5M_0^2\right)(5+\tau[3])\mathrm{II}[6+d]\left[\{\{0,m_\pi\},1\},\{\{q,m_\pi\},2\},\{\{-p+q,m_\Delta\},2\}\right]\right) -$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^4M_0}\Big(64ie^2\mathrm{g\pi N\Delta}^2\pi^3M_0\Big((-2+d)^2M_0\left(-5Q^2-s+M_0^2\right)\left(-m_\pi^2+M_0^2\right) - 2m_\Delta^3\left(-9\left((-5+2d)Q^2+(-3+d)s\right)+(-55+17d)M_0^2\right) -$$
$$m_\Delta^2M_0\left((84+(-52+d)d)Q^2+(32-3d(4+d))s+(-56+d(16+7d))M_0^2\right) +$$
$$(-2+d)m_\Delta\left(\left(Q^2+s\right)\left(37Q^2+10s\right) - \left(60Q^2+19s\right)M_0^2 + M_0^4 + 2m_\pi^2\left(-15Q^2-6s+10M_0^2\right)\right)\Big)$$
$$(5+\tau[3])\mathrm{II}[6+d]\left[\{\{0,m_\pi\},1\},\{\{q,m_\pi\},2\},\{\{-p+q,m_\Delta\},3\}\right]\Big) +$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^3M_0}\left(64i(-2+d)e^2\mathrm{g\pi N\Delta}^2\pi^3M_0\left(7Q^2-3s+3M_0^2\right)(5+\tau[3])\mathrm{II}[6+d]\left[\{\{0,m_\pi\},1\},\{\{q,m_\pi\},3\},\{\{-p,m_\Delta\},1\}\right]\right) +$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^4M_0}$$
$$\Big(32ie^2\mathrm{g\pi N\Delta}^2\pi^3M_0\Big(4(-2+d)^2Q^2\left(m_\pi-M_0\right)M_0\left(m_\pi+M_0\right) - 2(-2+d)m_\Delta^3\left(5\left(Q^2+s\right)+3M_0^2\right) + 4m_\Delta^2M_0\left(-(-2+d)\left(Q^2+s+ds\right)+(8+(-5+d)d)M_0^2\right) + (-2+d)$$
$$m_\Delta\left(3\left(Q^2+s\right)\left(3Q^2+s\right) - 2\left(3Q^2+2s\right)M_0^2 - 7M_0^4 + 2m_\pi^2\left(3\left(Q^2+s\right)+M_0^2\right)\right)\Big)(5+\tau[3])\mathrm{II}[6+d]\left[\{\{0,m_\pi\},1\},\{\{q,m_\pi\},3\},\{\{-p,m_\Delta\},2\}\right]\Big) +$$
$$\frac{1}{2(-1+d)^2F^2m_\Delta^3M_0}\left(64i(-2+d)e^2\mathrm{g\pi N\Delta}^2\pi^3M_0\left(-Q^2-s+M_0^2\right)(5+\tau[3])\mathrm{II}[6+d]\left[\{\{0,m_\pi\},1\},\{\{q,m_\pi\},3\},\{\{-p+q,m_\Delta\},1\}\right]\right) +$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^4M_0}\Big(32ie^2\mathrm{g\pi N\Delta}^2\pi^3M_0\Big(4(-2+d)^2Q^2M_0\left(-m_\pi^2+M_0^2\right) + 2m_\Delta^3\left((84-33d)Q^2+3(16-5d)s+(-76+23d)M_0^2\right) + 4m_\Delta^2M_0$$
$$\left(-7(-5+3d)Q^2+13s-d(5+d)s+(-21+d(9+d))M_0^2\right) + (-2+d)m_\Delta\left(-\left(Q^2+s\right)\left(67Q^2+13s\right) + 2\left(39Q^2+8s\right)M_0^2 + 5M_0^4 + m_\pi^2\left(18\left(3Q^2+s\right)-26M_0^2\right)\right)\Big)$$
$$(5+\tau[3])\mathrm{II}[6+d]\left[\{\{0,m_\pi\},1\},\{\{q,m_\pi\},3\},\{\{-p+q,m_\Delta\},2\}\right]\Big) - \frac{1}{2(-1+d)^2F^2m_\Delta^3M_0}$$
$$\Big(64ie^2\mathrm{g\pi N\Delta}^2\pi^3M_0\Big(-2(-2+d)m_\pi^2\left(2Q^2-s+M_0^2\right) + 2(-2+d)m_\Delta^2\left(3Q^2-2s+2M_0^2\right) + 2m_\Delta M_0\left(-3Q^2+2dQ^2+5s-2ds+(-5+2d)M_0^2\right) +$$
$$(-2+d)\left(-6Q^4-Q^2s+s^2+\left(Q^2-4s\right)M_0^2+3M_0^4\right)\Big)(5+\tau[3])\mathrm{II}[6+d]\left[\{\{0,m_\pi\},1\},\{\{q,m_\pi\},4\},\{\{-p,m_\Delta\},1\}\right]\Big) + \frac{1}{2(-1+d)^2F^2m_\Delta^3M_0}$$
$$\Big(64ie^2\mathrm{g\pi N\Delta}^2\pi^3M_0\Big(2(-2+d)m_\pi^2\left(4Q^2+s-M_0^2\right) + 4m_\Delta M_0\left(5Q^2+2s-d\left(3Q^2+s\right)+(-2+d)M_0^2\right) + 2m_\Delta^2\left(13Q^2-5dQ^2+7s-2ds+(-7+2d)M_0^2\right) +$$
$$(-2+d)\left(-\left(Q^2+s\right)\left(10Q^2+s\right)+7Q^2M_0^2+M_0^4\right)\Big)(5+\tau[3])\mathrm{II}[6+d]\left[\{\{0,m_\pi\},1\},\{\{q,m_\pi\},4\},\{\{-p+q,m_\Delta\},1\}\right]\Big) -$$
$$\frac{1}{2(-1+d)^2F^2m_\Delta^3M_0}\left(1024i(-2+d)e^2\mathrm{g\pi N\Delta}^2\pi^4M_0\left(-3Q^2-s+M_0^2\right)\left(-Q^2-s+3M_0^2\right)(5+\tau[3])\mathrm{II}[8+d]\left[\{\{0,m_\pi\},1\},\{\{q,m_\pi\},1\},\{\{p,m_\Delta\},5\}\right]\right) -$$
$$\frac{1}{2(-1+d)^2F^2m_\Delta^3M_0}\left(256i(-2+d)e^2\mathrm{g\pi N\Delta}^2\pi^4M_0\left(\left(Q^2+s\right)\left(11Q^2+3s\right) - 10\left(3Q^2+s\right)M_0^2 + 7M_0^4\right)(5+\tau[3])\mathrm{II}[8+d]\left[\{\{0,m_\pi\},1\},\{\{q,m_\pi\},2\},\{\{-p+q,m_\Delta\},4\}\right]\right) -$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^3M_0}\left(512i(-2+d)e^2\mathrm{g\pi N\Delta}^2\pi^4M_0\left(-\left(Q^2+s\right)^2+M_0^4\right)(5+\tau[3])\mathrm{II}[8+d]\left[\{\{0,m_\pi\},1\},\{\{q,m_\pi\},3\},\{\{-p,m_\Delta\},3\}\right]\right) -$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^3M_0}\left(512i(-2+d)e^2\mathrm{g\pi N\Delta}^2\pi^4M_0\left(3\left(Q^2+s\right)\left(5Q^2+s\right) - 8\left(4Q^2+s\right)M_0^2 + 5M_0^4\right)(5+\tau[3])\mathrm{II}[8+d]\left[\{\{0,m_\pi\},1\},\{\{q,m_\pi\},3\},\{\{-p+q,m_\Delta\},3\}\right]\right) -$$

$$\frac{1}{2(-1+d)^2F^2m_\Delta^3M_0}\left(256i(-2+d)e^2\text{g}\pi\text{N}\Delta^2\pi^4M_0\left(-\left(3Q^2-s\right)\left(Q^2+s\right)-2\left(Q^2+s\right)M_0^2+M_0^4\right)(5+\tau[3])\right.$$
$$\left.\text{II}[8+d]\left[\left\{\left\{0,m_\pi\right\},1\right\},\left\{\left\{q,m_\pi\right\},4\right\},\left\{\left\{-p,m_\Delta\right\},2\right\}\right]\right)-\frac{1}{2(-1+d)^2F^2m_\Delta^3M_0}$$
$$\left(256i(-2+d)e^2\text{g}\pi\text{N}\Delta^2\pi^4M_0\left(\left(Q^2+s\right)\left(9Q^2+s\right)-2\left(7Q^2+s\right)M_0^2+M_0^4\right)(5+\tau[3])\text{II}[8+d]\left[\left\{\left\{0,m_\pi\right\},1\right\},\left\{\left\{q,m_\pi\right\},4\right\},\left\{\left\{-p+q,m_\Delta\right\},2\right\}\right]\right)+$$
$$\frac{1}{2(-1+d)^2F^2m_\Delta^3M_0}\left(2048i(-2+d)e^2\text{g}\pi\text{N}\Delta^2\pi^4Q^2M_0\left(Q^2-s+M_0^2\right)(5+\tau[3])\text{II}[8+d]\left[\left\{\left\{0,m_\pi\right\},1\right\},\left\{\left\{q,m_\pi\right\},5\right\},\left\{\left\{-p,m_\Delta\right\},1\right\}\right]\right)-$$
$$\frac{1}{2(-1+d)^2F^2m_\Delta^3M_0}$$
$$\left(2048i(-2+d)e^2\text{g}\pi\text{N}\Delta^2\pi^4Q^2M_0\left(Q^2+s-M_0^2\right)(5+\tau[3])\text{II}[8+d]\left[\left\{\left\{0,m_\pi\right\},1\right\},\left\{\left\{q,m_\pi\right\},5\right\},\left\{\left\{-p+q,m_\Delta\right\},1\right\}\right]\right)$$

Diagram g)

$$\mathbf{A}(\nu,Q^2)=-\left(ie^2\text{g}\pi\text{N}\Delta^2\left(2d^8s^3M_0^4\left(18m_\pi^2m_\Delta^6M_0^2+4m_\pi^8\left(Q^2+s+2m_\Delta M_0+M_0^2\right)-8m_\pi^6\left(m_\Delta^2+M_0^2\right)\left(Q^2+s+2m_\Delta M_0+M_0^2\right)+\right.\right.\right.$$
$$3m_\Delta^6\left(m_\Delta^4+4m_\Delta^2M_0^2-5M_0^4\right)+m_\pi^4\left(-3m_\Delta^6+8m_\Delta^5M_0-16m_\Delta^3M_0^3+8m_\Delta M_0^5+4m_\Delta^4\left(Q^2+s+M_0^2\right)-8m_\Delta^2M_0^2\left(Q^2+s+M_0^2\right)+4M_0^4\left(Q^2+s+M_0^2\right)\right)\Big)+$$
$$64sm_\pi^2\left(m_\pi-m_\Delta-M_0\right)\left(m_\pi+m_\Delta-M_0\right)M_0^2\left(m_\pi-m_\Delta+M_0\right)\left(m_\pi+m_\Delta+M_0\right)\Big(-12sm_\Delta^3M_0\left(-s\left(Q^2+s\right)+\left(Q^2+6s\right)M_0^2+M_0^4\right)+4s^2m_\Delta M_0\Big(-\left(Q^2+s\right)^2-2sM_0^2+15M_0^4\Big)+$$
$$2m_\pi^2\Big(2s^2\left(Q^2+s\right)^2-6s^2\left(Q^2+s\right)m_\Delta M_0+\left(-Q^6+Q^4s+33Q^2s^2+35s^3\right)M_0^2+2\left(Q^2-16s\right)sm_\Delta M_0^3-\left(Q^4-39s^2\right)M_0^4+2sm_\Delta M_0^5+\left(Q^2-5s\right)M_0^6+M_0^8\Big)-$$
$$2m_\Delta^2\Big(2s^2\left(Q^2+s\right)^2-\left(Q^6+4Q^4s+33Q^2s^2+26s^3\right)M_0^2-\left(Q^4-4Q^2s+s^2\right)M_0^4+\left(Q^2+4s\right)M_0^6+M_0^8\Big)+$$
$$M_0^2\Big(s^4-M_0^2\left(Q^2+M_0^2\right)^3+s^3\left(11Q^2+10M_0^2\right)+s\left(Q^2+M_0^2\right)\left(3Q^4-5Q^2M_0^2+6M_0^4\right)+s^2\left(13Q^4+15Q^2M_0^2+56M_0^4\right)\Big)\Big)+$$
$$d^7s^2\Big(-2sm_\pi^{10}\Big(\left(Q^2+s\right)^2-4\left(Q^2+s\right)M_0^2+3M_0^4\Big)+2sm_\pi^8\Big(5\left(Q^2+s\right)^2m_\Delta^2+2\left(Q^2+s\right)^2m_\Delta M_0+\left(Q^2+s\right)\left(7Q^2+5s-16m_\Delta^2\right)M_0^2-8\left(Q^2+s\right)m_\Delta M_0^3+$$
$$\left(-66Q^2-64s+11m_\Delta^2\right)M_0^4-138m_\Delta M_0^5-69M_0^6\Big)+m_\pi^6\Big(-20s\left(Q^2+s\right)^2m_\Delta^4-16s\left(Q^2+s\right)^2m_\Delta^3M_0-8s\left(Q^2+s\right)m_\Delta^2\left(5Q^2+3s-6m_\Delta^2\right)M_0^2-$$
$$8s\left(Q^2+s\right)m_\Delta\left(3Q^2+2s-6m_\Delta^2\right)M_0^3+\left(-4s\left(11Q^4+14Q^2s+5s^2\right)+16s\left(16Q^2+15s\right)m_\Delta^2-9\left(Q^2+7s\right)m_\Delta^4\right)M_0^4+$$
$$8sm_\Delta\left(3Q^2+2s+68m_\Delta^2\right)M_0^5+\left(8s\left(31Q^2+29s\right)+296sm_\Delta^2-9m_\Delta^4\right)M_0^6+592sm_\Delta M_0^7+316sM_0^8\Big)-$$
$$\left(m_\Delta+M_0\right)\left(2sm_\Delta M_0^8\left(-7Q^4-4Q^2s-s^2+2Q^2M_0^2+M_0^4\right)+2sM_0^9\left(-7Q^4-4Q^2s-s^2+2Q^2M_0^2+M_0^4\right)-4sm_\Delta^3M_0^6\left(-3Q^4-9Q^2s-2s^2+5Q^2M_0^2+2M_0^4\right)-\right.$$
$$4sm_\Delta^2M_0^7\left(-3Q^4-9Q^2s-2s^2+5Q^2M_0^2+2M_0^4\right)+m_\Delta^5M_0^4\left(-4s^2\left(Q^2+3s\right)-20Q^2sM_0^2+\left(9Q^2-17s\right)M_0^4+9M_0^6\right)+2m_\Delta^9$$
$$\Big(-s\left(Q^2+s\right)^2+9\left(Q^2+4s\right)M_0^4+9M_0^6\Big)-m_\Delta^4M_0^5\left(4s^2\left(Q^2+3s\right)+20Q^2sM_0^2+\left(9Q^2+23s\right)M_0^4+9M_0^6\right)-2m_\Delta^8M_0\Big(s\left(Q^2+s\right)^2+\left(9Q^2+66s\right)M_0^4+9M_0^6\Big)-$$
$$m_\Delta^7M_0^2\left(-4s\left(Q^2+s\right)\left(Q^2+2s\right)-4Q^2sM_0^2+\left(27Q^2-287s\right)M_0^4+27M_0^6\right)+m_\Delta^6M_0^3\left(4s\left(Q^2+s\right)\left(Q^2+2s\right)+4Q^2sM_0^2+\left(27Q^2-175s\right)M_0^4+27M_0^6\right)\Big)+$$
$$m_\pi^4\Big(-8sm_\Delta^5M_0\Big(-3\left(Q^2+s\right)^2+6\left(Q^2+s\right)M_0^2+25M_0^4\Big)-8sm_\Delta M_0^5\left(-8Q^4-9Q^2s-3s^2+\left(Q^2-2s\right)M_0^2+45M_0^4\right)-4sM_0^6\left(-17Q^4-16Q^2s-\right.$$
$$5s^2+\left(34Q^2+28s\right)M_0^2+49M_0^4\Big)+4sm_\Delta^6\Big(5\left(Q^2+s\right)^2-8\left(Q^2+s\right)M_0^2+51M_0^4\Big)+8sm_\Delta^3M_0^3\left(\left(Q^2+s\right)\left(5Q^2+2s\right)+\left(Q^2+4s\right)M_0^2+62M_0^4\right)+$$
$$4sm_\Delta^2M_0^4\left(13Q^4+10Q^2s+3s^2+\left(48Q^2+52s\right)M_0^2+65M_0^4\right)+m_\Delta^4M_0^2\left(12s\left(Q^2+s\right)\left(3Q^2+s\right)-24s\left(5Q^2+4s\right)M_0^2+\left(27Q^2-67s\right)M_0^4+27M_0^6\right)\Big)+$$
$$m_\pi^2\left(-8sm_\Delta^2M_0^6\left(3Q^4-4Q^2s-s^2+2Q^2M_0^2+5M_0^4\right)+8sm_\Delta M_0^7\left(-9Q^4-7Q^2s-2s^2+\left(Q^2-2s\right)M_0^2+6M_0^4\right)-16sm_\Delta^3M_0^5\right.$$
$$\left(-s\left(3Q^2+s\right)-\left(2Q^2+3s\right)M_0^2+6M_0^4\right)-16sm_\Delta^7M_0\Big(\left(Q^2+s\right)^2-\left(Q^2+s\right)M_0^2+8M_0^4\Big)-8sm_\Delta^5M_0^3\left(\left(Q^2-2s\right)\left(Q^2+s\right)+3\left(Q^2+2s\right)M_0^2+10M_0^4\right)+$$
$$2sM_0^8\left(-25Q^4-18Q^2s-5s^2+8Q^2M_0^2+13M_0^4\right)+4m_\Delta^6M_0^2\left(-2\left(Q^2-s\right)s\left(Q^2+s\right)-4s^2M_0^2+\left(9Q^2-95s\right)M_0^4+9M_0^6\right)+$$
$$m_\Delta^8\Big(-10s\left(Q^2+s\right)^2+8s\left(Q^2+s\right)M_0^2+\left(27Q^2-85s\right)M_0^4+27M_0^6\Big)-m_\Delta^4M_0^4\left(-4s\left(-Q^4+6Q^2s+s^2\right)+8\left(Q^2-s\right)sM_0^2+\left(27Q^2+101s\right)M_0^4+27M_0^6\right)\Big)\Big)-$$

$$8d^2 (8sm_\Delta^{11} M_0 (s^2 (Q^2+s) - 4s^2 M_0^2 + (Q^2+6s) M_0^4 + M_0^6) - 2s^2 M_0^8 (s (Q^2+s) (3Q^4+2s^2) + (-6Q^6+19Q^4 s+7Q^2 s^2+3s^3) M_0^2 + (-2Q^4+34Q^2 s+17s^2)$$
$$M_0^4 + (5Q^2-23s) M_0^6 + M_0^8) - 4sm_\Delta^9 M_0 \Big(4s^2 (Q^2+s)^2 + 2s^2 (26Q^2+27s) M_0^2 - (3Q^4+3Q^2 s+38s^2) M_0^4 + (-2Q^2+33s) M_0^6 + M_0^8\Big) +$$
$$4sm_\Delta^5 M_0^5 (s (5Q^6+9Q^4 s-17Q^2 s^2-35s^3) - (2Q^6+12Q^4 s+59Q^2 s^2-15s^3) M_0^2 - (Q^4+10Q^2 s+42s^2) M_0^4 + (4Q^2-s) M_0^6 + 3M_0^8) -$$
$$4sm_\Delta^7 M_0^3 (-4s^2 (Q^2+s)(Q^2+5s) - (Q^6+8Q^4 s+111Q^2 s^2+164s^3) M_0^2 + 4 (Q^4+28s^2) M_0^4 + (9Q^2-26s) M_0^6 + 4M_0^8) +$$
$$4m_\pi^{10} \Big(3s^3 (Q^2+s)^2 - 2s^3 (Q^2+s) m_\Delta M_0 - s^3 (23Q^2+19s) M_0^2 - (Q^6+Q^4 s+3Q^2 s^2-13s^3) M_0^4 - (Q^2-3s)(Q^2+2s) M_0^6 + (Q^2-4s) M_0^8 + M_0^{10}\Big) +$$
$$2sm_\Delta^2 M_0^6 (2s^2 (Q^2+s)(3Q^4+2s^2) - s (5Q^6+41Q^4 s-12Q^2 s^2+3s^3) M_0^2 +$$
$$(3Q^6+23Q^4 s-44Q^2 s^2+21s^3) M_0^4 + (7Q^4+34Q^2 s-11s^2) M_0^6 + 5 (Q^2-2s) M_0^8 + M_0^{10}) -$$
$$2m_\Delta^{10} \Big(6s^3 (Q^2+s)^2 + 2s^3 (23Q^2+27s) M_0^2 - (2Q^6+5Q^4 s+17Q^2 s^2+143s^3) M_0^4 + (-2Q^4+2Q^2 s-17s^2) M_0^6 + (2Q^2-5s) M_0^8 + 2M_0^{10}\Big) -$$
$$2m_\Delta^6 M_0^4 (s^2 (-7Q^6+17Q^4 s+56Q^2 s^2+49s^3) - s (9Q^6+36Q^4 s+71Q^2 s^2+25s^3) M_0^2 - 2 (Q^6+7Q^4 s+22Q^2 s^2-104s^3) M_0^4 +$$
$$(-2Q^4+25Q^2 s-49s^2) M_0^6 + 2 (Q^2+9s) M_0^8 + 2M_0^{10}) + m_\Delta^8 M_0^2 (2s^3 (Q^2+s)(8Q^2+29s) + s (-6Q^6-9Q^4 s+67Q^2 s^2+166s^3) M_0^2 -$$
$$(8Q^6+22Q^4 s+71Q^2 s^2-96s^3) M_0^4 - 4 (2Q^4-9Q^2 s+28s^2) M_0^6 + 4 (2Q^2+s) M_0^8 + 8M_0^{10}) + sm_\Delta^4 M_0^4 (-2s^2 (Q^2+s)(3Q^4+2s^2) +$$
$$2s (-8Q^6+75Q^4 s+122Q^2 s^2+32s^3) M_0^2 - (18Q^6+113Q^4 s+153Q^2 s^2-84s^3) M_0^4 - (30Q^4+109Q^2 s+190s^2) M_0^6 + 2 (4Q^2+s) M_0^8 + 20M_0^{10}) +$$
$$2m_\pi^8 \Big(-30s^3 (Q^2+s)^2 m_\Delta^2 + 4s^3 (Q^2+s) m_\Delta (-2 (Q^2+s) + 5m_\Delta^2) M_0 + s^3 (-52Q^4-83Q^2 s-31s^2+2 (69Q^2+49s) m_\Delta^2) M_0^2 +$$
$$2s^3 m_\Delta (31Q^2+29s-8m_\Delta^2) M_0^3 + (s (11Q^6+27Q^4 s+304Q^2 s^2+221s^3) + (10Q^6+13Q^4 s+33Q^2 s^2-38s^3) m_\Delta^2) M_0^4 +$$
$$2sm_\Delta (Q^4+2Q^2 s+214s^2+2 (Q^2-2s) m_\Delta^2) M_0^5 + (4Q^6+9Q^4 s+11Q^2 s^2+173s^3 + (10Q^4-10Q^2 s-59s^2) m_\Delta^2) M_0^6 +$$
$$4sm_\Delta (Q^2+s+m_\Delta^2) M_0^7 + (4Q^4-10Q^2 (2s+m_\Delta^2) + s (3s+37m_\Delta^2)) M_0^8 + 2sm_\Delta M_0^9 - 2 (2Q^2-3s+5m_\Delta^2) M_0^{10} - 4M_0^{12}) +$$
$$4s^2 m_\Delta M_0^9 (-7s^3 - Q^2 (-5Q^2+M_0^2)(Q^2+M_0^2) - 3s^2 (7Q^2+5M_0^2) + s (-23Q^4-25Q^2 M_0^2+20M_0^4)) -$$
$$4sm_\Delta^3 M_0^7 \Big(-26s^4 - Q^2 M_0^2 (Q^2+M_0^2)^2 - 6s^3 (13Q^2+4M_0^2) + s^2 (-14Q^4+33Q^2 M_0^2+32M_0^4) + 2s (5Q^6-4Q^2 M_0^4+2M_0^6)\Big) +$$
$$2m_\pi^6 (-8sm_\Delta^5 M_0 (5s^2 (Q^2+s) - 8s^2 M_0^2 + 2Q^2 M_0^4 + 2M_0^6) -$$
$$2sm_\Delta M_0^3 (-2s^2 (Q^2+s)(17Q^2+9s) + (Q^2-2s)(Q^4-4Q^2 s-15s^2) M_0^2 + 2 (2Q^4+Q^2 s+262s^2) M_0^4 + (5Q^2+6s) M_0^6 + 2M_0^8) -$$
$$2sm_\Delta^3 M_0 \Big(-16s^2 (Q^2+s)^2 + s^2 (41Q^2+33s) M_0^2 + (6Q^4+21Q^2 s+536s^2) M_0^4 + (16Q^2+s) M_0^6 + 10M_0^8\Big) + 2M_0^4 (s^2 (6Q^6+93Q^4 s+116Q^2 s^2+43s^3) -$$
$$s (13Q^6+7Q^4 s+217Q^2 s^2+179s^3) M_0^2 - (Q^6+11Q^4 s-7Q^2 s^2+260s^3) M_0^4 - (Q^4-13Q^2 s+18s^2) M_0^6 + (Q^2+5s) M_0^8 + M_0^{10}) +$$
$$4m_\Delta^4 \Big(15s^3 (Q^2+s)^2 - s^3 (23Q^2+3s) M_0^2 - (5Q^6+8Q^4 s+11Q^2 s^2+12s^3) M_0^4 + (-5Q^4+5Q^2 s+31s^2) M_0^6 + (5Q^2-17s) M_0^8 + 5M_0^{10}\Big) +$$
$$m_\Delta^2 M_0^2 (4s^3 (Q^2+s)(37Q^2+16s) + s (-30Q^6-67Q^4 s-103Q^2 s^2+34s^3) M_0^2 - (8Q^6+28Q^4 s+35Q^2 s^2+734s^3) M_0^4 +$$
$$(-8Q^4+42Q^2 s-32s^2) M_0^6 + 8 (Q^2-s) M_0^8 + 8M_0^{10})) + m_\pi^4 (16sm_\Delta^7 M_0 (5s^2 (Q^2+s) - 12s^2 M_0^2 + 3 (Q^2+2s) M_0^4 + 3M_0^6) +$$
$$sm_\Delta^4 M_0^2 (-12s^2 (Q^2+s)(22Q^2+s) + (48Q^6+97Q^4 s-1249Q^2 s^2-1180s^3) M_0^2 + (36Q^4-77Q^2 s+1298s^2) M_0^4 + (-12Q^2+78s) M_0^6) +$$
$$4sm_\Delta M_0^5 (-s (-5Q^6+79Q^4 s+101Q^2 s^2+31s^3) + 2 (Q^6-4Q^4 s-62Q^2 s^2-17s^3) M_0^2 + (5Q^4-3Q^2 s+426s^2) M_0^4 + (4Q^2+6s) M_0^6 + M_0^8) +$$
$$4sm_\Delta^3 M_0^3 (-16s^2 (Q^2+s)(4Q^2+s) + (3Q^6-4Q^4 s-271Q^2 s^2-144s^3) M_0^2 + 4 (3Q^4+8Q^2 s+3s^2) M_0^4 + (17Q^2+2s) M_0^6 + 8M_0^8) +$$
$$4sm_\Delta^5 M_0 \Big(-24s^2 (Q^2+s)^2 - 3s^2 (21Q^2+25s) M_0^2 + 3 (4Q^4+13Q^2 s+156s^2) M_0^4 + (28Q^2+27s) M_0^6 + 16M_0^8\Big) -$$
$$2m_\Delta^2 M_0^4 (s^2 (17Q^6+229Q^4 s+216Q^2 s^2+77s^3) + s (-5Q^6+81Q^4 s+1204Q^2 s^2+805s^3) M_0^2 -$$
$$(6Q^6+34Q^4 s+25Q^2 s^2+199s^3) M_0^4 + (-6Q^4-43Q^2 s+65s^2) M_0^6 + (6Q^2-50s) M_0^8 + 6M_0^{10}) - 2sM_0^4 (s^2 (Q^2+s)(3Q^4+2s^2) +$$
$$s (18Q^6+259Q^4 s+270Q^2 s^2+106s^3) M_0^2 - (19Q^6-51Q^4 s+106Q^2 s^2+188s^3) M_0^4 + (-25Q^4+42Q^2 s-440s^2) M_0^6 - 14sM_0^8 + 6M_0^{10}) -$$
$$2m_\Delta^6 \Big(60s^3 (Q^2+s)^2 + 4s^3 (23Q^2+43s) M_0^2 - (20Q^6+38Q^4 s+30Q^2 s^2-103s^3) M_0^4 + (-20Q^4+20Q^2 s+102s^2) M_0^6 + (20Q^2-62s) M_0^8 + 20M_0^{10}\Big)\Big) +$$

$$
\begin{aligned}
&2m_\pi^2 \left(-4sm_\Delta^9 M_0 \left(5s^2 \left(Q^2+s\right) - 16s^2 M_0^2 + 4\left(Q^2+4s\right) M_0^4 + 4M_0^6\right) - 2sm_\Delta^5 M_0^3 \left(-2\left(13Q^2-11s\right) s^2 \left(Q^2+s\right) + \right.\right.\\
&\left(3Q^6+10Q^4 s - 153Q^2 s^2 - 10s^3\right) M_0^2 + 2\left(2Q^4 - 9Q^2 s - 88s^2\right) M_0^4 + \left(3Q^2 - 26s\right) M_0^6 + 2M_0^8\big) -\\
&2sm_\Delta^3 M_0^5 \left(2s\left(5Q^6+9Q^4 s+29Q^2 s^2+11s^3\right) + \left(9Q^2-23s\right) s\left(4Q^2+s\right) M_0^2 + \left(4Q^4+31Q^2 s - 12s^2\right) M_0^4 + \left(8Q^2-7s\right) M_0^6 + 4M_0^8\right) -\\
&2sm_\Delta^7 M_0 \left(-16s^2\left(Q^2+s\right)^2 - s^2\left(125Q^2+133s\right) M_0^2 + \left(10Q^4+23Q^2 s+184s^2\right) M_0^4 + \left(16Q^2-5s\right) M_0^6 + 6M_0^8\right) -\\
&sm_\Delta^2 M_0^4 \left(-2s^2\left(Q^2+s\right)\left(3Q^4+2s^2\right) + 2s\left(25Q^6+18Q^4 s+70Q^2 s^2+29s^3\right) M_0^2 + \left(6Q^6+151Q^4 s - 541Q^2 s^2 - 412s^3\right) M_0^4 + \right.\\
&\left(-2Q^4+81Q^2 s+128s^2\right) M_0^6 + \left(-8Q^2+10s\right) M_0^8\big) + 2sM_0^6 \left(s^2\left(Q^2+s\right)\left(3Q^4+2s^2\right) + s^2\left(69Q^4+58Q^2 s+24s^2\right) M_0^2 + \right.\\
&2\left(-Q^6+9Q^4 s+26Q^2 s^2+s^3\right) M_0^4 + \left(-5Q^4+14Q^2 s-71s^2\right) M_0^6 - 4\left(Q^2-s\right) M_0^8 - M_0^{10}\big) - 2m_\Delta^6 M_0^2 \left(-14\left(Q^2-2s\right) s^3 \left(Q^2+s\right) + \right.\\
&s\left(Q^6+2Q^4 s - 195Q^2 s^2 - 126s^3\right) M_0^2 - \left(4Q^6+6Q^4 s - 23Q^2 s^2 + 158s^3\right) M_0^4 + \left(-4Q^4+17Q^2 s+49s^2\right) M_0^6 + 4Q^2 M_0^8 + 4M_0^{10}\big) +\\
&2m_\Delta^8 \left(15s^3\left(Q^2+s\right)^2 + s^3\left(69Q^2+89s\right) M_0^2 - \left(5Q^6+11Q^4 s+15Q^2 s^2 - 10s^3\right) M_0^4 + \left(-5Q^4+5Q^2 s+4s^2\right) M_0^6 + \left(5Q^2-14s\right) M_0^8 + 5M_0^{10}\right) -\\
&2sm_\Delta M_0^7 \left(-24s^4 + Q^2 M_0^2\left(Q^2+M_0^2\right)^2 - 26s^3\left(3Q^2+2M_0^2\right) + s^2\left(-72Q^4 - 113Q^2 M_0^2 + 136M_0^4\right) + 2s\left(5Q^6+Q^4 M_0^2 - 2Q^2 M_0^4 + M_0^6\right)\right) +\\
&m_\Delta^4 M_0^4 \left(40s^5 + 6M_0^4\left(-Q^2+M_0^2\right)\left(Q^2+M_0^2\right)^2 - 8s^4\left(-5Q^2+73M_0^2\right) - 2sM_0^2\left(-2Q^2+7M_0^2\right)\left(3Q^4+4Q^2 M_0^2+3M_0^4\right) + \right.\\
&\left.\left.\left. s^3\left(60Q^4 - 187Q^2 M_0^2 + 292M_0^4\right) + s^2\left(-2Q^6+21Q^4 M_0^2 - 9Q^2 M_0^4 + 164M_0^6\right)\right)\right)\right) +\\
&16d\left(4s^2 m_\Delta M_0^9\left(2Q^6 - Q^4 s - 3s^3 + \left(2Q^4 - 11Q^2 s - 15s^2\right) M_0^2 + 12sM_0^4\right) - 4s^2 m_\Delta^3 M_0^7 \left(\left(Q^2+s\right)\left(4Q^4+5Q^2 s - 8s^2\right) - s\left(23Q^2+56s\right) M_0^2 + \right.\right.\\
&\left(-5Q^2+79s\right) M_0^4 + M_0^6\big) - 4sm_\Delta^{11} M_0 \left(-3s^2\left(Q^2+s\right) + s^2 M_0^2 + 3\left(Q^2+3s\right) M_0^4 + 3M_0^6\right) +\\
&4s^2 m_\Delta^5 M_0^5 \left(2\left(Q^6+7Q^4 s+5Q^2 s^2 - 4s^3\right) - 2\left(3Q^4+52Q^2 s+47s^2\right) M_0^2 + \left(-11Q^2+149s\right) M_0^4 + 21M_0^6\right) -\\
&s^2 M_0^8 \left(s\left(Q^2+s\right)\left(3Q^4 - 2Q^2 s + s^2\right) - \left(5Q^6+6Q^4 s+11Q^2 s^2 - 4s^3\right) M_0^2 - \left(Q^2-18s\right)\left(3Q^2+s\right) M_0^4 + 3\left(Q^2-8s\right) M_0^6 + M_0^8\right) -\\
&4sm_\Delta^7 M_0^3 \left(\left(3Q^2-4s\right) s^2\left(Q^2+s\right) - s\left(4Q^4+136Q^2 s+143s^2\right) M_0^2 + s\left(-7Q^2+170s\right) M_0^4 + 3\left(Q^2+16s\right) M_0^6 + 3M_0^8\right) +\\
&4sm_\Delta^9 M_0 \left(-s^2\left(Q^2+s\right)^2 - s^2\left(59Q^2+53s\right) M_0^2 + s\left(-Q^2+49s\right) M_0^4 + \left(6Q^2+37s\right) M_0^6 + 6M_0^8\right) +\\
&m_\Delta^6 M_0^4 \left(-s^2\left(-5Q^6+31Q^4 s+62Q^2 s^2+36s^3\right) + s\left(4Q^6 - 36Q^4 s + 389Q^2 s^2 + 166s^3\right) M_0^2 + \left(4Q^6+26Q^4 s+239Q^2 s^2 - 142s^3\right) M_0^4 + \right.\\
&4\left(2Q^4 - 3Q^2 s - 8s^2\right) M_0^6 + \left(4Q^2-26s\right) M_0^8\big) + 2m_\pi^{10}\left(2s^3\left(Q^2+s\right)^2 - 6s^3\left(Q^2+s\right) m_\Delta M_0 - 4s^3\left(5Q^2+4s\right) M_0^2 - \right.\\
&2s^3 m_\Delta M_0^3 + \left(-Q^6+Q^4 s - 3Q^2 s^2 + 9s^3\right) M_0^4 + 2\left(Q^2-3s\right) sm_\Delta M_0^5 - \left(Q^4-9s^2\right) M_0^6 + 2sm_\Delta M_0^7 + \left(Q^2-5s\right) M_0^8 + M_0^{10}\big) -\\
&2m_\Delta^{10}\left(2s^3\left(Q^2+s\right)^2 + 2s^3\left(21Q^2+23s\right) M_0^2 - \left(Q^6+4Q^4 s+53Q^2 s^2+107s^3\right) M_0^4 - \left(Q^4 - 4Q^2 s + 15s^2\right) M_0^6 + \left(Q^2+4s\right) M_0^8 + M_0^{10}\right) +\\
&sm_\Delta^2 M_0^6 \left(2s^2\left(Q^2+s\right)\left(3Q^4 - 2Q^2 s + s^2\right) - s\left(5Q^6+87Q^4 s+64Q^2 s^2+4s^3\right) M_0^2 + \right.\\
&\left(2Q^6+16Q^4 s+165Q^2 s^2+60s^3\right) M_0^4 + \left(6Q^4+45Q^2 s - 136s^2\right) M_0^6 + \left(6Q^2-8s\right) M_0^8 + 2M_0^{10}\big) + m_\Delta^8 M_0^2\\
&\left(18s^4\left(Q^2+s\right) + s\left(-Q^6+29Q^4 s+21Q^2 s^2+99s^3\right) M_0^2 - \left(5Q^6+22Q^4 s+259Q^2 s^2+214s^3\right) M_0^4 + \left(-7Q^4+21Q^2 s - 34s^2\right) M_0^6 + \left(Q^2+26s\right) M_0^8 + 3M_0^{10}\right) -\\
&m_\Delta^4 M_0^4 \left(s^3\left(Q^2+s\right)\left(3Q^4 - 2Q^2 s + s^2\right) - s^2\left(-5Q^6+116Q^4 s+145Q^2 s^2+30s^3\right) M_0^2 + s\left(5Q^6+12Q^4 s+478Q^2 s^2+103s^3\right) M_0^4 + \right.\\
&\left(Q^6+18Q^4 s+128Q^2 s^2 - 142s^3\right) M_0^6 + \left(3Q^4+7Q^2 s - 45s^2\right) M_0^8 + 3\left(Q^2-2s\right) M_0^{10} + M_0^{12}\big) +\\
&m_\pi^8\left(-4sm_\Delta^3 M_0\left(-15s^2\left(Q^2+s\right) - 3s^2 M_0^2 + \left(7Q^2-15s\right) M_0^4 + 7M_0^6\right) - 4sm_\Delta M_0\left(s^2\left(Q^2+s\right)^2 - s^2\left(35Q^2+41s\right) M_0^2 + 2\left(6Q^2-79s\right) sM_0^4 + \right.\right.\\
&2\left(Q^2-s\right) M_0^6 + 2M_0^8\big) + M_0^2\left(-2s^3\left(Q^2+s\right)\left(32Q^2+23s\right) - s\left(-19Q^6+11Q^4 s+245Q^2 s^2+325s^3\right) M_0^2 + \right.\\
&\left(3Q^6+10Q^4 s+29Q^2 s^2 - 424s^3\right) M_0^4 + \left(Q^4 - 15Q^2 s + 22s^2\right) M_0^6 + \left(-7Q^2+10s\right) M_0^8 - 5M_0^{10}\big) -\\
&2m_\Delta^2\left(10s^3\left(Q^2+s\right)^2 - 2s^3\left(19Q^2+9s\right) M_0^2 - 5\left(Q^6+2Q^2 s^2+7s^3\right) M_0^4 - \left(Q^2-2s\right)\left(5Q^2+6s\right) M_0^6 + \left(5Q^2-16s\right) M_0^8 + 5M_0^{10}\right)\big) +\\
&m_\pi^2\left(-4s^2 m_\Delta M_0^7\left(4Q^6 - 18Q^4 s - 25Q^2 s^2 - 15s^3 - 3s\left(12Q^2+17s\right) M_0^2 - \left(Q^2-151s\right) M_0^4 + M_0^6\right) + 4s^2 m_\Delta^3 M_0^5\left(-2\left(2Q^6+12Q^4 s+23Q^2 s^2+7s^3\right) - \right.\right.\\
&2\left(10Q^4 - 5Q^2 s + 26s^2\right) M_0^2 + 3\left(5Q^2+131s\right) M_0^4 + 3M_0^6\big) + 4sm_\Delta^9 M_0\left(-15s^2\left(Q^2+s\right) + 3s^2 M_0^2 + \left(13Q^2+21s\right) M_0^4 + 13M_0^6\right) +
\end{aligned}
$$

$$4sm_\Delta^5 M_0^3 \left(3s^2 \left(Q^2+s\right)\left(8Q^2+s\right)+s\left(-4Q^4+29Q^2s+10s^2\right)M_0^2-s\left(61Q^2+196s\right)M_0^4+\left(7Q^2-32s\right)M_0^6+7M_0^8\right)-$$
$$4sm_\Delta^7 M_0 \left(-4s^2\left(Q^2+s\right)^2-2s^2\left(71Q^2+59s\right)M_0^2+s\left(-29Q^2+41s\right)M_0^4+\left(8Q^2+27s\right)M_0^6+8M_0^8\right)+$$
$$2sM_0^6\left(s^2\left(Q^2+s\right)\left(3Q^4-2Q^2s+s^2\right)+s\left(-12Q^6-29Q^4s-26Q^2s^2+13s^3\right)M_0^2+\left(2Q^6+20Q^4s+35Q^2s^2-15s^3\right)M_0^4+\right.$$
$$\left(6Q^4+4Q^2s-253s^2\right)M_0^6+6\left(Q^2-2s\right)M_0^8+2M_0^{10}\right)+2m_\Delta^4 M_0^4\left(-s^2\left(12Q^6+34Q^4s+51Q^2s^2+11s^3\right)+\right.$$
$$s\left(19Q^6+29Q^4s+297Q^2s^2+190s^3\right)M_0^2-\left(4Q^6-7Q^4s+117Q^2s^2+119s^3\right)M_0^4-\left(6Q^4+7Q^2s+56s^2\right)M_0^6-7sM_0^8+2M_0^{10}\right)+$$
$$2m_\Delta^8\left(10s^3\left(Q^2+s\right)^2+4s^3\left(37Q^2+42s\right)M_0^2-\left(5Q^6+15Q^4s+125Q^2s^2+234s^3\right)M_0^4+\left(-5Q^4+16Q^2s-25s^2\right)M_0^6+\left(5Q^2+11s\right)M_0^8+5M_0^{10}\right)+$$
$$m_\Delta^2 M_0^4\left(2s^3\left(Q^2+s\right)\left(3Q^4-2Q^2s+s^2\right)+2s^2\left(4Q^6+127Q^4s+107Q^2s^2+12s^3\right)M_0^2-s\left(30Q^6+199Q^4s+67Q^2s^2+14s^3\right)M_0^4+\right.$$
$$\left(Q^2-20s\right)\left(2Q^4-4Q^2s-49s^2\right)M_0^6+2\left(3Q^4-13Q^2s+53s^2\right)M_0^8+6\left(Q^2-2s\right)M_0^{10}+2M_0^{12}\right)-m_\Delta^6 M_0^2\left(8s^5+4M_0^4\left(-3Q^2+M_0^2\right)\left(Q^2+M_0^2\right)^2+\right.$$
$$4s^4\left(-14Q^2+33M_0^2\right)+5s^2M_0^2\left(23Q^4-13Q^2M_0^2+20M_0^4\right)-s^3\left(64Q^4+47Q^2M_0^2+360M_0^4\right)+16sM_0^2\left(Q^6-Q^4M_0^2+Q^2M_0^4+M_0^6\right)\right))+$$
$$m_\pi^6\left(8sm_\Delta^5 M_0\left(-15s^2\left(Q^2+s\right)-s^2M_0^2+9\left(Q^2-s\right)M_0^4+9M_0^6\right)+4sm_\Delta M_0^3\left(s^2\left(Q^2+s\right)\left(18Q^2+11s\right)+s\left(4Q^4-47Q^2s-52s^2\right)M_0^2+\right.\right.$$
$$5\left(5Q^2-88s\right)sM_0^4+\left(Q^2+4s\right)M_0^6+M_0^8\right)+4sm_\Delta^3 M_0\left(4s^2\left(Q^2+s\right)^2-2s^2\left(23Q^2+35s\right)M_0^2+\left(51Q^2-259s\right)sM_0^4+\left(8Q^2-5s\right)M_0^6+8M_0^8\right)+$$
$$2m_\Delta^4\left(20s^3\left(Q^2+s\right)^2+8s^3\left(6Q^2+11s\right)M_0^2-\left(10Q^6+10Q^4s+30Q^2s^2+197s^3\right)M_0^4+2\left(-5Q^4+8Q^2s+s^2\right)M_0^6+2\left(5Q^2-7s\right)M_0^8+10M_0^{10}\right)+$$
$$m_\Delta^2 M_0^2\left(24s^3\left(Q^2+s\right)\left(8Q^2+5s\right)+s\left(-56Q^6-35Q^4s+611Q^2s^2+764s^3\right)M_0^2+\left(-4Q^6-48Q^4s+69Q^2s^2+464s^3\right)M_0^4+\right.$$
$$4\left(Q^4+14Q^2s-36s^2\right)M_0^6+4\left(5Q^2+4s\right)M_0^8+12M_0^{10}\right)+2M_0^4\left(49s^5+2M_0^6\left(Q^2+M_0^2\right)^2+s^4\left(83Q^2+340M_0^2\right)+\right.$$
$$7s^3\left(5Q^4+41Q^2M_0^2+24M_0^4\right)+sM_0^2\left(-17Q^6-7Q^4M_0^2+21Q^2M_0^4+7M_0^6\right)-s^2\left(7Q^6-34Q^4M_0^2+19Q^2M_0^4+62M_0^6\right)\right))+$$
$$m_\pi^4\left(-8s^2m_\Delta M_0^5\left(-Q^6+17Q^4s+26Q^2s^2+11s^3+\left(3Q^4+5Q^2s+11s^2\right)M_0^2+\left(7Q^2-211s\right)M_0^4+M_0^6\right)-8sm_\Delta^7 M_0\right.$$
$$\left(-15s^2\left(Q^2+s\right)+s^2M_0^2+\left(11Q^2+3s\right)M_0^4+11M_0^6\right)-4sm_\Delta^5 M_0\left(6s^2\left(Q^2+s\right)^2+36s^2\left(2Q^2+s\right)M_0^2+\left(67Q^2-93s\right)sM_0^4+\left(4Q^2+7s\right)M_0^6+4M_0^8\right)-$$
$$4sm_\Delta^3 M_0^3\left(3s^2\left(Q^2+s\right)\left(13Q^2+6s\right)+s\left(4Q^4+118Q^2s+101s^2\right)M_0^2+s\left(27Q^2+218s\right)M_0^4+5\left(Q^2-4s\right)M_0^6+5M_0^8\right)-$$
$$m_\Delta^2 M_0^4\left(s^2\left(-33Q^6-29Q^4s+2Q^2s^2+40s^3\right)-s\left(-8Q^6+66Q^4s+1031Q^2s^2+1174s^3\right)M_0^2+\right.$$
$$\left(-4Q^6+26Q^4s+205Q^2s^2-1838s^3\right)M_0^4+2s\left(8Q^2+63s\right)M_0^6+2\left(6Q^2-13s\right)M_0^8+8M_0^{10}\right)-$$
$$2m_\Delta^6\left(20s^3\left(Q^2+s\right)^2+4s^3\left(43Q^2+53s\right)M_0^2-5\left(2Q^6+4Q^4s+19Q^2s^2+56s^3\right)M_0^4+\left(-10Q^4+24Q^2s-11s^2\right)M_0^6+2\left(5Q^2+2s\right)M_0^8+10M_0^{10}\right)-$$
$$2m_\Delta^4 M_0^2\left(42s^5+3M_0^4\left(Q^2+M_0^2\right)^3+s^4\left(138Q^2+203M_0^2\right)+2s^2M_0^2\left(-33Q^4+46Q^2M_0^2+6M_0^4\right)+sM_0^2\left(Q^2+M_0^2\right)\left(-27Q^4+5Q^2M_0^2+18M_0^4\right)+\right.$$
$$s^3\left(96Q^4+217Q^2M_0^2+65M_0^4\right)\right)+M_0^4\left(-s^6+s^5\left(Q^2-78M_0^2\right)-M_0^6\left(Q^2+M_0^2\right)^3-sM_0^4\left(Q^2+M_0^2\right)\left(-11Q^4+21Q^2M_0^2+18M_0^4\right)-\right.$$
$$s^4\left(Q^4+23Q^2M_0^2+275M_0^4\right)+s^2M_0^2\left(33Q^6-100Q^4M_0^2+10Q^2M_0^4+109M_0^6\right)+3s^3\left(-Q^6+14Q^4M_0^2-102Q^2M_0^4+184M_0^6\right)\right)))+$$
$$2d^4\left(4s^2m_\Delta M_0^9\left(19Q^6-57Q^4s-55Q^2s^2-25s^3+\left(16Q^4-101Q^2s-75s^2\right)M_0^2+\left(-3Q^2+82s\right)M_0^4\right)-\right.$$
$$8sm_\Delta^{11}M_0\left(-6s^2\left(Q^2+s\right)+11s^2M_0^2+\left(Q^2+s\right)M_0^4+M_0^6\right)-$$
$$2s^2M_0^8\left(s\left(Q^2+s\right)\left(6Q^4-14Q^2s+s^2\right)+\left(-23Q^6+31Q^4s+14Q^2s^2+29s^3\right)M_0^2+\left(-9Q^4+165Q^2s+51s^2\right)M_0^4+\left(18Q^2-85s\right)M_0^6+4M_0^8\right)+$$
$$4sm_\Delta^5 M_0^5\left(s\left(19Q^6+51Q^4s-35Q^2s^2-113s^3\right)-2\left(3Q^6+22Q^4s+212Q^2s^2+81s^3\right)M_0^2+\left(-5Q^4-67Q^2s+280s^2\right)M_0^4+8\left(Q^2+4s\right)M_0^6+7M_0^8\right)+$$
$$4sm_\Delta^9 M_0\left(-12s^2\left(Q^2+s\right)^2-s^2\left(277Q^2+271s\right)M_0^2+\left(7Q^4-8Q^2s+298s^2\right)M_0^4+6\left(3Q^2+2s\right)M_0^6+11M_0^8\right)-$$
$$4sm_\Delta^7 M_0^3\left(-2s^2\left(Q^2+s\right)\left(2Q^2+31s\right)-\left(3Q^6+30Q^4s+641Q^2s^2+810s^3\right)M_0^2+\left(8Q^4-44Q^2s+886s^2\right)M_0^4+3\left(9Q^2+10s\right)M_0^6+16M_0^8\right)+$$
$$2m_\Delta^6 M_0^4\left(-s^2\left(-7Q^6+145Q^4s+301Q^2s^2+201s^3\right)+s\left(-5Q^6-24Q^4s+551Q^2s^2+248s^3\right)M_0^2+\right.$$
$$\left(4Q^6+19Q^4s+441Q^2s^2-592s^3\right)M_0^4+\left(8Q^4-31Q^2s-56s^2\right)M_0^6+\left(4Q^2-59s\right)M_0^8\right)+$$

$$4m_\pi^{10}\Big(10s^3\left(Q^2+s\right)^2-12s^3\left(Q^2+s\right)m_\Delta M_0-s^3\left(85Q^2+69s\right)M_0^2-2s^3m_\Delta M_0^3-\left(Q^6-7Q^4s+3Q^2s^2-48s^3\right)M_0^4+$$

$$2\left(Q^2-3s\right)sm_\Delta M_0^5-\left(Q^2-3s\right)\left(Q^2+6s\right)M_0^6+2sm_\Delta M_0^7+\left(Q^2-8s\right)M_0^8+M_0^{10}\Big)-$$

$$2m_\Delta^{10}\Big(20s^3\left(Q^2+s\right)^2+32s^3\left(7Q^2+8s\right)M_0^2-\left(2Q^6+6Q^4s+99Q^2s^2+649s^3\right)M_0^4+\left(-2Q^4+2Q^2s-25s^2\right)M_0^6+2\left(Q^2+6s\right)M_0^8+2M_0^{10}\Big)+$$

$$2sm_\Delta^2M_0^6\left(2s^2\left(Q^2+s\right)\left(6Q^4-14Q^2s+s^2\right)-s\left(39Q^6+275Q^4s+87Q^2s^2+s^3\right)M_0^2+\left(11Q^6+66Q^4s+115Q^2s^2+146s^3\right)M_0^4+\right.$$

$$\left(27Q^4+167Q^2s-214s^2\right)M_0^6+3\left(7Q^2-6s\right)M_0^8+5M_0^{10}\big)+m_\Delta^8M_0^2\left(2s^3\left(Q^2+s\right)\left(24Q^2+101s\right)+2s\left(8Q^6+54Q^4s+175Q^2s^2+372s^3\right)M_0^2-\right.$$

$$\left(10Q^6+10Q^4s+607Q^2s^2+623s^3\right)M_0^4+\left(-14Q^4+58Q^2s-45s^2\right)M_0^6+2\left(Q^2+50s\right)M_0^8+6M_0^{10}\big)+$$

$$2m_\pi^8\Big(-100s^3\left(Q^2+s\right)^2m_\Delta^2-24s^3\left(Q^2+s\right)m_\Delta\left(Q^2+s-5m_\Delta^2\right)M_0+s^3\left(-11\left(Q^2+s\right)\left(22Q^2+15s\right)+8\left(57Q^2+37s\right)m_\Delta^2\right)M_0^2+$$

$$2s^3m_\Delta\left(209Q^2+215s-14m_\Delta^2\right)M_0^3+\left(-18s^3\left(25s+3m_\Delta^2\right)+2Q^6\left(8s+5m_\Delta^2\right)-10Q^4s\left(11s+5m_\Delta^2\right)+Q^2s^2\left(-197s+26m_\Delta^2\right)\right)M_0^4+$$

$$2sm_\Delta\left(3Q^4-2Q^2\left(18s+5m_\Delta^2\right)+2s\left(566s+11m_\Delta^2\right)\right)M_0^5+\left(3Q^6-35Q^4s+134Q^2s^2-571s^3+2\left(5Q^4+11Q^2s-52s^2\right)m_\Delta^2\right)M_0^6+$$

$$4sm_\Delta\left(Q^2-4s-5m_\Delta^2\right)M_0^7+\left(Q^4-5Q^2\left(3s+2m_\Delta^2\right)+s\left(-5s+52m_\Delta^2\right)\right)M_0^8-2sm_\Delta M_0^9+\left(-7Q^2+28s-10m_\Delta^2\right)M_0^{10}-5M_0^{12}\Big)-$$

$$m_\Delta^4M_0^4\left(2s^3\left(Q^2+s\right)\left(6Q^4-14Q^2s+s^2\right)-2s^2\left(9Q^6+447Q^4s+581Q^2s^2+150s^3\right)M_0^2+2s\left(14Q^6+105Q^4s+716Q^2s^2+51s^3\right)M_0^4+\right.$$

$$\left(2Q^6+94Q^4s+771Q^2s^2+49s^3\right)M_0^6+\left(6Q^4+34Q^2s-151s^2\right)M_0^8+\left(6Q^2-32s\right)M_0^{10}+2M_0^{12}\big)+$$

$$m_\pi^4\Big(-400s^3\left(Q^2+s\right)^2m_\Delta^6+96s^3\left(Q^2+s\right)m_\Delta^5\left(-3\left(Q^2+s\right)+5m_\Delta^2\right)M_0-4s^3m_\Delta^4\left(327Q^4+423Q^2s+96s^2+332Q^2m_\Delta^2+492sm_\Delta^2\right)M_0^2-$$

$$8s^3m_\Delta^3\left(142Q^4+197Q^2s+55s^2+6\left(17Q^2+14s\right)m_\Delta^2+62m_\Delta^4\right)M_0^3+2\left(-s^3\left(Q^2+s\right)\left(6Q^4-14Q^2s+s^2\right)-\right.$$

$$s^2\left(-125Q^6+99Q^4s+135Q^2s^2+147s^3\right)m_\Delta^2+2s\left(36Q^6+38Q^4s-1021Q^2s^2-997s^3\right)m_\Delta^4+2\left(10Q^6-10Q^4s+65Q^2s^2+327s^3\right)m_\Delta^6\big)M_0^4+$$

$$4sm_\Delta\left(-s\left(-19Q^6+309Q^4s+443Q^2s^2+161s^3\right)+\left(9Q^6-22Q^4s-1029Q^2s^2-682s^3\right)m_\Delta^2+2\left(15Q^4-82Q^2s+627s^2\right)m_\Delta^4+4\left(-5Q^2+3s\right)m_\Delta^6\right)$$

$$M_0^5+\left(-2s^2\left(-141Q^6+139Q^4s+331Q^2s^2+304s^3\right)+2s\left(25Q^6+176Q^4s+237Q^2s^2+1116s^3\right)m_\Delta^2+\left(-12Q^6+80Q^4s-325Q^2s^2+3437s^3\right)m_\Delta^4+\right.$$

$$4\left(10Q^4+6Q^2s-73s^2\right)m_\Delta^6\big)M_0^6+8sm_\Delta\left(3\left(Q^6-6Q^4s-57Q^2s^2-26s^3\right)+\left(16Q^4-18Q^2s-435s^2\right)m_\Delta^2+6\left(5Q^2-7s\right)m_\Delta^4-10m_\Delta^6\right)M_0^7+$$

$$\left(-2s\left(-14Q^6+361Q^4s+808Q^2s^2+609s^3\right)+2\left(4Q^6+Q^4s-469Q^2s^2+5224s^3\right)m_\Delta^2-\left(36Q^4+44Q^2s+473s^2\right)m_\Delta^4-8\left(5Q^2+2s\right)m_\Delta^6\right)M_0^8+$$

$$4sm_\Delta\left(15Q^4-55Q^2s+2794s^2+\left(31Q^2-2s\right)m_\Delta^2+30m_\Delta^4\right)M_0^9-2\left(Q^6+25Q^4s+98Q^2s^2-2486s^3-3\left(17Q^2-144s\right)sm_\Delta^2+2\left(9Q^2-5s\right)m_\Delta^4+20m_\Delta^6\right)$$

$$M_0^{10}-16sm_\Delta\left(-3Q^2+s-2m_\Delta^2\right)M_0^{11}-2\left(3Q^4+57Q^2s-79s^2+3\left(4Q^2-21s\right)m_\Delta^2+6m_\Delta^4\right)M_0^{12}+12sm_\Delta M_0^{13}-2\left(3Q^2+18s+8m_\Delta^2\right)M_0^{14}-2M_0^{16}\Big)-$$

$$4sm_\Delta^3M_0^7\Big(-88s^4-3Q^2M_0^2\left(Q^2+M_0^2\right)^2-2s^3\left(104Q^2+99M_0^2\right)+s^2\left(-14Q^4+11Q^2M_0^2+264M_0^4\right)+2s\left(19Q^6+Q^4M_0^2-17Q^2M_0^4+6M_0^6\right)\Big)+$$

$$m_\pi^6\left(16sm_\Delta^5M_0\left(-30s^2\left(Q^2+s\right)+19s^2M_0^2+\left(5Q^2-7s\right)M_0^4+5M_0^6\right)-\right.$$

$$8sm_\Delta^3M_0\Big(-24s^2\left(Q^2+s\right)^2+s^2\left(175Q^2+187s\right)M_0^2+2\left(4Q^4-33Q^2s+544s^2\right)M_0^4+6\left(2Q^2-5s\right)M_0^6+4M_0^8\Big)-$$

$$4sm_\Delta M_0^3\left(-2s^2\left(Q^2+s\right)\left(72Q^2+43s\right)+\left(3Q^6-26Q^4s+169Q^2s^2+258s^3\right)M_0^2+2\left(6Q^4-40Q^2s+1525s^2\right)M_0^4+13\left(Q^2-2s\right)M_0^6+4M_0^8\right)+$$

$$2M_0^4\left(s^2\left(-59Q^6+377Q^4s+689Q^2s^2+345s^3\right)+s\left(-31Q^6+350Q^4s+957Q^2s^2+1230s^3\right)M_0^2+s\left(37Q^4-149Q^2s-536s^2\right)M_0^4+\right.$$

$$\left(4Q^4+63Q^2s-106s^2\right)M_0^6+\left(8Q^2-s\right)M_0^8+4M_0^{10}\big)+4m_\Delta^2M_0^2\left(s^3\left(Q^2+s\right)\left(351Q^2+197s\right)+s\left(-28Q^6+77Q^4s+749Q^2s^2+986s^3\right)M_0^2-\right.$$

$$\left(2Q^6-15Q^4s+88Q^2s^2+253s^3\right)M_0^4+\left(2Q^4+24Q^2s-7s^2\right)M_0^6+\left(10Q^2-11s\right)M_0^8+6M_0^{10}\big)+m_\Delta^4\Big(400s^3\left(Q^2+s\right)^2-$$

$$8\left(31Q^2-49s\right)s^3M_0^2-\left(40Q^6-120Q^4s+85Q^2s^2+1339s^3\right)M_0^4+\left(-40Q^4-56Q^2s+339s^2\right)M_0^6+8\left(5Q^2-12s\right)M_0^8+40M_0^{10}\Big)\Big)+$$

$$m_\pi^2\left(8sm_\Delta^9M_0\left(-30s^2\left(Q^2+s\right)+43s^2M_0^2+\left(5Q^2+s\right)M_0^4+5M_0^6\right)+4sm_\Delta^5M_0^3\left(2\left(68Q^2-19s\right)s^2\left(Q^2+s\right)+\left(-9Q^6-34Q^4s+557Q^2s^2+130s^3\right)M_0^2-\right.\right.$$

$$2\left(6Q^4+76Q^2s+441s^2\right)M_0^4+\left(9Q^2-58s\right)M_0^6+12M_0^8\big)-8sm_\Delta^7M_0\Big(-24s^2\left(Q^2+s\right)^2-s^2\left(311Q^2+299s\right)M_0^2+$$

$$2\left(6Q^4-19Q^2s+127s^2\right)M_0^4+2\left(14Q^2-5s\right)M_0^6+16M_0^8\Big)-4m_\Delta^6M_0^2\left(-\left(85Q^2-69s\right)s^3\left(Q^2+s\right)+s\left(20Q^6+87Q^4s-283Q^2s^2-50s^3\right)M_0^2-\right.$$

$$\left(6Q^6-15Q^4s+57Q^2s^2+560s^3\right)M_0^4+\left(-10Q^4+20Q^2s+58s^2\right)M_0^6+\left(-2Q^2+33s\right)M_0^8+2M_0^{10}\big)+2sM_0^6\left(2s^2\left(Q^2+s\right)\left(6Q^4-14Q^2s+s^2\right)+\right.$$

$$s\left(-105Q^6+15Q^4s+23Q^2s^2+133s^3\right)M_0^2+\left(Q^6+112Q^4s+383Q^2s^2+18s^3\right)M_0^4+\left(9Q^4+137Q^2s-1560s^2\right)M_0^6+15Q^2M_0^8+7M_0^{10}\big)+m_\Delta^8$$

$$
\begin{aligned}
&\Big(200s^3\left(Q^2+s\right)^2+4s^3\left(363Q^2+443s\right)M_0^2-\left(20Q^6+20Q^4s+413Q^2s^2+1351s^3\right)M_0^4+\left(-20Q^4+4Q^2s+39s^2\right)M_0^6+4\left(5Q^2+16s\right)M_0^8+20M_0^{10}\Big)+\\
&4m_\Delta^2M_0^4\left(s^3\left(Q^2+s\right)\left(6Q^4-14Q^2s+s^2\right)+s^2\left(29Q^6+406Q^4s+275Q^2s^2+45s^3\right)M_0^2+s\left(-24Q^6-256Q^4s+485Q^2s^2+233s^3\right)M_0^4+\right.\\
&\left.\left(Q^6-36Q^4s-172Q^2s^2+951s^3\right)M_0^6+\left(3Q^2-17s\right)\left(Q^2-6s\right)M_0^8+\left(3Q^2-23s\right)M_0^{10}+M_0^{12}\right)-\\
&8sm_\Delta^3M_0^5\Big(63s^4+5M_0^4\left(Q^2+M_0^2\right)^2+s^3\left(161Q^2+81M_0^2\right)-s^2\left(-71Q^4+95Q^2M_0^2+830M_0^4\right)+s\left(19Q^6+72Q^4M_0^2+3Q^2M_0^4-34M_0^6\right)\Big)-\\
&4sm_\Delta M_0^7\Big(-112s^4+3Q^2M_0^2\left(Q^2+M_0^2\right)^2-2s^3\left(146Q^2+143M_0^2\right)+s^2\left(-234Q^4-415Q^2M_0^2+956M_0^4\right)+2s\left(19Q^6+3Q^4M_0^2-7Q^2M_0^4+4M_0^6\right)\Big)+\\
&m_\Delta^4M_0^4\Big(6s^5+8M_0^4\left(-2Q^2+M_0^2\right)\left(Q^2+M_0^2\right)^2-2s^4\left(253Q^2+542M_0^2\right)+s^3\left(-266Q^4+1638Q^2M_0^2+391M_0^4\right)-\\
&2sM_0^2\left(-11Q^6+57Q^4M_0^2+83Q^2M_0^4+3M_0^6\right)-s^2\left(146Q^6+28Q^4M_0^2+253Q^2M_0^4+395M_0^6\right)\Big)\Big)\Big)+\\
&d^5\left(16s^3m_\Delta^{11}M_0\left(-Q^2-s+2M_0^2\right)-4s^2m_\Delta M_0^9\left(19Q^6-133Q^4s-105Q^2s^2-29s^3+2\left(7Q^4-40Q^2s-30s^2\right)M_0^2-5\left(Q^2-17s\right)M_0^4\right)-\right.\\
&4sm_\Delta^9M_0\Big(-23s^2\left(Q^2+s\right)^2-4s^2\left(37Q^2+41s\right)M_0^2+\left(2Q^4+69s^2\right)M_0^4+4Q^2M_0^6+2M_0^8\Big)+\\
&4sm_\Delta^7M_0^3\left(-2s^2\left(Q^2+s\right)\left(21Q^2+58s\right)-\left(5Q^6+18Q^4s+483Q^2s^2+588s^3\right)M_0^2-6\left(Q^4+3Q^2s-64s^2\right)M_0^4+\left(3Q^2+4s\right)M_0^6+4M_0^8\right)+\\
&s^2M_0^8\left(\left(3Q^2-5s\right)\left(3Q^2-s\right)s\left(Q^2+s\right)+\left(-45Q^6+280Q^4s+175Q^2s^2+66s^3\right)M_0^2+\left(-11Q^4+169Q^2s+80s^2\right)M_0^4+\left(41Q^2-158s\right)M_0^6+7M_0^8\right)+\\
&2m_\pi^{10}\Big(-30s^3\left(Q^2+s\right)^2+8s^3\left(Q^2+s\right)m_\Delta M_0+2s^3\left(97Q^2+83s\right)M_0^2+\left(Q^6-5Q^4s+3Q^2s^2-127s^3\right)M_0^4+\left(Q^2-3s\right)\left(Q^2+5s\right)M_0^6-\left(Q^2-7s\right)M_0^8-M_0^{10}\Big)+\\
&2m_\Delta^{10}\Big(30s^3\left(Q^2+s\right)^2+2s^3\left(69Q^2+83s\right)M_0^2-\left(Q^6+3Q^4s+152Q^2s^2+920s^3\right)M_0^4-\left(Q^4+2Q^2s+118s^2\right)M_0^6+\left(Q^2+s\right)M_0^8+M_0^{10}\Big)-\\
&2sm_\Delta^2M_0^6\left(\left(3Q^2-5s\right)\left(3Q^2-s\right)s^2\left(Q^2+s\right)+s\left(-36Q^6-129Q^4s+256Q^2s^2+57s^3\right)M_0^2+\left(12Q^6+68Q^4s-193Q^2s^2+25s^3\right)M_0^4+\right.\\
&\left.\left(26Q^4+132Q^2s-37s^2\right)M_0^6+4\left(4Q^2-s\right)M_0^8+2M_0^{10}\right)-m_\Delta^8M_0^2\left(20s^3\left(Q^2+s\right)\left(5Q^2+14s\right)+\right.\\
&\left.s\left(9Q^6+19Q^4s+443Q^2s^2+661s^3\right)M_0^2-\left(Q^6+760Q^2s^2+1115s^3\right)M_0^4+\left(5Q^4+17Q^2s-535s^2\right)M_0^6+13\left(Q^2+2s\right)M_0^8+7M_0^{10}\right)+\\
&m_\pi^4\Big(600s^3\left(Q^2+s\right)^2m_\Delta^6+8s^3\left(Q^2+s\right)m_\Delta^5\left(69\left(Q^2+s\right)-20m_\Delta^2\right)M_0+8s^3m_\Delta^4\left(9\left(Q^2+s\right)\left(19Q^2+4s\right)+\left(13Q^2+83s\right)m_\Delta^2\right)M_0^2+\\
&8s^3m_\Delta^3\left(173Q^4+235Q^2s+62s^2-6\left(13Q^2+5s\right)m_\Delta^2+24m_\Delta^4\right)M_0^3+\left(\left(3Q^2-5s\right)\left(3Q^2-s\right)s^3\left(Q^2+s\right)+\right.\\
&2s^2\left(-63Q^6+581Q^4s+469Q^2s^2+197s^3\right)m_\Delta^2+6s\left(-13Q^6-7Q^4s+321Q^2s^2+447s^3\right)m_\Delta^4+4\left(-5Q^6+Q^4s-36Q^2s^2+362s^3\right)m_\Delta^6\Big)M_0^4+\\
&4sm_\Delta\left(s\left(-19Q^6+452Q^4s+595Q^2s^2+200s^3\right)+\left(-15Q^6+42Q^4s+943Q^2s^2+660s^3\right)m_\Delta^2-6\left(2Q^4-4Q^2s+311s^2\right)m_\Delta^4\right)M_0^5+\\
&\left(s^2\left(-153Q^6+1672Q^4s+1887Q^2s^2+838s^3\right)+2s\left(-37Q^6-4Q^4s+1825Q^2s^2+1162s^3\right)m_\Delta^2-\left(18Q^6-16Q^4s-79Q^2s^2+5857s^3\right)m_\Delta^4-\right.\\
&\left.4\left(5Q^4+10Q^2s-54s^2\right)m_\Delta^6\right)M_0^6+8sm_\Delta\left(-5Q^6+23Q^4s+253Q^2s^2+130s^3-\left(17Q^4-13Q^2s-580s^2\right)m_\Delta^2-12\left(Q^2-s\right)m_\Delta^4\right)M_0^7+\\
&\left(-s\left(49Q^6-534Q^4s+458Q^2s^2+585s^3\right)-2\left(6Q^6+59Q^4s-235Q^2s^2+2536s^3\right)m_\Delta^2+\left(-54Q^4+162Q^2s+209s^2\right)m_\Delta^4+4\left(5Q^2-11s\right)m_\Delta^6\right)M_0^8-\\
&4sm_\Delta\left(22Q^4-57Q^2s+2978s^2+\left(23Q^2+4s\right)m_\Delta^2+12m_\Delta^4\right)M_0^9-\\
&\left(3Q^6+28Q^4s-378Q^2s^2+5832s^3+2\left(12Q^4+55Q^2s-202s^2\right)m_\Delta^2+2\left(27Q^2-34s\right)m_\Delta^4-20m_\Delta^6\right)M_0^{10}-\\
&8sm_\Delta\left(7Q^2+2m_\Delta^2\right)M_0^{11}-\left(9Q^4-63Q^2s+65s^2+6\left(2Q^2+11s\right)m_\Delta^2+18m_\Delta^4\right)M_0^{12}-8sm_\Delta M_0^{13}+\left(-9Q^2+42s\right)M_0^{14}-3M_0^{16}\Big)+\\
&4sm_\Delta^3M_0^7\Big(-128s^4-5Q^2M_0^2\left(Q^2+M_0^2\right)^2-4s^3\left(113Q^2+34M_0^2\right)+s^2\left(-86Q^4+73Q^2M_0^2+184M_0^4\right)+2s\left(19Q^6+5Q^4M_0^2-14Q^2M_0^4+2M_0^6\right)\Big)-\\
&4sm_\Delta^5M_0^5\Big(-192s^4+2M_0^2\left(-5Q^2+M_0^2\right)\left(Q^2+M_0^2\right)^2+s^3\left(-107Q^2+8M_0^2\right)-2s^2\left(-14Q^4+269Q^2M_0^2+45M_0^4\right)+s\left(19Q^6-22Q^4M_0^2-41Q^2M_0^4+8M_0^6\right)\Big)+\\
&2m_\Delta^6M_0^4\Big(199s^5+2M_0^4\left(Q^2+M_0^2\right)^2\left(Q^2+2M_0^2\right)+s^4\left(199Q^2+226M_0^2\right)+s^3\left(83Q^4+167Q^2M_0^2+4M_0^4\right)+\\
&sM_0^2\left(Q^2+M_0^2\right)\left(-3Q^4-14Q^2M_0^2+21M_0^4\right)-s^2\left(9Q^6+60Q^4M_0^2+395Q^2M_0^4+170M_0^6\right)\Big)-\\
&m_\Delta^4M_0^4\Big(-5s^6+3M_0^6\left(Q^2+M_0^2\right)^3+13s^5\left(Q^2+10M_0^2\right)+sM_0^4\left(Q^2+M_0^2\right)\left(-39Q^4-53Q^2M_0^2+14M_0^4\right)+s^4\left(9Q^4+1049Q^2M_0^2+441M_0^4\right)-\\
&s^2M_0^2\left(-9Q^6+286Q^4M_0^2+557Q^2M_0^4+26M_0^6\right)-s^3\left(9Q^6-664Q^4M_0^2+526Q^2M_0^4+1089M_0^6\right)\Big)+
\end{aligned}
$$

$$m_\pi^8 \Big(16s^3m_\Delta^3M_0\left(-5\left(Q^2+s\right)+2M_0^2\right)-4sm_\Delta M_0\Big(-23s^2\left(Q^2+s\right)^2+8s^2\left(25Q^2+23s\right)M_0^2+\left(2Q^4-8Q^2s+1465s^2\right)M_0^4+4\left(Q^2-2s\right)M_0^6+2M_0^8\Big)+$$

$$M_0^2\left(4s^3\left(Q^2+s\right)\left(139Q^2+94s\right)-s\left(17Q^6-93Q^4s+1595Q^2s^2+1053s^3\right)M_0^2-\right.$$

$$\left.\left(7Q^6-48Q^4s+167Q^2s^2+1050s^3\right)M_0^4+\left(-13Q^4+71Q^2s-72s^2\right)M_0^6+\left(-5Q^2+6s\right)M_0^8+M_0^{10}\right)+$$

$$2m_\Delta^2\Big(150s^3\left(Q^2+s\right)^2-2s^3\left(319Q^2+249s\right)M_0^2+\left(-5Q^6+17Q^4s-15Q^2s^2+359s^3\right)M_0^4+\left(-5Q^4-10Q^2s+59s^2\right)M_0^6+\left(5Q^2-27s\right)M_0^8+5M_0^{10}\Big)\Big)+$$

$$m_\pi^6\Big(-32s^3m_\Delta^5M_0\left(-5\left(Q^2+s\right)+4M_0^2\right)+16sm_\Delta^3M_0\Big(-23s^2\left(Q^2+s\right)^2+s^2\left(113Q^2+97s\right)M_0^2+2\left(Q^4-3Q^2s+387s^2\right)M_0^4+\left(4Q^2-6s\right)M_0^6+2M_0^8\Big)+$$

$$4sm_\Delta M_0^3\left(-2s^2\left(Q^2+s\right)\left(97Q^2+60s\right)+\left(5Q^6-30Q^4s+79Q^2s^2+168s^3\right)M_0^2+2\left(7Q^4-19Q^2s+1822s^2\right)M_0^4+\left(13Q^2-12s\right)M_0^6+4M_0^8\right)+$$

$$4m_\Delta^2M_0^2\left(-4s^3\left(Q^2+s\right)\left(98Q^2+53s\right)+s\left(15Q^6-32Q^4s+266Q^2s^2-61s^3\right)M_0^2+\left(5Q^6-20Q^4s+100Q^2s^2+1306s^3\right)M_0^4+\right.$$

$$\left.\left(11Q^4-49Q^2s+52s^2\right)M_0^6+7\left(Q^2-2s\right)M_0^8+M_0^{10}\right)-m_\Delta^4\Big(600s^3\left(Q^2+s\right)^2-8s^3\left(153Q^2+83s\right)M_0^2+\left(-20Q^6+36Q^4s+47Q^2s^2+373s^3\right)M_0^4+$$

$$\left(-20Q^4-40Q^2s+343s^2\right)M_0^6+4\left(5Q^2-19s\right)M_0^8+20M_0^{10}\Big)+2M_0^4\Big(-415s^5+2M_0^4\left(Q^2+M_0^2\right)^2\left(2Q^2+M_0^2\right)+s^4\left(-1007Q^2+624M_0^2\right)-$$

$$sM_0^2\left(Q^2+M_0^2\right)\left(-25Q^4+44Q^2M_0^2+29M_0^4\right)+5s^3\left(-141Q^4+193Q^2M_0^2+492M_0^4\right)+s^2\left(27Q^6-206Q^4M_0^2+35Q^2M_0^4+102M_0^6\right)\Big)\Big)+$$

$$m_\pi^2\Big(-16s^3m_\Delta^9M_0\left(-5\left(Q^2+s\right)+8M_0^2\right)+8sm_\Delta^3M_0^5\Big(s\left(19Q^6+76Q^4s+225Q^2s^2+92s^3\right)+2s\left(31Q^4+55Q^2s+77s^2\right)M_0^2+\left(2Q^4+31Q^2s-552s^2\right)M_0^4+$$

$$4\left(Q^2-3s\right)M_0^6+2M_0^8\Big)+16sm_\Delta^7M_0\Big(-23s^2\left(Q^2+s\right)^2-s^2\left(61Q^2+77s\right)M_0^2+2\left(Q^4-Q^2s+38s^2\right)M_0^4+\left(4Q^2-2s\right)M_0^6+2M_0^8\Big)-$$

$$4sm_\Delta^5M_0^3\left(2\left(55Q^2-56s\right)s^2\left(Q^2+s\right)+\left(-15Q^6-6Q^4s+539Q^2s^2+240s^3\right)M_0^2-2\left(13Q^4+15Q^2s+78s^2\right)M_0^4-\left(7Q^2+12s\right)M_0^6+4M_0^8\right)+$$

$$2sM_0^6\left(-\left(3Q^2-5s\right)\left(3Q^2-s\right)s^2\left(Q^2+s\right)-s\left(-72Q^6+519Q^4s+430Q^2s^2+195s^3\right)M_0^2+\left(8Q^6-102Q^4s-217Q^2s^2-11s^3\right)M_0^4+\right.$$

$$\left.\left(14Q^4-164Q^2s+1187s^2\right)M_0^6+\left(4Q^2-22s\right)M_0^8-2M_0^{10}\right)+4m_\Delta^6M_0^2\left(-4\left(16Q^2-29s\right)s^3\left(Q^2+s\right)+\right.$$

$$\left.s\left(11Q^6+24Q^4s-238Q^2s^2-181s^3\right)M_0^2+\left(Q^6+4Q^4s+137Q^2s^2-707s^3\right)M_0^4+\left(7Q^4-5Q^2s+185s^2\right)M_0^6+\left(11Q^2+2s\right)M_0^8+5M_0^{10}\right)-$$

$$m_\Delta^8\Big(300s^3\left(Q^2+s\right)^2+4s^3\left(179Q^2+249s\right)M_0^2-\left(10Q^6+14Q^4s+519Q^2s^2+301s^3\right)M_0^4-5\left(2Q^4+4Q^2s+55s^2\right)M_0^6+2\left(5Q^2-3s\right)M_0^8+10M_0^{10}\Big)+$$

$$4sm_\Delta M_0^7\Big(-132s^4+5Q^2M_0^2\left(Q^2+M_0^2\right)^2-4s^3\left(108Q^2+77M_0^2\right)+s^2\left(-414Q^4-469Q^2M_0^2+884M_0^4\right)+2s\left(19Q^6-Q^4M_0^2-16Q^2M_0^4+2M_0^6\right)\Big)-$$

$$m_\Delta^4M_0^4\Big(-38s^5+12M_0^6\left(Q^2+M_0^2\right)^2-6s^4\left(113Q^2+372M_0^2\right)-2sM_0^2\left(Q^2+M_0^2\right)\left(15Q^4+80Q^2M_0^2+41M_0^4\right)+s^3\left(-82Q^4+170Q^2M_0^2+2723M_0^4\right)+$$

$$s^2\left(-90Q^6-12Q^4M_0^2+407Q^2M_0^4+397M_0^6\right)\Big)+2m_\Delta^2M_0^4\Big(-5s^6+3M_0^6\left(Q^2+M_0^2\right)^3+s^5\left(13Q^2+126M_0^2\right)+sM_0^4\left(Q^2+M_0^2\right)\left(21Q^4-5Q^2M_0^2+2M_0^4\right)-$$

$$s^4\left(-9Q^4-221Q^2M_0^2+839M_0^4\right)+s^2M_0^2\left(9Q^6+424Q^4M_0^2+420Q^2M_0^4+M_0^6\right)-s^3\left(9Q^6+432Q^4M_0^2+1196Q^2M_0^4+648M_0^6\right)\Big)\Big)+$$

$$d^6\Big(2s^3m_\pi^{10}\Big(9\left(Q^2+s\right)^2-2\left(23Q^2+21s\right)M_0^2+33M_0^4\Big)-8s^3m_\Delta^9M_0\Big(4\left(Q^2+s\right)^2+\left(7Q^2+9s\right)M_0^2+35M_0^4\Big)+$$

$$4s^2m_\Delta M_0^9\left(3Q^6-50Q^4s-33Q^2s^2-8s^3+2\left(Q^4-2Q^2s-5s^2\right)M_0^2-\left(Q^2-18s\right)M_0^4\right)-$$

$$2s^2m_\Delta^{10}\Big(9s\left(Q^2+s\right)^2+2s\left(7Q^2+9s\right)M_0^2-2\left(31Q^2+121s\right)M_0^4-62M_0^6\Big)+$$

$$4sm_\Delta^3M_0^7\left(6s\left(-Q^6+6Q^4s+25Q^2s^2+6s^3\right)+\left(Q^6-2Q^4s-43Q^2s^2+24s^3\right)M_0^2+2\left(Q^4+2Q^2s-22s^2\right)M_0^4+Q^2M_0^6\right)+$$

$$4sm_\Delta^7M_0^3\left(4s^2\left(Q^2+s\right)\left(4Q^2+9s\right)+\left(Q^2+s\right)\left(Q^4+Q^2s+72s^2\right)M_0^2+2\left(Q^4+Q^2s+50s^2\right)M_0^4+Q^2M_0^6\right)-$$

$$s^2M_0^8\Big(\left(Q^2-s\right)^2s\left(Q^2+s\right)+\left(-7Q^6+112Q^4s+69Q^2s^2+18s^3\right)M_0^2-\left(Q^2-2s\right)\left(Q^2+5s\right)M_0^4+\left(7Q^2-30s\right)M_0^6+M_0^8\Big)+2sm_\Delta^2M_0^6$$

$$\Big(\left(Q^2-s\right)^2s^2\left(Q^2+s\right)+s\left(-5Q^6-7Q^4s+125Q^2s^2+27s^3\right)M_0^2+\left(2Q^6+11Q^4s-79Q^2s^2-12s^3\right)M_0^4+\left(4Q^4+17Q^2s-s^2\right)M_0^6+\left(2Q^2+s\right)M_0^8\Big)-$$

$$4sm_\Delta^5M_0^5\Big(56s^4+2Q^2M_0^2\left(Q^2+M_0^2\right)^2+s^3\left(25Q^2+4M_0^2\right)+Q^2s\left(Q^2+M_0^2\right)\left(-3Q^2+5M_0^2\right)+2s^2\left(-3Q^4+70Q^2M_0^2+34M_0^4\right)\Big)+$$

$$m_\Delta^8M_0^2\Big(70s^5+Q^2sM_0^2\left(Q^2+M_0^2\right)^2+M_0^4\left(Q^2+M_0^2\right)^3+s^4\left(96Q^2+101M_0^2\right)-s^2M_0^2\left(Q^2+M_0^2\right)\left(-3Q^2+292M_0^2\right)+s^3\left(26Q^4+91Q^2M_0^2+64M_0^4\right)\Big)-$$

$$2m_\Delta^6 M_0^4 \Big(34s^5 - Q^2 s M_0^2 (Q^2+M_0^2)^2 + M_0^4 (Q^2+M_0^2)^3 + s^4 (2Q^2+93M_0^2) + s^3 (-2Q^4+117Q^2M_0^2+75M_0^4) - s^2 (Q^2+M_0^2)(2Q^4+7Q^2M_0^2+106M_0^4)\Big) +$$
$$m_\Delta^4 M_0^4 \Big(-s^6 + s^5 (Q^2-20M_0^2) - 7Q^2 s M_0^4 (Q^2+M_0^2)^2 + M_0^6 (Q^2+M_0^2)^3 - s^2 M_0^2 (Q^2+M_0^2)(Q^4+43Q^2M_0^2+45M_0^4) +$$
$$s^4 (Q^4+243Q^2M_0^2+123M_0^4) - s^3 (Q^6-114Q^4M_0^2+154Q^2M_0^4+394M_0^6)) +$$
$$m_\pi^8 \Big(-2s^3 m_\Delta^2 \Big(45(Q^2+s)^2 - 10(17Q^2+15s)M_0^2 + 121M_0^4\Big) + 8s^3 m_\Delta M_0 \Big(-4(Q^2+s)^2 + (23Q^2+21s)M_0^2 + 229M_0^4\Big) + M_0^2 (-98s^5 +$$
$$M_0^4 (Q^2+M_0^2)^3 - sM_0^2 (Q^2+M_0^2)^2 (-Q^2+8M_0^2) + s^2 M_0^2 (Q^2+M_0^2)(-5Q^2+22M_0^2) + s^4 (-240Q^2+677M_0^2) + s^3 (-142Q^4+779Q^2M_0^2+734M_0^4)\Big)\Big) +$$
$$2m_\pi^6 \Big(-8s^3 m_\Delta^3 M_0 \Big(-8(Q^2+s)^2 + (31Q^2+27s)M_0^2 + 238M_0^4\Big) + s^2 m_\Delta^4 \Big(90s(Q^2+s)^2 - 20s(11Q^2+9s)M_0^2 + (29Q^2+229s)M_0^4 + 29M_0^6\Big) -$$
$$2sm_\Delta M_0^3 (-4s^2 (Q^2+s)(14Q^2+9s) + (Q^2+s)(Q^4-7Q^2s+32s^2)M_0^2 + 2(Q^4-3Q^2s+530s^2)M_0^4 + Q^2M_0^6) -$$
$$2m_\Delta^2 M_0^2 \Big(-56s^5 + M_0^4 (Q^2+M_0^2)^3 - sM_0^2 (Q^2+M_0^2)^2 (-Q^2+6M_0^2) + s^2 M_0^2 (Q^2+M_0^2)(-3Q^2+16M_0^2) + s^4 (-156Q^2+233M_0^2) +$$
$$s^3 (-100Q^4+305Q^2M_0^2+504M_0^4)) + M_0^4 \Big(106s^5 - M_0^4 (Q^2+M_0^2)^3 + sM_0^2 (Q^2+M_0^2)^2 (-3Q^2+8M_0^2) -$$
$$3s^4 (-94Q^2+187M_0^2) - s^2 (Q^2+M_0^2)(2Q^4-23Q^2M_0^2+18M_0^4) - s^3 (-214Q^4+649Q^2M_0^2+1006M_0^4))) +$$
$$2m_\pi^2 \Big(8s^3 m_\Delta^7 M_0 \Big(8(Q^2+s)^2 - (Q^2-3s)M_0^2 + 26M_0^4\Big) + 4s^2 m_\Delta^3 M_0^5 (-3Q^6 - 14Q^4s - 63Q^2s^2 - 24s^3 - 2(5Q^4+30Q^2s+29s^2)M_0^2 + (-7Q^2+116s)M_0^4) +$$
$$s^2 m_\Delta^8 \Big(45s(Q^2+s)^2 + 10s(Q^2+3s)M_0^2 + (-95Q^2+188s)M_0^4 - 95M_0^6\Big) -$$
$$2sm_\Delta M_0^7 (-2s(-3Q^6+70Q^4s+63Q^2s^2+18s^3) + (Q^2+s)(Q^4-3Q^2s-72s^2)M_0^2 + 2(Q^4-4Q^2s+90s^2)M_0^4 + Q^2M_0^6) -$$
$$2sm_\Delta^5 M_0^3 (-12(2Q^2-3s)s^2(Q^2+s) + (Q^2+s)(3Q^4-5Q^2s-96s^2)M_0^2 + 2(3Q^4-Q^2s-58s^2)M_0^4 + 3Q^2M_0^6) + sM_0^6 \Big((Q^2-s)^2 s^2 (Q^2+s) +$$
$$s(-9Q^6+211Q^4s+165Q^2s^2+53s^3)M_0^2 + Q^2(-2Q^4+15Q^2s-11s^2)M_0^4 + (-4Q^4+29Q^2s-203s^2)M_0^6 + (-2Q^2+5s)M_0^8\Big) -$$
$$2m_\Delta^6 M_0^2 \Big(28s^5 + M_0^4 (Q^2+M_0^2)^3 - sM_0^2 (Q^2+M_0^2)^2 (-Q^2+2M_0^2) - 3s^4 (-4Q^2+17M_0^2) + s^2 M_0^2 (Q^2+M_0^2)(Q^2+74M_0^2) - s^3 (16Q^4+35Q^2M_0^2+353M_0^4)\Big) +$$
$$m_\Delta^4 M_0^4 \Big(-18s^5 + M_0^4 (Q^2+M_0^2)^3 - sM_0^2 (Q^2+M_0^2)^2 (5Q^2+8M_0^2) - s^4 (114Q^2+199M_0^2) + s^2 (Q^2+M_0^2)(-6Q^4-7Q^2M_0^2+97M_0^4) +$$
$$s^3 (2Q^4-39Q^2M_0^2+535M_0^4)) - m_\Delta^2 M_0^4 \Big(-s^6 + M_0^6 (Q^2+M_0^2)^3 - sM_0^4 (Q^2+M_0^2)^2 (-Q^2+4M_0^2) + s^5 (Q^2+56M_0^2) +$$
$$s^2 M_0^2 (Q^2+M_0^2)(7Q^4+49Q^2M_0^2+17M_0^4) - s^4 (-Q^4-163Q^2M_0^2+165M_0^4) - s^3 (Q^6+74Q^4M_0^2+214Q^2M_0^4+72M_0^6))) +$$
$$m_\pi^4 \Big(16s^3 m_\Delta^5 M_0 \Big(-12(Q^2+s)^2 + 6(4Q^2+3s)M_0^2 + 115M_0^4\Big) + 2s^2 m_\Delta^6 \Big(-90s(Q^2+s)^2 + 20s(5Q^2+3s)M_0^2 + (4Q^2-571s)M_0^4 + 4M_0^6\Big) +$$
$$4sm_\Delta^3 M_0^3 (-12s^2(Q^2+s)(8Q^2+3s) + (Q^2+s)(3Q^4-13Q^2s-136s^2)M_0^2 + 2(3Q^4-5Q^2s-282s^2)M_0^4 + 3Q^2M_0^6) + 2m_\Delta^2 M_0^4$$
$$\Big(-54s^5 + 7Q^2sM_0^2 (Q^2+M_0^2)^2 + M_0^4 (Q^2+M_0^2)^3 - s^4 (166Q^2+627M_0^2) - s^2 (Q^2+M_0^2)(-6Q^4+23Q^2M_0^2+10M_0^4) - s^3 (218Q^4+675Q^2M_0^2+174M_0^4)\Big) +$$
$$4sm_\Delta M_0^5 \Big(-56s^4 + Q^2s(3Q^2-13M_0^2)(Q^2+M_0^2) + 2Q^2M_0^2 (Q^2+M_0^2)^2 - s^3 (169Q^2+72M_0^2) + 2s^2 (-69Q^4-46Q^2M_0^2+382M_0^4)\Big) +$$
$$2m_\Delta^4 M_0^2 \Big(-42s^5 + 3M_0^4 (Q^2+M_0^2)^3 - 3sM_0^2 (Q^2+M_0^2)^2 (-Q^2+4M_0^2) - s^4 (216Q^2+25M_0^2) - s^2 M_0^2 (Q^2+M_0^2)(3Q^2+49M_0^2) +$$
$$s^3 (-174Q^4+105Q^2M_0^2+589M_0^4)) + M_0^4 \Big(-s^6 + s^5 (Q^2-220M_0^2) + M_0^6 (Q^2+M_0^2)^3 - sM_0^4 (Q^2+M_0^2)^2 (-9Q^2+8M_0^2) +$$
$$s^2 M_0^2 (Q^2+M_0^2)(15Q^4-83Q^2M_0^2+5M_0^4) + s^4 (Q^4-621Q^2M_0^2+539M_0^4) + s^3 (-Q^6-614Q^4M_0^2+630Q^2M_0^4+1588M_0^6)))) +$$
$$4d^3 M_0 (-4s^2 m_\Delta M_0^8 (3Q^6+26Q^4s+21Q^2s^2-4s^3+(Q^2-7s)(4Q^2+5s)M_0^2+(Q^2+21s)M_0^4) + 4sm_\Delta^{11} (-9s^2(Q^2+s)+7s^2M_0^2+(7Q^2+11s)M_0^4+7M_0^6) +$$
$$sm_\Delta^{10} M_0 (4s^2(55Q^2+59s) - (4Q^4+241Q^2s+165s^2)M_0^2 + (12Q^2-109s)M_0^4 + 40M_0^6) +$$
$$4sm_\Delta^3 M_0^6 (s(6Q^6+47Q^4s+91Q^2s^2+2s^3) + (Q^6-4Q^4s-63Q^2s^2-133s^3)M_0^2 + (2Q^4-11Q^2s+190s^2)M_0^4 + (Q^2+5s)M_0^6) +$$

$$4sm_\pi^{10}\left(9s^2(Q^2+s)m_\Delta+s^2(13Q^2+9s)M_0+3s^2m_\Delta M_0^2-3(2Q^4+s^2)M_0^3-3(Q^2-3s)m_\Delta M_0^4+3(Q^2-3s)M_0^5-3m_\Delta M_0^6+3M_0^7\right)+$$

$$2s^2M_0^7\left(s(Q^2+s)(Q^4-8Q^2s-2s^2)-(4Q^6+47Q^4s+33Q^2s^2-9s^3)M_0^2+(-4Q^4+78Q^2s+13s^2)M_0^4+(Q^2-21s)M_0^6+M_0^8\right)+$$

$$4sm_\Delta^7M_0^2\left(7s^2(Q^2+s)(3Q^2+2s)+(Q^6-12Q^4s-306Q^2s^2-330s^3)M_0^2+(6Q^4-Q^2s+397s^2)M_0^4+4(4Q^2+31s)M_0^6+11M_0^8\right)-$$

$$4sm_\Delta^9\left(2s^2(Q^2+s)^2-s^2(161Q^2+139s)M_0^2+(2Q^4+6Q^2s+127s^2)M_0^4+2(9Q^2+38s)M_0^6+16M_0^8\right)+$$

$$m_\Delta^8M_0(2s^3(Q^2+s)(12Q^2+s)-2s(7Q^6+80Q^4s+15Q^2s^2+103s^3)M_0^2+$$

$$(4Q^6+6Q^4s+601Q^2s^2+229s^3)M_0^4+(12Q^4-28Q^2s+79s^2)M_0^6+12(Q^2-8s)M_0^8+4M_0^{10})-$$

$$sm_\Delta^2M_0^5(4s^2(Q^2+s)(Q^4-8Q^2s-2s^2)-s(21Q^6+317Q^4s+386Q^2s^2+30s^3)M_0^2+(2Q^6+10Q^4s+617Q^2s^2+212s^3)M_0^4+$$

$$(10Q^4+115Q^2s-448s^2)M_0^6+2(7Q^2-6s)M_0^8+6M_0^{10})+$$

$$2m_\pi^8(-2s^3(Q^2+s)m_\Delta(2(Q^2+s)+45m_\Delta^2)+s^3(54Q^4+97Q^2s+43s^2+6Q^2m_\Delta^2+46sm_\Delta^2)M_0-2s^3m_\Delta(83Q^2+105s+5m_\Delta^2)M_0^2+$$

$$s(-5Q^6+99Q^4s+808Q^2s^2+837s^3+2(23Q^4+6Q^2s-90s^2)m_\Delta^2)M_0^3+2sm_\Delta((40Q^2-251s)s+(19Q^2-41s)m_\Delta^2)M_0^4+(2Q^6+23Q^4s-$$

$$71Q^2s^2+913s^3-2s(9Q^2+s)m_\Delta^2)M_0^5+2sm_\Delta(6Q^2-2s+19m_\Delta^2)M_0^6+(6Q^4-20Q^2s+21s^2-4sm_\Delta^2)M_0^7+12sm_\Delta M_0^8+6(Q^2-4s)M_0^9+2M_0^{11})+$$

$$m_\Delta^4M_0^3(2s^3(Q^2+s)(Q^4-8Q^2s-2s^2)-2s^2(9Q^6+188Q^4s+186Q^2s^2+56s^3)M_0^2+2s(-5Q^6-58Q^4s+688Q^2s^2+248s^3)M_0^4+$$

$$(4Q^6+18Q^4s+343Q^2s^2-699s^3)M_0^6+(12Q^4+24Q^2s-169s^2)M_0^8+4(3Q^2-s)M_0^{10}+4M_0^{12})+$$

$$m_\pi^4(-24s^3(Q^2+s)m_\Delta^5(2(Q^2+s)+15m_\Delta^2)+4s^3m_\Delta^4(99Q^4+165Q^2s+66s^2+278Q^2m_\Delta^2+318sm_\Delta^2)M_0+$$

$$4s^3m_\Delta^3(61Q^4+101Q^2s+40s^2+6(39Q^2+17s)m_\Delta^2+30m_\Delta^4)M_0^2+s(2s^2(Q^2+s)(Q^4-8Q^2s-2s^2)-$$

$$s(203Q^6+735Q^4s+550Q^2s^2+42s^3)m_\Delta^2-4(18Q^6+33Q^4s-367Q^2s^2-258s^3)m_\Delta^4+(72Q^4-307Q^2s-1739s^2)m_\Delta^6)M_0^3+$$

$$4sm_\Delta(s(-3Q^6+20Q^4s+57Q^2s^2+36s^3)+(3Q^6+226Q^2s^2+214s^3)m_\Delta^2+2(-3Q^4+107Q^2s+117s^2)m_\Delta^4-18(-3Q^2+s)m_\Delta^6)M_0^4+$$

$$(-2s^2(108Q^6+477Q^4s+364Q^2s^2+4s^3)-s(-10Q^6+472Q^4s+4915Q^2s^2+5008s^3)m_\Delta^2+(24Q^6-68Q^4s+445Q^2s^2+1875s^3)m_\Delta^4+$$

$$(24Q^2-283s)sm_\Delta^6)M_0^5+4sm_\Delta(2(Q^6+4Q^4s-40Q^2s^2+5s^3)+(2Q^4+81Q^2s+557s^2)m_\Delta^2-2(2Q^2-19s)m_\Delta^4+54m_\Delta^6)M_0^6+$$

$$(10s(3Q^6+31Q^4s+174Q^2s^2+166s^3)+(8Q^6+142Q^4s+827Q^2s^2-6488s^3)m_\Delta^2+(72Q^4-12Q^2s+445s^2)m_\Delta^4+192sm_\Delta^6)M_0^7+$$

$$4sm_\Delta(4Q^4+33Q^2s-920s^2+4(2Q^2-5s)m_\Delta^2+2m_\Delta^4)M_0^8+2(2Q^6+55Q^4s-58Q^2s^2-600s^3+(12Q^4+35Q^2s+224s^2)m_\Delta^2+4(9Q^2-2s)m_\Delta^4)$$

$$M_0^9+4sm_\Delta(2Q^2+10s+9m_\Delta^2)M_0^{10}+2(6Q^4+44Q^2s-70s^2+12Q^2m_\Delta^2+5sm_\Delta^2+12m_\Delta^4)M_0^{11}+4(3Q^2+2(s+m_\Delta^2))M_0^{13}+4M_0^{15})-$$

$$4sm_\Delta^5M_0^4\left(18s^4+2M_0^2(Q^2+M_0^2)^3-19s^3(-3Q^2+14M_0^2)+2s^2(20Q^4-75Q^2M_0^2+207M_0^4)+s(3Q^6-20Q^4M_0^2-19Q^2M_0^4+64M_0^6)\right)-$$

$$m_\Delta^6M_0^3\Big(-62s^5+8M_0^4(Q^2+M_0^2)^3+2s^4(-97Q^2+358M_0^2)-s^3(129Q^4-1273Q^2M_0^2+605M_0^4)-$$

$$2sM_0^2(13Q^6-5Q^4M_0^2+3Q^2M_0^4+33M_0^6)-s^2(5Q^6+294Q^4M_0^2-590Q^2M_0^4+185M_0^6))+$$

$$m_\pi^6(8sm_\Delta^5(45s^2(Q^2+s)-5s^2M_0^2+(-23Q^2+29s)M_0^4-23M_0^6)-sm_\Delta^4M_0(8s^2(71Q^2+91s)+(128Q^4+3Q^2s-1187s^2)M_0^2-$$

$$3(8Q^2+55s)M_0^4+88M_0^6)+8sm_\Delta^3\left(4s^2(Q^2+s)^2+44s^2(Q^2+2s)M_0^2+(Q^4-85Q^2s+98s^2)M_0^4-(8Q^2+11s)M_0^6-9M_0^8\right)-$$

$$4sm_\Delta M_0^2(s^2(Q^2+s)(20Q^2+13s)+(Q^6+6Q^4s-158Q^2s^2-152s^3)M_0^2+(2Q^4+77Q^2s-820s^2)M_0^4+4(Q^2+5s)M_0^6+3M_0^8)-$$

$$m_\Delta^2M_0(4s^3(Q^2+s)(87Q^2+65s)+s(-44Q^6+311Q^4s+3201Q^2s^2+3188s^3)M_0^2+(16Q^6+32Q^4s+67Q^2s^2+2860s^3)M_0^4+$$

$$(48Q^4-52Q^2s-4s^2)M_0^6+16(3Q^2-2s)M_0^8+16M_0^{10})+4M_0^3\Big(-29s^5-2M_0^4(Q^2+M_0^2)^3-10s^4(-Q^2+85M_0^2)+$$

$$2s^2(-Q^2+M_0^2)(-13Q^4+44Q^2M_0^2+10M_0^4)+sM_0^4(-19Q^4-2Q^2M_0^2+11M_0^4)-s^3(-67Q^4+830Q^2M_0^2+534M_0^4)))+$$

$$m_\pi^2(-4sm_\Delta^9(-45s^2(Q^2+s)+25s^2M_0^2+(31Q^2+19s)M_0^4+31M_0^6)-sm_\Delta^8M_0(4s^2(207Q^2+227s)+(8Q^4-527Q^2s-1089s^2)M_0^2+3(12Q^2-89s)M_0^4+$$

$$148M_0^6)+8sm_\Delta^3M_0^4(s(3Q^6+24Q^4s+28Q^2s^2+5s^3)+s(22Q^4-75Q^2s+42s^2)M_0^2+(Q^4-34Q^2s-597s^2)M_0^4+(2Q^2-17s)M_0^6+M_0^8)+$$

$$8sm_\Delta^7\left(4s^2(Q^2+s)^2-4s^2(50Q^2+39s)M_0^2+(3Q^4-39Q^2s-26s^2)M_0^4+(16Q^2+31s)M_0^6+13M_0^8\right)-$$

$$4sm_\Delta^5 M_0^2 (s^2 (Q^2+s)(62Q^2+41s)+3(Q^6-6Q^4s+26Q^2s^2+12s^3) M_0^2+(6Q^4-221Q^2s-802s^2) M_0^4+20(Q^2-7s) M_0^6+17M_0^8)+$$
$$4sM_0^5 (s^2 (Q^2+s)(-Q^4+8Q^2s+2s^2)+s(30Q^6+168Q^4s+140Q^2s^2+5s^3) M_0^2+(-5Q^6-11Q^4s-61Q^2s^2+s^3) M_0^4+$$
$$(-14Q^4-4Q^2s+391s^2) M_0^6-13(Q^2-s) M_0^8-4M_0^{10})-m_\Delta^6 M_0 (4s^3 (Q^2+s)(45Q^2+23s)-s(52Q^6+405Q^4s+147Q^2s^2+688s^3) M_0^2+$$
$$(16Q^6-48Q^4s+149Q^2s^2+334s^3) M_0^4+2(24Q^4-14Q^2s-59s^2) M_0^6+16(3Q^2-8s) M_0^8+16M_0^{10})+$$
$$4sm_\Delta M_0^6 \Big(-25s^4+s^3(Q^2-101M_0^2)-Q^2M_0^2(Q^2+M_0^2)^2+s^2(28Q^4-27Q^2M_0^2+369M_0^4)+s(6Q^6+2Q^4M_0^2+5Q^2M_0^4+3M_0^6)\Big)+$$
$$m_\Delta^4 M_0^3 \Big(96s^5+8M_0^4(Q^2+M_0^2)^3-4s^4(-79Q^2+442M_0^2)+s^3(338Q^4-2008Q^2M_0^2+1137M_0^4)-4sM_0^2(9Q^6+14Q^4M_0^2+17Q^2M_0^4+30M_0^6)+$$
$$s^2(94Q^6+142Q^4M_0^2+623Q^2M_0^4+911M_0^6)\Big)-m_\Delta^2 M_0^3 \Big(-8s^6+8M_0^6(Q^2+M_0^2)^3+8s^5(-5Q^2+46M_0^2)-4sM_0^4(Q^2+M_0^2)(17Q^4+17Q^2M_0^2+21M_0^4)-$$
$$4s^4(7Q^4-363Q^2M_0^2+215M_0^4)+s^2M_0^2(202Q^6-273Q^4M_0^2+61Q^2M_0^4+428M_0^6)+s^3(4Q^6+1226Q^4M_0^2-737Q^2M_0^4+3652M_0^6)\Big)\Big)\Big)$$
$$(9+5\tau[3])\mathrm{II}[d][\{\{0,m_\pi\},1\}]) / (768(-1+d)^4 d(1+d)F^2s^3M_0$$
$$m_\Delta^6 (m_\pi-m_\Delta-M_0)(m_\pi+m_\Delta-M_0) M_0^4 (m_\pi-m_\Delta+M_0)(m_\pi+m_\Delta+M_0))-$$
$$\Big(ie^2 \mathrm{g\pi N\Delta}^2 \Big(-2(-2+d)m_\pi^{10} \Big(3(-2+d)^4(Q^2+s)^2 m_\Delta^2+(Q^2+s) m_\Delta ((-2+d)^3(-1+d)(Q^2+s)-24(-3+d)(-2+d)m_\Delta^2) M_0+$$
$$((-2+d)^2(Q^2+s)((18+d(-14+3d))Q^2+(9+2(-4+d)d)s)-$$
$$2((116+d(-242+d(169+d(-49+5d))))Q^2+(68+d(-170+d(133+d(-43+5d))))s) m_\Delta^2) M_0^2+$$
$$2m_\Delta(-(-2+d)((-56+d(47+d(-17+2d)))Q^2+(-4+d)(17+d(-7+2d))s)+4(-4+(-2+d)d)m_\Delta^2) M_0^3+$$
$$(-(-2+d)^2((125+d(-78+11d))Q^2+2(45+d(-29+5d))s)+(-24+d(-188+7d(30+(-10+d)d)))m_\Delta^2) M_0^4+$$
$$(-2+d)(-1+d)(36+d(-22+3d))m_\Delta M_0^5+(-2+d)^2(81-50d+8d^2) M_0^6)+$$
$$m_\pi^8 \Big(15(-2+d)^5(Q^2+s)^2 m_\Delta^4+10(-2+d)^2(Q^2+s) m_\Delta^3((-2+d)^2(-1+d)(Q^2+s)-12(-3+d)m_\Delta^2) M_0+$$
$$2(-2+d)m_\Delta^2((280+d(-480+d(312+d(-94+11d))))Q^4+(380+d(-660+d(439+d(-138+17d))))Q^2s-$$
$$5(-8+(-4+d)(-2+d)d(-11+4d))Q^2m_\Delta^2+s((100+d(-180+d(127-44d+6d^2)))s+5(56+d(16+d(-62+(29-4d)d)))m_\Delta^2)) M_0^2+$$
$$2m_\Delta((-2+d)^2(Q^2+s)((-36+d(52+d(-29+5d)))Q^2+(-1+d)(8+3(-4+d)d)s)-$$
$$2((-400+d(936+d(-876+d(409+d(-93+8d)))))Q^2+(-640+d(1136+d(-856+d(359+d(-83+8d)))))s) m_\Delta^2+$$
$$20(-2+d)(-2+d(-7+2d))m_\Delta^4) M_0^3+((-2+d)^3(5(24+d(-16+3d))Q^4+2(67+8(-5+d)d)Q^2s+5(6+(-4+d)d)s^2)-$$
$$2(-2+d)((424+d(-1222+d(961+29(-10+d)d)))Q^2+2(96+d(-362+d(302+d(-101+12d))))s) m_\Delta^2+$$
$$(2992+d(-2576+d(-132+d(946+d(-359+38d)))))m_\Delta^4) M_0^4+$$
$$2m_\Delta(-2(-2+d)^2((-208+3d(73+d(-28+3d)))Q^2+(-238+d(175+d(-65+8d)))s)+$$
$$(368+d(480+d(-1088+d(606+d(-137+11d)))))m_\Delta^2) M_0^5+$$
$$2(-(-2+d)^3((317+d(-185+23d))Q^2+5(42+d(-25+4d))s)+(-2+d)(-468+d(-264+d(557+18(-11+d)d)))m_\Delta^2) M_0^6+$$
$$2(-2+d)^2(-228+d(270+d(-115+13d)))m_\Delta M_0^7+5(-2+d)^3(78+d(-46+7d))M_0^8)+$$
$$m_\pi^6 \Big(-20(-2+d)^5(Q^2+s)^2 m_\Delta^6+20(-2+d)^2(Q^2+s) m_\Delta^5(-(-2+d)^2(-1+d)(Q^2+s)+8(-3+d)m_\Delta^2) M_0-$$
$$4(-2+d)m_\Delta^4((200+d(-320+d(194+d(-58+7d))))Q^4+(220+d(-340+d(203-66d+9d^2)))Q^2s-$$
$$10(-44+d(22+d(9+(-7+d)d)))Q^2m_\Delta^2+s((20+d(-20+d(9+2(-4+d)d)))s-10(-60+d(46+d(-3+(-5+d)d)))m_\Delta^2)) M_0^2-$$
$$8m_\Delta^3((-2+d)(Q^2+s)((58+d(-99+d(70+3(-8+d)d)))Q^2+(-1+d)(-2+d(5+(-5+d)d))s)-$$
$$((320+d(-188+d(-192+d(203+d(-61+6d)))))Q^2+(80+d(12+d(-172+3d(51+d(-17+2d)))))s) m_\Delta^2+20(-4+d)(-2+d)dm_\Delta^4)$$
$$M_0^3+m_\Delta^2(-4(-2+d)((234+d(-378+d(230+d(-66+7d))))Q^4+2(91+2d(-62+d(32+(-9+d)d)))Q^2s+(6+(-4+d)d^3)s^2)+$$

$$4\left((1432+d(-1536+d(189+d(271+d(-105+11d)))))Q^2+2(512+d(-621+d(218+(-6+d)d(-1+3d))))s\right)m_\Delta^2+$$
$$(-5792+d(5000+d(-156+d(-1906+d(907+3(-44+d)d)))))m_\Delta^4\Big)M_0^4-$$
$$4m_\Delta\left((-2+d)^2\left((-52+(-4+d)d(-17+5d))Q^4+2(-16+d(19+d(-13+2d)))Q^2s+\left(4+(-5+d)d^2\right)s^2\right)-\right.$$
$$4\left((88+d(42+d(-193+d(135+d(-35+3d)))))Q^2+(-48+d(136+d(-135+d(73+2(-10+d)d))))s\right)m_\Delta^2+$$
$$(1216+d(-816+d(-272+d(410+d(-137+15d)))))m_\Delta^4\Big)M_0^5-4\left((-2+d)^3\left((44+d(-22+3d))Q^4+(39+(-10+d)d)Q^2s+5s^2\right)-\right.$$
$$2(-2+d)\left((-115+d(-82+d(139+d(-47+4d))))Q^2+(-102+d(6+d(45+d(-23+3d))))s\right)m_\Delta^2+$$
$$\left(1984+d\left(-2054+d\left(598+d\left(81-62d+6d^2\right)\right)\right)\right)m_\Delta^4\Big)M_0^6+8m_\Delta\left((-2+d)^2(-188+7(-6+d)(-5+d)d)Q^2+\right.$$
$$(-2+d)^2(-196+d(145+d(-55+6d)))s+(-596+d(336+d(239+d(-212-3(-16+d)d))))m_\Delta^2\Big)M_0^7+$$
$$4(-2+d)\left(11(-2+d)^2(19+(-10+d)d)Q^2+10(-2+d)^2(13+(-7+d)d)s+(770-d(292+d(170+d(-82+5d))))m_\Delta^2\right)M_0^8-$$
$$4(-2+d)^2(-236+(-5+d)d(-50+11d))m_\Delta M_0^9-20(-2+d)^3(25+2(-7+d)d)M_0^{10}\Big)+$$
$$m_\pi^4\Big(15(-2+d)^5\left(Q^2+s\right)^2m_\Delta^8+20(-2+d)^2\left(Q^2+s\right)m_\Delta^7\left((-2+d)^2(-1+d)\left(Q^2+s\right)-6(-3+d)m_\Delta^2\right)M_0+$$
$$2(-2+d)m_\Delta^6\left(2\left(Q^2+s\right)\left((120+d(-160+d(76+d(-22+3d))))Q^2+(-60+d(140+d(-109-2(-14+d)d)))s\right)-\right.$$
$$5\left((-256+d(220+d(-44+d(-7+2d))))Q^2+(-304+d(292+d(-80+d(-1+2d))))s\right)m_\Delta^2\Big)M_0^2+$$
$$4m_\Delta^5\left(3(-2+d)\left(Q^2+s\right)\left((44+d(-58+d(30+(-9+d)d)))Q^2-(-1+d)(-12+d(22+(-8+d)d))s\right)-\right.$$
$$2\left((1040+d(-1312+d(492+d(-3+d(-29+4d)))))Q^2+(800+d(-1112+d(512+d(-53+d(-19+4d)))))s\right)m_\Delta^2+$$
$$10(-2+d)(2+d(-17+4d))m_\Delta^4\Big)M_0^3-$$
$$m_\Delta^4\left(2(-2+d)\left((-236+d(364+d(-200+(56-5d)d)))Q^4+2(-6+d(-64+d(89+4(-8+d)d)))Q^2s+\right.\right.$$
$$(68+d(-144+d(102+(-24+d)d)))s^2\Big)+4\left((1904+d(-2752+d(1261+d(-223+d(11+d)))))Q^2+720s-2d\right.$$
$$(569+d(-339+d(119+2(-12+d)d)))s)m_\Delta^2+(-5024+d(3800+d(28+d(-1934+d(1019+d(-161+3d))))))m_\Delta^4\Big)M_0^4+$$
$$4m_\Delta^3\left((-2+d)\left((116+d(-190+d(122+d(-35+3d))))Q^4-2d(-3+2d)(14+(-7+d)d)Q^2s-(-1+d)(-20+d(26+(-8+d)d))s^2\right)+\right.$$
$$2\left((-896+d(1408+d(-798+d(195+(-22+d)d))))Q^2+(-440+d(676+d(-480+d(195+d(-43+4d)))))s\right)m_\Delta^2+$$
$$(1824+d(-2008+d(608+(-2+d)d(-131+27d))))m_\Delta^4\Big)M_0^5+$$
$$m_\Delta^2\left(4(-2+d)(138+d(-266+d(190+d(-58+5d))))Q^4-4\left(-36+d\left(98+d\left(-66+9d+d^3\right)\right)\right)Q^2s-\right.$$
$$12(-2+d)(14+d(-20+7d))s^2+4(-1748+d(2702+d(-1457+d(373+d(-58+5d)))))Q^2m_\Delta^2+$$
$$4(-616+d(942+d(-584+d(196+d(-33+2d)))))sm_\Delta^2+(1184+d(176+d(-1012+d(1278+d(-695+3(56-5d)d)))))m_\Delta^4\Big)M_0^6+$$
$$4m_\Delta\left((-2+d)^2\left((-44+d(44+(-25+d)d))Q^4-2(2+d(11+(-2+d)d))Q^2s-(-1+d)(16+(-4+d)d)s^2\right)+\right.$$
$$2\left((-976+d(1500+d(-992+d(344+5(-13+d)d))))Q^2+(-4+d)(184+d(-234+d(135+d(-37+4d))))s\right)m_\Delta^2+$$
$$(1144+d(-928+d(126+d(62+3(-9+d)d))))m_\Delta^4\Big)M_0^7+\left((-2+d)^3\left((108-d(4+9d))Q^4-2(-42+d(-20+7d))Q^2s-5(-4+d)ds^2\right)+\right.$$
$$4(-2+d)\left((706+d(-518+d(159+5(-6+d)d)))Q^2+2(254+d(-254+d(106+d(-19+2d))))s\right)m_\Delta^2-$$
$$2(-2440+d(2632+d(-1360+d(716+d(-241+30d)))))m_\Delta^4\Big)M_0^8-4(-2+d)m_\Delta$$
$$\left(2(-2+d)\left((-175+d(193+4(-16+d)d))Q^2+(-169+d(120+d(-45+4d)))s\right)+(1308+d(-818+d(144+d(-33+7d))))m_\Delta^2\right)M_0^9-$$
$$2(-2+d)\left((-2+d)^2\left((302+d(-135+8d))Q^2+5(-4+d)(-9+2d)s\right)+2\left(1026+d\left(-728+d\left(111+2d+4d^2\right)\right)\right)m_\Delta^2\right)M_0^{10}+$$
$$4(-2+d)^2(-234+d(230+d(-95+9d)))m_\Delta M_0^{11}+5(-4+d)(-2+d)^3(-18+5d)M_0^{12}\Big)+$$
$$(-m_\Delta+M_0)(m_\Delta+M_0)\Big(-(-2+d)^5\left(Q^2+s\right)^2m_\Delta^{10}+2(-2+d)^2\left(Q^2+s\right)m_\Delta^9\left(-(-2+d)^2(-1+d)\left(Q^2+s\right)+4(-3+d)m_\Delta^2\right)M_0+$$
$$(-2+d)m_\Delta^8\left(72s\left(Q^2+s\right)+d^4\left(Q^2+s\right)\left(Q^2+3s\right)-16\left(21Q^2+23s\right)m_\Delta^2-2d^3\left(6Q^4+22Q^2s+16s^2-7Q^2m_\Delta^2-9sm_\Delta^2\right)-\right.$$
$$8d\left(4Q^4+23Q^2s+19s^2-2\left(22Q^2+25s\right)m_\Delta^2\right)+2d^2\left(20Q^4+77Q^2s+57s^2-2\left(31Q^2+37s\right)m_\Delta^2\right)\Big)M_0^2+$$

$$4m_\Delta^7 \left((-2+d)\left(Q^2+s\right)\left((-12+d(-2+d(16+(-7+d)d)))Q^2+16s+2(-3+d)d(8+(-5+d)d)s\right)+\right.$$
$$\left((472+d(-680+d(358+d(-81+7d))))Q^2+(424+d(-640+d(362+d(-91+9d))))s\right)m_\Delta^2-2(-2+d)(2+d(-9+2d))m_\Delta^4\Big)M_0^3+$$
$$m_\Delta^6\left(2(-2+d)\left((-20+d(36+d(-10+(-4+d)d)))Q^4+2(-26+d(36+d(-5+(-7+d)d)))Q^2s-(52+d(-80+d(36+(-4+d)d)))s^2\right)+\right.$$
$$2\left((-128+d(228+(-10+d)d(27+(-9+d)d)))Q^2+(-560+d(852+d(-572+(176-21d)d)))s\right)m_\Delta^2-$$
$$\left.(912+d(-1944+d(2088+d(-980+3d(70+(-10+d)d)))))m_\Delta^4\right)M_0^4-$$
$$2m_\Delta^5\left(2(-2+d)\left(2(-16+d(10+(-1+d)d))Q^4+(-36+d(14+d(12+(-3+d)d)))Q^2s+(-1+d)(-28+d(54+d(-22+3d)))s^2\right)-\right.$$
$$2\left((-936+d(1292+d(-762+d(222+(-29+d)d))))Q^2-4\left(292+d\left(-469+d\left(298-84d+9d^2\right)\right)\right)s\right)m_\Delta^2+$$
$$\left.(448+d(-736+5d(92+d(6+d(-17+3d)))))m_\Delta^4\right)M_0^5+m_\Delta^4$$
$$\left(2(-2+d)\left((120+d(-176+d(74+d(-16+3d))))Q^4+2(84+d(-136+d(74+d(-21+4d))))Q^2s-(-32+d(8+d(38+(-20+d)d)))s^2\right)-\right.$$
$$4\left((512+d(-566+d(-11+2d(72+(-20+d)d))))Q^2+(264-d(344+d(54+d(-124+27d))))s\right)m_\Delta^2-$$
$$\left.(-1056+d(656+d(-472+d(136+d(179+2d(-47+6d))))))m_\Delta^4\right)M_0^6+$$
$$4m_\Delta^3\left((-2+d)\left((12+d(-62+d(52+d(-17+3d))))Q^4+(60+d(-202+d(186+d(-65+9d))))Q^2s+2(-1+d)^2(12+(-6+d)d)s^2\right)-\right.$$
$$\left((-632+d(844+d(-694+d(330+d(-73+5d)))))Q^2-2(296+d(-206+d(2+d(11+d))))s\right)m_\Delta^2+$$
$$\left.2(584+d(-926+d(574+d(-73+d(-32+7d)))))m_\Delta^4\right)M_0^7+m_\Delta^2$$
$$\left((-2+d)\left(-(152+d(-152+d(2+d)(6+d)))Q^4+2(-12+d(-104+d(162+d(-66+7d))))Q^2s+(-8+d(-48+(-4+d)d(-20+3d)))s^2\right)-\right.$$
$$4\left((-540+d(616+d(-200+d(37+d(-19+5d)))))Q^2+(-112+d(-6+d(200+d(-122+21d))))s\right)m_\Delta^2+$$
$$\left.(800+d(-2576+d(1456+d(-32+d(80+3d(-31+5d))))))m_\Delta^4\right)M_0^8-$$
$$2m_\Delta\left((-2+d)^2\left((-28+d(60+d(-31+7d)))Q^4+4(-5+d(11+(-5+d)d))Q^2s+(-1+d)(8+(-4+d)d)s^2\right)+\right.$$
$$2\left((112+d(44+d(-124+d(80+d(-29+5d)))))Q^2+2(204+d(-262+d(145+d(-43+6d))))s\right)m_\Delta^2+$$
$$\left.2(952+d(-860+d(158+d(92+d(-39+5d)))))m_\Delta^4\right)M_0^9-$$
$$\left((-2+d)^3\left((8+d(-24+7d))Q^4+2(1+2(-4+d)d)Q^2s+(2+(-4+d)d)s^2\right)+2(-2+d)((-80+d(-118+d(126+d(-41+4d))))\right.$$
$$\left.Q^2-(128+d(-104+d(14+5d)))s\right)m_\Delta^2+(368+d(168+d(-1004+d(930+d(-297+29d)))))m_\Delta^4\Big)M_0^{10}+$$
$$4m_\Delta\left((-2+d)^2(28+d(-25+d(4+d)))Q^2+(-2+d)^2(26+5(-3+d)d)s-2(-284+d(358+d(-183+d(57+(-12+d)d))))m_\Delta^2\right)M_0^{11}+$$
$$(-2+d)\left(2(-2+d)^2\left((-17+d(3+d))Q^2+(-10+3d)s\right)-(472+d(-368+d(76+3(-2+d)d)))m_\Delta^2\right)M_0^{12}+$$
$$\left.2(-2+d)^2(-44+d(38+(-15+d)d))m_\Delta M_0^{13}+(-2+d)^3(22+(-10+d)d)M_0^{14}\right)+$$
$$m_\pi^2\Big(-6(-2+d)^5\left(Q^2+s\right)^2m_\Delta^{10}+2(-2+d)^2\left(Q^2+s\right)m_\Delta^9\left(-5(-2+d)^2(-1+d)\left(Q^2+s\right)+24(-3+d)m_\Delta^2\right)M_0+$$
$$2(-2+d)m_\Delta^8\Big(\left(-40+d^2(42+(-14+d)d)\right)Q^4+(100+d(-300+d(269+d(-78+7d))))Q^2s+2(-380+d(374+d(-115+d(7+d))))Q^2m_\Delta^2+$$
$$s\left(\left(140+d\left(-300+d\left(227-64d+6d^2\right)\right)\right)s+2(-428+d(446+d(-151+d(13+d))))m_\Delta^2\right)\Big)M_0^2+$$
$$4m_\Delta^7\left(2(-2+d)\left(Q^2+s\right)\left((-30+d(1+d)(17+(-7+d)d))Q^2+26s+(-3+d)d(25+3(-5+d)d)s\right)+\right.$$
$$\left((1760+d(-2436+d(1176+d(11+d)(-19+2d))))Q^2+(1520+d(-2236+d(1196+d(-259+d(13+2d)))))s\right)m_\Delta^2-$$
$$\left.4(-2+d)(4+d(-22+5d))m_\Delta^4\right)M_0^3+$$
$$m_\Delta^6\left(4(-2+d)\left((-14+d(30+d(-10+(-2+d)d)))Q^4+2(-1+2d(-4+d(11+(-7+d)d)))Q^2s-(34+d(-56+d(28+(-4+d)d)))s^2\right)-\right.$$
$$2\left((-632+d(940+d(-204+d(-21+(-2+d)d))))Q^2+2(600+d(-870+d(526+d(-109+d(-5+3d)))))s\right)m_\Delta^2-$$
$$\left.(2816+d(-3544+d(2580+d(-534+d(-129+d(26+3d))))))m_\Delta^4\right)M_0^4+$$
$$2m_\Delta^5\left(2(-2+d)\left((32+d(-16+(-6+d)(-1+d)d))Q^4+2(48+d(-78+d(57+d(-21+2d))))Q^2s-(-1+d)(-32+d(56+d(-22+3d)))s^2\right)-\right.$$
$$8\left((-4+d)(-50+d(31+d(-3+(-3+d)d)))Q^2+(368+d(-568+d(313+d(-53+2(-3+d)d))))s\right)m_\Delta^2-$$

$$(2192+d(-3096+d(1552+d(162+d(-281+47d))))m_\Delta^4\big)\,M_0^5+$$
$$2m_\Delta^4\left(2(-2+d)\left(-\left(4+(-2+d)d\left(4+d^2\right)\right)Q^4+(44+d(-152+d(159+d(-58+5d))))Q^2s+(-24+d(20+(9-8d)d))s^2\right)-\right.$$
$$4\left((278+d(-275+d(-17+d(91+2(-14+d)d))))Q^2+404s-d(646+d(-256+d(9+d(3+d))))s\right)m_\Delta^2+$$
$$(4056+d(-7640+d(6234+d(-2527+d(555+d(-62+3d))))))m_\Delta^4\big)\,M_0^6+$$
$$8m_\Delta^3\left((-2+d)\left((-6+d(57+d(-62+(22-3d)d)))Q^4-2d(-15+d(15+(-5+d)d))Q^2s-(-1+d)(-14+d(19+(-7+d)d))s^2\right)+\right.$$
$$\left((-528+d(512+d(-112+(-2+d)(-1+d)d)))Q^2+(-640+d(1044+d(-800+d(339+d(-73+6d)))))s\right)m_\Delta^2+$$
$$(948+d(-1480+d(829+d(-80+d(-46+9d)))))m_\Delta^4\big)\,M_0^7+$$
$$m_\Delta^2\left(2(-2+d)\left((20+d(148+d(-220-7(-12+d)d)))Q^4-2(-62+d(-16+d(108+d(-52+5d))))Q^2s+\left(44-(-8+d)d^2(-8+3d)\right)s^2\right)-\right.$$
$$4\left((184+d(392+d(-695+d(347+3(-23+d)d))))Q^2+368s-2d\left(201+d\left(-34+d\left(-14+d+d^2\right)\right)\right)s\right)m_\Delta^2-$$
$$(-4528+d(3696+d(1268+d(-946+d(59+d(4+3d))))))m_\Delta^4\big)\,M_0^8+$$
$$2m_\Delta\left((-2+d)^2\left((-12+d(64+d(-31+11d)))Q^4+2(-20+d(54+d(-23+5d)))Q^2s+(-1+d)(28+3(-4+d)d)s^2\right)-\right.$$
$$8\left((-392+d(648+d(-496+d(197+d(-37+2d)))))Q^2+2(-210+d(316+d(-214+d(79+(-15+d)d))))s\right)m_\Delta^2-$$
$$2(-1872+d(2608+d(-1436+d(334+(-35+d)d))))m_\Delta^4\big)\,M_0^9+$$
$$2\left((-2+d)^3\left((-6+d(-26+9d))Q^4+(-7+d(-26+7d))Q^2s+(3+2(-4+d)d)s^2\right)-2(-2+d)((454+d(-294+d(75+(-7+d)d)))\right.$$
$$Q^2+(360+d(-382+d(151+d(-25+3d))))s\big)\,m_\Delta^2+2(344+d(-354+d(-170+d(413+2d(-82+9d)))))m_\Delta^4\big)\,M_0^{10}+$$
$$4m_\Delta\left((-2+d)^2\left(-5\left(32-33d+9d^2\right)Q^2+(-148+d(97+d(-35+2d)))s\right)+2(-1116+d(1456+d(-775+d(250+d(-54+5d)))))m_\Delta^2\right)M_0^{11}+$$
$$2\left(-(-2+d)^3\left((-113+d(38+d))Q^2-2(33+(-13+d)d)s\right)+(-2+d)\left(1236+d\left(-1044+d\left(286-42d+9d^2\right)\right)\right)m_\Delta^2\right)M_0^{12}-$$
$$2(-2+d)^2(-228+d(210+d(-85+7d)))m_\Delta M_0^{13}-2(-2+d)^3\left(69-34d+4d^2\right)M_0^{14}\big)+(-2+d)^2m_\pi^{12}$$
$$\left(8m_\Delta M_0\left(-(-3+d)\left(Q^2+s\right)+M_0^2\right)+(-2+d)\left((-2+d)^2\left(Q^2+s\right)^2-2\left((-4+d)(-5+2d)Q^2+(16+d(-11+2d))s\right)M_0^2+(28+3(-6+d)d)M_0^4\right)\right)\Big)$$
$$(9+5\tau[3])\mathrm{II}[d]\left[\{\{0,m_\pi\},1\},\{\{p,m_\Delta\},1\}\right]\Big/\left(384(-1+d)^4F^2m_\Delta^6M_0\left(m_\pi-m_\Delta-M_0\right)(m_\pi+\right.$$
$$\left.m_\Delta-M_0)\,M_0^4\left(m_\pi-m_\Delta+M_0\right)\left(m_\pi+m_\Delta+M_0\right)\right)-$$
$$\frac{1}{768(-1+d)^4(1+d)F^2s^3m_\Delta^6M_0}\left(ie^2\mathrm{g\pi N\Delta}^2\left(\left(Q^2+s\right)\left((-2+d)^3s^4\left((-6+(-3+d)d)Q^4-2(-2+(-3+d)d)Q^2s+\left(-2+d+d^2\right)s^2\right)-\right.\right.\right.$$
$$(-2+d)s^3\left((-2+d)(56+d(-20+13(-3+d)d))Q^4+2(80+d(60+d(-82+5d(3+d))))Q^2s+(40-d(60+d(74+d(-83+15d))))s^2\right)m_\Delta^2+$$
$$s^2\left((-2+d)(-48+d(56+d(-10+3(-1+d)d)))Q^4-2(688+d(-316+d(-660+d(528+d(-129+11d)))))Q^2s-\right.$$
$$\left.(-2+d)(88+d(516+d(234+d(-237+65d))))s^2\right)m_\Delta^4+s\left((-2+d)^3(6+(-1+d)d)Q^4+\right.$$
$$\left.2(192+d(36+d(-284+d(84+(-3+d)d))))Q^2s+(-2160+d(-976+d(1564+d(476+d(-145+d(-19+6d))))))s^2\right)m_\Delta^6-$$
$$2\left((-2+d)^3(2+d)Q^4+2(-2+d)\left(12+(-2+d)d^2\right)Q^2s+(-800+d(112+d(474+d(-190+d(150+d(-62+9d))))))s^2\right)m_\Delta^8\big)+$$
$$4(1+d)sm_\Delta\left((-2+d)^2s^2\left((-2+d)dQ^6-2(8+(-3+d)d)Q^4s+(-20+d(2+d))Q^2s^2-2(-2+d)s^3\right)+\right.$$
$$2(-2+d)s\left((-2+d)^3Q^6+(-80+d(126+d(-65+12d)))Q^4s+(-48+d(110+7(-8+d)d))Q^2s^2+(8+(4-5d)d)s^3\right)m_\Delta^2+$$
$$\left((-2+d)^3dQ^6+2(-2+d)^2(8+(-5+d)d)Q^4s+(800+d(-1208+d(620+d(-146+17d))))Q^2s^2-2(-128+d(292+d(-144+19d)))s^3\right)m_\Delta^4-$$
$$2\left((-6+d)(-2+d)dQ^4+2(4+d)(-2+3d)Q^2s+(-240+d(200+d(-62+7d)))s^2\right)m_\Delta^6-$$
$$\left.4(-6+d)\left((-2+d)Q^2+(-6+d)s\right)m_\Delta^8\right)M_0+$$
$$\left((-2+d)^3(1+d)Q^6\left((-14+17d)s^3+3(-2+d)s^2m_\Delta^2+3(-2+d)sm_\Delta^4+(2+d)m_\Delta^6\right)+\right.$$
$$2(-2+d)Q^4\left((-2+d)^2(10+d(12+d))s^4+(-400+d(252+d(352+d(-277+59d))))s^3m_\Delta^2+\right.$$
$$\left.(-112+d(56+d(52+d(-51+11d))))s^2m_\Delta^4+(32+d(4+d(4+(-3+d)d)))sm_\Delta^6-(-2+d)^2(2+d)m_\Delta^8\right)+$$
$$Q^2s\left(-(-2+d)^3(-26+d(-19+3d))s^4+(-2+d)(32+d(792+7d(6+d(-65+17d))))s^3m_\Delta^2-\right.$$

$$(-5472+d(2616+d(4160+d(-3358+d(1070+d(-145+3d))))))s^2m_\Delta^4+$$
$$(-624+d(-8+d(428+d(-414+d(156+d(-41+9d))))))sm_\Delta^6-4(-2+d)^2(2+d)(4+d)m_\Delta^8)+$$
$$s^2(-2(-2+d)^3(2+5d)s^4+8(-2+d)(2+d(4+d(-17+8d)))s^3m_\Delta^2-(-960+d(1680+d(772+d(-1842+d(845+3(-50+d)d)))))s^2m_\Delta^4+$$
$$(1120+d(-1408+d(-256+d(1430+d(-271+d(-62+15d))))))sm_\Delta^6-2(-240+d(136+d(218+d(-50+d(118+d(-62+9d))))))m_\Delta^8))$$
$$M_0^2+8(1+d)sm_\Delta(4(-2+d)^2s^4-2(-16+d(2+d))s^3m_\Delta^2-2(164+d(-132+5d))s^2m_\Delta^4+2(88+d(-66+5d))sm_\Delta^6-$$
$$2(-6+d)(-2+d)m_\Delta^8+(-2+d)^3Q^4((6-5d)s^2-2sm_\Delta^2+dm_\Delta^4)+$$
$$(-2+d)Q^2((-2+d)(-6+d^2)s^3-2(-6+d)^2(-1+d)s^2m_\Delta^2+(-20+d(10+(-6+d)d))sm_\Delta^4-2(-6+d)dm_\Delta^6))M_0^3+$$
$$(-(-2+d)^3(1+d)Q^4((-46+41d)s^3+(-6+7d)s^2m_\Delta^2-(-18+5d)sm_\Delta^4-3(2+d)m_\Delta^6)+$$
$$(-2+d)Q^2((-2+d)^2(-38+5(-7+d)d)s^4-(-1024+d(280+d(890-429d+57d^2)))s^3m_\Delta^2+$$
$$(-272+(-4+d)d(14+d(-43+19d)))s^2m_\Delta^4+(-56+d(100+d(58+(-25+d)d)))sm_\Delta^6+2(-2+d)^2(2+d)m_\Delta^8)+$$
$$s(-2(-2+d)^3(-2+(-4+d)d)s^4-2(-2+d)(64+d(140+d(-56+d(-17+7d))))s^3m_\Delta^2-$$
$$(1120+d(112+d(-1012+d(642+d(-289+d(40+3d))))))s^2m_\Delta^4+$$
$$(544+d(432+d(-716+d(158+d(61+d(-44+9d))))))sm_\Delta^6+2(-2+d)(32+d(36+(-22+d)d))m_\Delta^8))M_0^4+$$
$$4(-2+d)(1+d)sm_\Delta(s-m_\Delta^2)^2((-2+d)^2(dQ^2+2s)-2(-6+d)dm_\Delta^2)M_0^5+(-2+d)$$
$$(s-m_\Delta^2)^2$$
$$(2(-2+d)^2(2+d)s^2-4(-4+5d)(-4+(-2+d)d)sm_\Delta^2+$$
$$2(-2+d)^2(2+d)m_\Delta^4+(-2+d)^2(1+d)Q^2((-6+7d)s+3(2+d)m_\Delta^2))$$
$$M_0^6+(-2+d)^3(1+d)(s-m_\Delta^2)^2((-2+d)s+(2+d)m_\Delta^2)M_0^8-(-2+d)$$
$$m_\pi^6$$
$$((-2+d)^2s(Q^2+s)((-18+(-9+d)d)Q^4+2(26+(31-3d)d)Q^2s+(-58+d(-41+9d))s^2)-$$
$$8((-2+d)^2(2+d)Q^6-(-2+d(10+3(-4+d)d))Q^4s+(14+d(2+d)(-10+3d))Q^2s^2+(20+d(-14+d(-18+7d)))s^3)m_\Delta^2+$$
$$8(1+d)sm_\Delta(-(-2+d)(Q^2-s)(d(Q^2-3s)+22s)-8(-3+d)(Q^2-2s)m_\Delta^2)M_0+$$
$$((-2+d)^2((1+d)(2+d)Q^6-2(4+3d(1+d))Q^4s+(-50+d(-49+17d))Q^2s^2+8(11+(5-3d)d)s^3)-$$
$$8((-2+d)^2(2+d)Q^4+2(-2+d)(2+d^2)Q^2s+(-6+d(44+(12-11d)d))s^2)m_\Delta^2)$$
$$M_0^2+16(1+d)sm_\Delta(-(-2+d)(d(Q^2-2s)+3s)-4(-3+d)m_\Delta^2)M_0^3+$$
$$((-2+d)^2(3(1+d)(2+d)Q^4-(-10+d(13+15d))Q^2s+2(-16+d(7+11d))s^2)+8((-2+d)^2(2+d)Q^2+(-22+d(18+(8-5d)d))s)m_\Delta^2)$$
$$M_0^4-8(-2+d)d(1+d)sm_\Delta M_0^5+(-2+d)^2(2+d)(3(1+d)Q^2-8ds+8m_\Delta^2)M_0^6+(-2+d)^2(1+d)(2+d)M_0^8)-$$
$$m_\pi^4(-(Q^2+s)((-2+d)^3s^2(3(-10+(-5+d)d)Q^4-2(-54+d(-47+19d))Q^2s+3(-3+d)(6+5d)s^2)+$$
$$(-2+d)s((-2+d)^2(-30+d(-19+3d))Q^4-2(-4+d(6+d(58+d(-19+5d))))Q^2s+(168+d(364+d(-102+d(-175+51d))))s^2)m_\Delta^2+$$
$$(-12(-2+d)^3(2+d)Q^4+24(-2+d)(-2+d(2+(-4+d)d))Q^2s+(640+d(-128+d(-324+d(234+d(47+d(-58+9d))))))s^2)m_\Delta^4)+$$
$$4(1+d)sm_\Delta\left((-2+d)^2\left(-d^2Q^2(Q^2-3s)^2+2d(Q^2-7s)(Q^2-s)(Q^2+s)+8s(3Q^4+11Q^2s-7s^2)\right)+\right.$$
$$2((-2+d)d(-10+3d)Q^4-2(-2+d)(76+d(-41+4d))Q^2s+(-144+d(176+d(-82+13d)))s^2)m_\Delta^2+$$
$$8(3(-4+d)(-2+d)Q^2+(-12+(16-3d)d)s)m_\Delta^4)M_0+$$
$$(-3(-2+d)^3(1+d)(2+d)Q^6(s+m_\Delta^2)+2Q^4((-2+d)^3(-16+d(-9+13d))s^2+(-3+d)(-2+d)d(4+5d^2)sm_\Delta^2+6(-2+d)^3(2+d)m_\Delta^4)+$$
$$(-2+d)Q^2s(-(-2+d)^2(-118+d(-67+75d))s^2-(528+d(360+d(-198+d(-139+35d))))sm_\Delta^2+24(-8+d^3)m_\Delta^4)+$$
$$s^2(4(-2+d)^3(-12+d(5+8d))s^2+8(-2+d)(134+d(108+d(-87+2d(-12+5d))))s$$

$$m_\Delta^2 + (-32 + d(-1232 + d(500 + d(406 + d(-255 + (58 - 9d)d)))))m_\Delta^4)) M_0^2 + 8(-2 + d)(1 + d)sm_\Delta$$
$$(-(-2 + d) ((-2 + d)dQ^4 - (10 + d(-4 + 3d))Q^2 s + 2(-1 + 4d)s^2) + 2 (-4s + d ((-10 + 3d)Q^2 + (17 - 4d)s)) m_\Delta^2 + 12(-4 + d)m_\Delta^4) M_0^3 +$$
$$(-2 + d) ((-2 + d)^2 s (-(1 + d)(18 + 5d)Q^4 + (26 + d(67 + 29d))Q^2 s - 6(4 + d)(1 + 3d)s^2) -$$
$$(9(-2 + d)^2(1 + d)(2 + d)Q^4 - (-24 + d(84 + d(-62 + d(-69 + 29d))))Q^2 s + 2(152 + d(84 + d(-208 + d(-11 + 21d))))s^2) m_\Delta^2 +$$
$$12 (-(-2 + d)^2(2 + d)Q^2 + (4 + d(-24 + d(2 + 3d)))s) m_\Delta^4) M_0^4 - 4(-2 + d)(1 + d)sm_\Delta (8s + d ((-2 + d)^2 Q^2 + 20m_\Delta^2 - 2d (s + 3m_\Delta^2))) M_0^5 +$$
$$(-2 + d) (12(-2 + d)^2(2 + 3d)s^2 + 8 (12 + d(-1 + 2d) (-8 + d^2)) sm_\Delta^2 - 12(-2 + d)^2(2 + d)m_\Delta^4 - (-2 + d)^2(1 + d)Q^2 ((18 + d)s + 9(2 + d)m_\Delta^2)) M_0^6 +$$
$$(-2 + d)^3(1 + d) ((-6 + d)s - 3(2 + d)m_\Delta^2) M_0^8) +$$
$$m_\pi^2 (- (Q^2 + s) ((-2 + d)^3 s^3 ((-38 + d(-11 + 19d))Q^4 - 2(-26 + d(-25 + 9d))Q^2 s + (-18 + d(-3 + 7d))s^2) + 2(-2 + d)s^2$$
$$((-20 + d(-6 + d(32 + 3(-5 + d)d)))Q^4 + 2(-258 + d(51 + d(206 + d(-122 + 17d))))Q^2 s + (-8 + d(164 + d(44 + d(-133 + 31d))))s^2) m_\Delta^2 +$$
$$s ((-2 + d)^3(-6 + d(-11 + 3d))Q^4 - 2(-392 + d(12 + d(414 + d(-184 + d(23 + d)))))Q^2$$
$$s + (2064 + d(352 + d(-1260 + d(418 - (-9 + d)d(-22 + 3d)))))s^2) m_\Delta^4 -$$
$$(8(-2 + d)^3(2 + d)Q^4 + 16(-2 + d)(7 + (-1 + d)d)Q^2 s + (-2080 + d (64 + d (1192 + d (-446 + d (215 - 66d + 9d^2))))) s^2) m_\Delta^6) -$$
$$8(1 + d)sm_\Delta ((-2 + d)^2 s ((-2 + d)dQ^6 - (28 + d(-23 + 8d))Q^4 s + (-38 + d(4 + 3d))Q^2 s^2 - (-14 + 5d)s^3) +$$
$$((-2 + d)^3 dQ^6 - 2(-3 + d)(-2 + d) (4 + d^2) Q^4 s + (-2 + d)(96 + d(-16 + d(-24 + 5d)))Q^2 s^2 - 2(16 + d(-8 + d(-12 + 5d)))s^3) m_\Delta^2 -$$
$$((-2 + d)d(-14 + 3d)Q^4 - 2(-100 + d(96 + d(-33 + 2d)))Q^2 s + (216 + d(20 + d(-110 + 17d)))s^2) m_\Delta^4 -$$
$$8 ((-5 + d)(-2 + d)Q^2 - 2(-6 + d)s) m_\Delta^6) M_0 +$$
$$(-3(-2 + d)^3(1 + d)Q^6 ((2 + d)s^2 + 2dsm_\Delta^2 + (2 + d)m_\Delta^4) + 2(-2 + d)Q^4 ((-2 + d)^2(-60 + d(-23 + 41d))s^3 + 2$$
$$(-72 + d(-6 + d(34 + (-13 + d)d)))s^2 m_\Delta^2 + (2 + d)(-24 + (-3 + d)(-2 + d)d)sm_\Delta^4 + 4(-2 + d)^2(2 + d)m_\Delta^6) +$$
$$Q^2 s (-(-2 + d)^3(6 + d(-3 + 7d))s^3 + 2(-2 + d)(-1376 + d(-168 + d(1028 + d(-247 + 5d))))s^2 m_\Delta^2 -$$
$$(-2080 + d(8 + d(1608 + d(-782 + d(98 + d(-23 + 9d))))))sm_\Delta^4 + 16(-2 + d)^2(6 + d(4 + d))m_\Delta^6) +$$
$$s^2 (8(-2 + d)^3(1 + d(5 + d))s^3 + 8(-2 + d)(18 + d(64 + d(20 + d(-45 + 8d))))s^2 m_\Delta^2 - 2(-880+$$
$$d(-368 + d(580 + d(-66 + d(141 + d(-46 + 3d))))))sm_\Delta^4 + (-320 + d(-400 + d(632 + d(178 + 3d(13 + d(-22 + 3d))))))m_\Delta^6)) M_0^2 +$$
$$16(1 + d)sm_\Delta (-(-2 + d)^2 s ((-2 + d)Q^4 + (-4 + d)(1 + d)Q^2 s + (3 + 2d)s^2) + (-(-2 + d)^3 dQ^4 + (-2 + d)(-16 + d(4 + (-2 + d)d))$$
$$Q^2 s + 2(-4 + d)(12 + d(-9 + 2d))s^2) m_\Delta^2 + (-92s + d ((-2 + d)(-14 + 3d)Q^2 + (48 + (9 - 2d)d)s)) m_\Delta^4 + 4(-5 + d)(-2 + d)m_\Delta^6) M_0^3 +$$
$$((-2 + d)^3 s^2 ((1 + d)(-26 + 7d)Q^4 + (118 + 87d - 39d^2) Q^2 s + 2(-20 + d(-29 + 3d))s^2) - 2(-2 + d)s$$
$$(5(-2 + d)^2 d(1 + d)Q^4 - (-4 + d(-50 + d(-12 + d(3 + 5d))))Q^2 s + 2(-176 + d(-64 + d(130 + (-35 + d)d)))s^2) m_\Delta^2 -$$
$$(9(-2 + d)^3(1 + d)(2 + d)Q^4 - (-2 + d)(2 + d)(36 + d(-72 + d(-23 + 13d)))Q^2 s + (-320+$$
$$d(896 + d(100 + d(-362 + d(73 + 3d(-8 + 3d))))))s^2) m_\Delta^4 + 8(-2 + d) (-(-2 + d)^2(2 + d)Q^2 + (-14 + d(-30 + d(12 + d)))s) m_\Delta^6) M_0^4 +$$
$$8(-2 + d)(1 + d)sm_\Delta ((-2 + d)s ((-2 + d)^2 Q^2 - (-4 + d)s) - (-2 + d) ((-2 + d)dQ^2 + 2(-6 + d)s) m_\Delta^2 + d(-14 + 3d)m_\Delta^4) M_0^5 +$$
$$(-2 + d) (-8(-2 + d)^2(-2 + (-2 + d)d)s^3 - 8(24 + d(8 + 5(-4 + d)d))s^2 m_\Delta^2 + 8(24 + d(-8 + d(-20 + d(4 + d))))sm_\Delta^4 - 8(-2 + d)^2(2 + d)m_\Delta^6 +$$
$$(-2 + d)^2(1 + d)Q^2 (11(-2 + d)s^2 - 2dsm_\Delta^2 - 9(2 + d)m_\Delta^4)) M_0^6 + (-2 + d)^3(1 + d) ((-2 + d)s^2 + 2dsm_\Delta^2 - 3(2 + d)m_\Delta^4) M_0^8) +$$
$$2(-2 + d)^2 m_\pi^8 (-8(1 + d)sm_\Delta M_0 (Q^2 - 3s + M_0^2) + (-2 + d) (-Q^2 - s + M_0^2) ((10 + 9d)s^2 + (2 + d) (Q^2 + M_0^2)^2 - 2s ((2 + 3d)Q^2 + (4 + 3d)M_0^2)))) (9 +$$
$$5\tau[3])\Pi[d] [\{\{0, m_\pi\}, 1\}, \{\{p + q, m_\Delta\}, 1\}]) - \frac{1}{384(-1+d)^3 F^2 m_\Delta^6 M_0} (ie^2 \mathrm{g\pi N\Delta}^2 (4(-2 + d)^3 Q^4 M_0^4 (-5Q^2 - s + 5M_0^2) +$$
$$32m_\Delta^7 M_0 ((-44 + d(17 + (-6 + d)d))Q^2 - 4(-5 + d)(-2 + d)s + 4(-5 + d)(-2 + d)M_0^2) +$$
$$m_\Delta^8 (-(368 + d(-1064 + d(420 + d(-86 + 9d)))) (Q^2 + s) + (720 + d(328 + d(20 + d(-130 + 27d))))M_0^2) +$$
$$m_\pi^4 ((Q^2 + s) (-4(-2 + d)^3 Q^4 + 4(-2 + d)(-18 + 7d)Q^2 m_\Delta^2 + (80 + d(112 + d(-4 + (2 - 3d)d)))m_\Delta^4) +$$

$$
\begin{aligned}
&16(-1+d)Q^2m_\Delta\left((-2+d)^2Q^2-2(-4+d)(-1+d)m_\Delta^2\right)M_0+\\
&\left(20(-2+d)^3Q^4-4(52+d(8+d(-33+8d)))Q^2m_\Delta^2+(464+d(-368+d(164+d(-82+15d))))m_\Delta^4\right)M_0^2\big)-\\
&16m_\Delta^5M_0\left(-(-2+d)(28+3(-5+d)d)Q^4-2(-50+d(64+d(-23+3d)))Q^2s+8(-4+d)(-1+d)s^2+\right.\\
&2\left((-3+d)(30+d(-14+3d))Q^2-4(8+(-5+d)d)s\right)M_0^2+32M_0^4\big)-\\
&16Q^2m_\Delta^3M_0\left((-2+d)\left(-(-2+d)^2Q^4-(8+d(-9+2d))Q^2s+2(-1+d)s^2\right)+\right.\\
&\left((-2+d)(38+3d(-9+2d))Q^2+2(-22+d(24+(-9+d)d))s\right)M_0^2+4(-4+d)(-2+d)M_0^4\big)+\\
&4(-2+d)Q^2m_\Delta^2M_0^2\left(\left(28-28d+6d^2\right)Q^4+2(24+d(-27+7d))Q^2s+4(-2+d)(-1+d)s^2+\right.\\
&\left(-5(38+3d(-9+2d))Q^2+(-70+(65-16d)d)s\right)M_0^2+(70+d(-29+4d))M_0^4\big)-\\
&2m_\Delta^6\left(-2\left(Q^2+s\right)\left((76+d(-76+13d))Q^2+4(22+d(-22+3d))s\right)+\right.\\
&2\left((388-d(18+(-3+d)d(-5+3d)))Q^2+2(316+d(-173+d(54+d(-20+3d))))s\right)M_0^2+\\
&\left(-640+d(1084+3d(-104+d(6+d))))M_0^4\right)+\\
&m_\Delta^4\left(-4(-2+d)\left(Q^2+s\right)\left((8+(-8+d)d)Q^4+4(-3+d)(-1+d)Q^2s+4(-1+d)^2s^2\right)+\right.\\
&4\left((-368+d(400+17(-8+d)d))Q^4+2(-276+d(335+d(-129+20d)))Q^2s+2\left(-32+d\left(65-33d+6d^2\right)\right)s^2\right)M_0^2+\\
&\left((3408-d(2744+3d(-324+d(42+d))))Q^2+(1008+d(-1024+d(580+3d(-54+5d))))s\right)M_0^4+\\
&\left(-240+d(328+d(-220+(50-3d)d))\right)M_0^6\big)+16(-2+d)^2Q^4m_\Delta M_0^3\left(s+(-2+d)\left(-Q^2+M_0^2\right)\right)+\\
&2m_\pi^2\left(-64(-5+d)dm_\Delta^5M_0\left(-s+M_0^2\right)-4(-2+d)^3Q^4M_0^2\left(-3Q^2-s+5M_0^2\right)-8(-2+d)^2Q^4m_\Delta M_0\left(-(-2+d)Q^2+s+(-3+2d)M_0^2\right)+\right.\\
&16Q^2m_\Delta^3M_0\left((-2+d)(11+d(-9+2d))Q^2+(-3+d)(6+(-4+d)d)s+\left(10+(-5+d)d^2\right)M_0^2\right)+\\
&m_\Delta^6\left(-2(136+d(-278+d(170+d(-38+3d))))\left(Q^2+s\right)+(720+d(12+d(220+3d(-46+7d))))M_0^2\right)+4(-2+d)Q^2m_\Delta^2\\
&\left(-\left(Q^2+s\right)\left((4+(-6+d)d)Q^2+2(-2+d)(-1+d)s\right)+\left((80+d(-56+11d))Q^2+4(11+d(-9+2d))s\right)M_0^2+2(-24+d(3+d))M_0^4\right)-\\
&m_\Delta^4\left(-4\left(Q^2+s\right)\left(2(16+d(-27+5d))Q^2+(20+d(-27+5d))s\right)+\right.\\
&\left((592+d(-364+d(108+(2-3d)d)))Q^2+2(248+d(-136+d(80+d(-28+3d))))s\right)M_0^2+2(64+d(-46+d(10+3(-4+d)d)))M_0^4\big)\big)\big)\\
&(9+5\tau[3])\mathrm{II}[d]\left[\{\{0,m_\pi\},1\},\{\{p+q,m_\Delta\},1\},\{\{p,m_\Delta\},1\}\right]\right)-\frac{1}{384(-1+d)^3F^2m_\Delta^6M_0}\\
&(ie^2\mathrm{g}\pi\mathrm{N}\Delta^2(32\left(-14+d-4d^2+d^3\right)Q^2m_\Delta^9M_0+8(-2+d)^3Q^6m_\Delta M_0\left(m_\pi^2-M_0^2\right)^2+\\
&m_\Delta^{10}\left(-(320+(-8+d)d(68+3(-6+d)d))\left(Q^2+s\right)+(576+d(-304+3d(68+5(-6+d)d)))M_0^2\right)+8(-2+d)^3Q^6\left(-m_\pi^2M_0+M_0^3\right)^2-32m_\Delta^7M_0\\
&\left((-5+d)\left((-2+d)Q^4+4(-1+d)Q^2s+4(-1+d)s^2\right)+2\left(-14+d-4d^2+d^3\right)Q^2m_\pi^2+\right.\\
&2\left((-24+d(13+(-6+d)d))Q^2-4(-5+d)(-1+d)s\right)M_0^2+4(-5+d)(-1+d)M_0^4\big)-\\
&8(-2+d)Q^4m_\Delta^2\left(m_\pi^4\left(Q^2+s-d\left(Q^2+s\right)+(-17+4d)M_0^2\right)+M_0^4\left((-3+2d)\left((-3+d)Q^2+(-5+2d)s\right)+(-31+(19-4d)d)M_0^2\right)+\right.\\
&m_\pi^2M_0^2\left(-2(3+(-5+d)d)Q^2+(-16+(17-4d)d)s+(48+d(-23+4d))M_0^2\right)\big)+\\
&8Q^2m_\Delta^5M_0\left((-2+d)\left((-2+d)^2Q^4+4(-3+d)(-1+d)Q^2s+4(-3+d)(-1+d)s^2\right)+4\left(-14+d-4d^2+d^3\right)m_\pi^4+\right.\\
&4\left(-(-2+d)(13+(-6+d)d)Q^2-2(-1+d)(16+(-7+d)d)s\right)M_0^2+8(-20+d(18+(-7+d)d))M_0^4-\\
&8m_\pi^2\left((-5+d)\left(dQ^2+2(-1+d)s\right)+(-24+d(13+(-6+d)d))M_0^2\right)\big)+\\
&2m_\Delta^8\left(2\left(Q^2+s\right)\left((16+(-16+d)d)Q^2+2(24+(-17+d)d)s\right)+\right.\\
&\left((96+d(-380+d(110+d(-26+3d))))Q^2-2(112+d(-88+d(34+3(-4+d)d)))s\right)M_0^2-\\
&2(64+3d(58+(-6+d)d(2+d)))M_0^4+m_\pi^2\left(-(320+(-8+d)d(68+3(-6+d)d))\left(Q^2+s\right)+(576+d(-304+3d(68+5(-6+d)d)))M_0^2\right)\big)+\\
&m_\Delta^6\left(8(-2+d)(-1+d)Q^2\left(Q^2+s\right)\left(Q^2+2s\right)+8\left((-54+(47-7d)d)Q^4-(140+d(-107+11d))Q^2s+(-80+d(39+d))s^2\right)M_0^2+\right.\\
&\left((1152-d(520+d(-244+d(22+3d))))Q^2+(1664+d(-1248+d(316+15(-6+d)d)))s\right)M_0^4+
\end{aligned}
$$

$$(-768+d(1176+d(-332-3(-14+d)d)))M_0^6+$$
$$m_\pi^4 (-(320+(-8+d)d(68+3(-6+d)d)) (Q^2+s)+(576+d(-304+3d(68+5(-6+d)d)))M_0^2)+$$
$$2m_\pi^2 (4 (Q^2+s) ((16+(-16+d)d)Q^2+(24+(-17+d)d)s)+$$
$$((864+d(-348+d(268+d(-74+3d))))Q^2-2(112+d(-88+d(34+3(-4+d)d)))s) M_0^2-2(64+3d(58+(-6+d)d(2+d)))M_0^4))+$$
$$4Q^2m_\Delta^4 (m_\pi^4 ((16+(-16+d)d) (Q^2+s)+(-64+d(48+d(-49+8d)))M_0^2)+2m_\pi^2 (2(-2+d)(-1+d) (Q^2+s)^2-$$
$$((8+d(-56+11d))Q^2+2(58+5(-9+d)d)s) M_0^2-4(-40+d(29+2(-7+d)d))M_0^4)+$$
$$M_0^2 (2(-2+d) ((6+(-6+d)d)Q^4+(18+d(-19+4d))Q^2s+4(-3+d)(-1+d)s^2)+$$
$$((216+d(-216+(67-8d)d))Q^2+(296+d(-316+(107-16d)d))s) M_0^2+(-296+d(260+d(-107+16d)))M_0^4))-$$
$$16(-2+d)Q^4m_\Delta^3M_0 (-2(-3+d)(-1+d)s (m_\pi-M_0) (m_\pi+M_0)-Q^2 ((2+(-4+d)d)m_\pi^2-(-2+d)^2M_0^2)+$$
$$2 (m_\pi-M_0) (m_\pi+M_0) (8M_0^2+(-5+d) (m_\pi^2+dM_0^2))))$$
$$(9+5\tau[3])\mathrm{II}[d] [\{\{0,m_\pi\},1\},\{\{p+q,m_\Delta\},1\},\{\{p,m_\Delta\},2\}]) - \frac{1}{96(-1+d)^3F^2m_\Delta^6M_0}(ie^2\mathrm{g}\pi\mathrm{N}\Delta^2\pi(-(944+d(-272+d(220+d(-130+21d))))m_\Delta^8 (Q^2+s+M_0^2)+8$$
$$(-2+d)^3Q^4 (m_\pi-M_0) (m_\pi+M_0) (Q^2+m_\pi^2-M_0^2) (Q^2+s+M_0^2)+$$
$$m_\Delta^6 (2 (Q^2+s) ((-272+d(-232+d(126+d(-22+3d))))Q^2+8(-34+d(-7+3d))s)+$$
$$((1440+d(-1760+d(1048+d(-334+45d))))Q^2-(992+d^2(292+d(-206+33d))) s) M_0^2+$$
$$(1984+d(-1296+d(796+d(-290+39d))))M_0^4-(224+d(-704+3d(92+(-14+d)d)))m_\pi^2 (Q^2+s+M_0^2))-$$
$$4(-2+d)Q^2m_\Delta^2 (-2Q^2 (Q^2+s) ((5+(-5+d)d)Q^2+2(-2+d)^2s)+((66+d(-41+10d))Q^4+(68+d(-59+12d))Q^2s+4(-4+d)(-1+d)s^2) M_0^2+$$
$$((52+d(-49+12d))Q^2+2d(-3+2d)s) M_0^4+2(-12+d)M_0^6+4(-1+(-3+d)d)m_\pi^4 (Q^2+s+M_0^2)+$$
$$m_\pi^2 (-(Q^2+s) ((48+d(-37+8d))Q^2+4(-4+d)(-1+d)s)+((-20+(47-12d)d)Q^2+2(2+(9-4d)d)s) M_0^2+(28+2(5-2d)d)M_0^4))+$$
$$m_\Delta^4 (8 (Q^2+s) ((-68+d(73+3(-8+d)d))Q^4+2(-49+d(56+3(-7+d)d))Q^2s-4(-3+d)(-1+d)s^2)+$$
$$((784-d(-80+3d(16+d(2+d))))Q^4-(-288+d(224+d(-40+3(-6+d)d)))Q^2s+8(36+d(-33+5d))s^2) M_0^2+$$
$$((1184-d(304+d(48+d(-46+15d))))Q^2+4 (124+d (-80+d (86-39d+6d^2))) s) M_0^4-$$
$$4(36+d(-50+d(48+d(-19+3d))))M_0^6+2(136+d(-64+d(40+d(-20+3d))))m_\pi^4 (Q^2+s+M_0^2)+$$
$$m_\pi^2 ((Q^2+s) ((-96+d(-152+d(80+3(-6+d)d)))Q^2+8d(-7+3d)s)+((-128+3d(-96+d(64+3(-6+d)d)))$$
$$Q^2-2(528+d(-352+d(228+d(-98+15d))))s) M_0^2+2(-16+d(-68+d(56+3(-6+d)d)))M_0^4)))$$
$$(9+5\tau[3])\mathrm{II}[2+d] [\{\{0,m_\pi\},1\},\{\{p,m_\Delta\},1\},\{\{p+q,m_\Delta\},2\}]) - \frac{1}{96(-1+d)^3F^2m_\Delta^6M_0}(ie^2\mathrm{g}\pi\mathrm{N}\Delta^2\pi(-64(-6+d)(-1+d)m_\Delta^7M_0+16(-2+d)^3$$
$$Q^2 (m_\pi-M_0) (m_\pi+M_0) (Q^2+m_\pi^2-M_0^2) (-Q^2-s+M_0^2)-32m_\Delta^5M_0 ((86+d(-44+7d))Q^2+2(29+d(-15+2d))s-4(-4+d)(-1+d)m_\pi^2+2(-17+d)M_0^2)+$$
$$2m_\Delta^6 (-(232+d(340+d(-468+d(182+3(-11+d)d)))) (Q^2+s)+(-712+d(1196+d(-392+d(258+d(-109+15d)))))M_0^2)+$$
$$32(-2+d)^2Q^2m_\Delta M_0 (-Q^2 (Q^2+s)+3m_\pi^4+2sM_0^2+M_0^4+m_\pi^2 (Q^2-2s-4M_0^2))-32m_\Delta^3M_0 ((36+d(-30+7d))Q^4+2(14+3(-4+d)d)Q^2s+2(-2+d)(-1+d)s^2+$$
$$2(-2+d)(-1+d)m_\pi^4-2 ((13+(-10+d)d)Q^2+(-1+d)s) M_0^2+2(-1+d)M_0^4-$$
$$2m_\pi^2 ((13+2(-4+d)d)Q^2+(-1+d)(-5+2d)s+(-1+d)M_0^2))+$$
$$m_\Delta^4 (-8 (Q^2+s) (2 (19+d (-10+d+d^2)) Q^2+(-1+d)(64+d(-45+11d))s)+$$
$$((-3424+d(3048+d(-980+d(142+3(-7+d)d))))Q^2+(-1904+d(2192+d(-1028+3d(82+(-7+d)d))))s) M_0^2+$$
$$(1552+d(-1608+d(868+d(-398+(113-15d)d))))M_0^4+$$
$$m_\pi^2 (-(-688+d(272+d(-164+d(86+3(-7+d)d)))) (Q^2+s)+(-848+d(560+d(-452+d(326+d(-113+15d)))))M_0^2))+$$
$$4(-2+d)m_\Delta^2 (-2Q^2 (Q^2+s) ((-1+(-2+d)d)Q^2+4(-2+d)(-1+d)s)+$$
$$((172+d(-129+25d))Q^4+(118+d(-131+37d))Q^2s+8(-2+d)(-1+d)s^2) M_0^2+$$

$$\left((-238+(139-21d)d)Q^2-8(-2+d)(-1+d)s\right)M_0^4+8(-2+d)(-1+d)m_\pi^4\left(Q^2+s-M_0^2\right)+$$
$$m_\pi^2\left(-\left(Q^2+s\right)\left((86+d(-59+13d))Q^2+8(-2+d)(-1+d)s\right)+(222+d(-91+5d))Q^2M_0^2+8(-2+d)(-1+d)M_0^4\right)\right))$$
$$(9+5\tau[3])\mathrm{II}[2+d]\left[\left\{\{0,m_\pi\},1\},\{\{p+q,m_\Delta\},1\},\{\{p,m_\Delta\},1\}\right]\right)-\frac{1}{96(-1+d)^3F^2m_\Delta^6M_0}(ie^2\mathrm{g}\pi\mathrm{N}\Delta^2\pi(32(-2+d)^2Q^4m_\Delta M_0^3\left(Q^2+s+M_0^2\right)+8(-2+d)^3Q^4M_0^4\left(-3Q^2-s+3M_0^2\right)$$
$$+128m_\Delta^7M_0\left(Q^2-5s-d\left((-4+d)Q^2+(-6+d)s\right)+(-5+d)(-1+d)M_0^2\right)+$$
$$m_\Delta^8\left(-(-1+d)(256+d(-640+d(244+3(-14+d)d)))\left(Q^2+s\right)+(-2400+d(1456+d(-1076+d(586+3d(-49+5d)))))M_0^2\right)+$$
$$+4m_\pi^4\left(2\left(Q^2+s\right)\left(-(-2+d)^3Q^4+4(-3+d)(-2+d)Q^2m_\Delta^2-4\left(2+d-4d^2+d^3\right)m_\Delta^4\right)+16Q^2m_\Delta\left((-2+d)^2Q^2-4(-3+d)m_\Delta^2\right)M_0+\right.$$
$$\left(6(-2+d)^3Q^4-8\left(8+(-4+d)d^2\right)Q^2m_\Delta^2+(152+d(-56+d(8+3(-4+d)d)))m_\Delta^4\right)M_0^2\right)-$$
$$32Q^2m_\Delta^3M_0\left((-2+d)\left(Q^2+s\right)\left(dQ^2+2(-1+d)s\right)+\left(-(44+5(-6+d)d)Q^2-2(8+(-5+d)d)s\right)M_0^2+4(-3+d)M_0^4\right)+8(-2+d)Q^2m_\Delta^2M_0^2$$
$$\left((7+d(-11+3d))Q^4+(21+d(-29+9d))Q^2s+4(-2+d)(-1+d)s^2-\right.$$
$$\left.\left((143+2d(-44+9d))Q^2+(46+3d(-15+4d))s\right)M_0^2+(42+d(-21+4d))M_0^4\right)+$$
$$m_\Delta^6\left(4\left(Q^2+s\right)\left((-48+d(88+(-25+d)d))Q^2+2(-1+d)(64+(-25+d)d)s\right)+\right.$$
$$\left((-768+d(824+3d(-288+d(134+(-23+d)d))))Q^2+(-2944+d(1392+d(-876+d(422+3(-27+d)d))))s\right)M_0^2+$$
$$\left.(4160+d(-3528+d(1388+d(-554+3(49-5d)d))))M_0^4\right)-$$
$$2m_\Delta^4\left(4(-2+d)\left(Q^2+s\right)\left((-1+2d)Q^4+2\left(-1+d^2\right)Q^2s+4(-1+d)^2s^2\right)+\right.$$
$$2\left((148+d(-246+(101-17d)d))Q^4-(-512+d(688+3d(-97+17d)))Q^2s-8(-16+d(29+3(-5+d)d))s^2\right)M_0^2+$$
$$\left(-2(1528+d(-896+d(277+3(-17+d)d)))Q^2+(-640+d(872+d(-476+(122-9d)d)))s\right)M_0^4+$$
$$\left.(144+d(-360+d(220+d(-50+3d))))M_0^6\right)+$$
$$m_\pi^2\left(-16(-2+d)^3Q^4M_0^2\left(-2Q^2-s+3M_0^2\right)-32(-2+d)^2Q^4m_\Delta M_0\left(Q^2+s+3M_0^2\right)-128m_\Delta^5M_0\left(7Q^2-5s-d\left((-2+d)Q^2+(-6+d)s\right)+(-5+d)(-1+d)M_0^2\right)+\right.$$
$$m_\Delta^6\left(-(-1+d)(448+d(-352+d(148+3(-14+d)d)))\left(Q^2+s\right)+(-1152+d(2224+d(-804+d(314+3d(-37+5d)))))M_0^2\right)+$$
$$8(-2+d)Q^2m_\Delta^2\left(-\left(Q^2+s\right)\left((-1+(-2+d)d)Q^2+4(-2+d)(-1+d)s\right)+\left(7(19+d(-11+2d))Q^2+(58+d(-49+12d))s\right)M_0^2+(-58+13d)M_0^4\right)-$$
$$2m_\Delta^4\left(-2\left(Q^2+s\right)\left((-32+(-16+d)(-1+d)d)Q^2+16(-5+d)(-1+d)s\right)+\right.$$
$$\left.\left(2(896+d(-300+d(57+d(-19+3d))))Q^2+(512+d(-584+d(316+9(-10+d)d)))s\right)M_0^2+(192+d(136+d(-124+d(10+3d))))M_0^4\right)-$$
$$64Q^2m_\Delta^3M_0\left(-(-4+(-1+d)d)s+(-4+d)\left((-3+d)Q^2+(-5+d)M_0^2\right)\right)+$$
$$\left.64m_\Delta^5M_0\left(-2(-5+d)(-1+d)s^2+Q^2\left((-2+(9-2d)d)Q^2+(62+3(-7+d)d)M_0^2\right)+s\left(-16Q^2+(-5+d)\left(-5dQ^2+2(-1+d)M_0^2\right)\right)\right)\right)(9+$$
$$5\tau[3])\mathrm{II}[2+d]\left[\{\{0,m_\pi\},1\},\{\{p+q,m_\Delta\},1\},\{\{p,m_\Delta\},2\}\right]\right)+\frac{1}{24(-1+d)^3F^2m_\Delta^6M_0}(ie^2\mathrm{g}\pi\mathrm{N}\Delta^2\pi M_0^2(2(64+d(60+d(-2+3(-4+d)d)))m_\Delta^{10}-4(-2+d)^3Q^6\left(m_\pi^2-M_0^2\right)^2+$$
$$m_\Delta^8\left(2(240+d(76+d(14+d(-20+3d))))Q^2+(544+3d(-88+d(48+d(-22+3d))))s+(-544+3d(88+d(-48+(22-3d)d)))M_0^2\right)+$$
$$4(-2+d)Q^4m_\Delta^2\left((-16+3d)m_\pi^4+M_0^2\left((12+d(-9+2d))Q^2+(-2+d)(-7+4d)s-2(15+d(-9+2d))M_0^2\right)+\right.$$
$$\left.m_\pi^2\left((-8+(9-2d)d)Q^2-(-2+d)(-7+4d)s+(46+d(-21+4d))M_0^2\right)\right)-$$
$$4Q^2m_\Delta^4\left((-2+d)\left((8+(-5+d)d)Q^4+(16+d(-17+4d))Q^2s+2(-1+d)(-5+2d)s^2\right)+4(-6+d(4+(-6+d)d))m_\pi^4+\right.$$
$$\left((144+d(-94+(25-4d)d))Q^2+(132+d(-138+(49-8d)d))s\right)M_0^2+(-136+d(116+d(-51+8d)))M_0^4+$$
$$\left.m_\pi^2\left(4(-8+7d)Q^2+(-92+(62-5d)d)s+(140+d(-94+(53-8d)d))M_0^2\right)\right)-$$
$$m_\Delta^6\left(4(10+d)(-8+3d)Q^4+4(-124+d(34+7d))Q^2s+8(-28+d(11+d))s^2+2(64+d(60+d(-2+3(-4+d)d)))m_\pi^4+\right.$$
$$\left(2(536+d(-24+d(48+d(-28+3d))))Q^2+\left(800+d\left(-760+d\left(256-66d+9d^2\right)\right)\right)s\right)M_0^2+(-448+d(792+d(-268-3(-14+d)d)))M_0^4-$$
$$\left.m_\pi^2\left(2(-160+d(76+d(-66+d(4+3d))))Q^2+\left(352+d\left(-584+d\left(272-66d+9d^2\right)\right)\right)s+(-96+d(824+d(-280+3d(6+d))))M_0^2\right)\right)\right)$$
$$(9+5\tau[3])\mathrm{II}[2+d]\left[\{\{0,m_\pi\},1\},\{\{p+q,m_\Delta\},1\},\{\{p,m_\Delta\},3\}\right]\right)-\frac{1}{96(-1+d)^3F^2m_\Delta^6M_0}(ie^2\mathrm{g}\pi\mathrm{N}\Delta^2\pi(-2(64+d(60+d(-2+3(-4+d)d)))m_\Delta^{10}\left(Q^2+s+M_0^2\right)+$$
$$2m_\pi^4\left(2(-2+d)^3Q^6-2(-2+d)(-16+3d)Q^4m_\Delta^2+8(-6+d(4+(-6+d)d))Q^2m_\Delta^4+(64+d(60+d(-2+3(-4+d)d)))m_\Delta^6\right)\left(Q^2+s+M_0^2\right)+4(-2+d)^3Q^6M_0^4$$

$$(Q^2+s+M_0^2) +$$
$$4(-2+d)Q^4 m_\Delta^2 M_0^2 (-2 (Q^2+s) ((5+(-5+d)d)Q^2+2(-3+d)(-1+d)s) + ((22+d(-7+2d))Q^2+12s-5ds) M_0^2 + (32+d(-17+4d))M_0^4) + m_\Delta^8$$
$$((Q^2+s) ((-32+d(-576+d(180+d(-26+3d))))Q^2+32(-3+d)(1+2d)s) +$$
$$((960+d(-1264+d(532+d(-158+21d))))Q^2-2(-8+3d)(-56+d(32+d(-14+3d)))s) M_0^2 + 2 (496+d (-344+d (176-66d+9d^2))) M_0^4) +$$
$$4Q^2 m_\Delta^4 ((-2+d) (Q^2+s) ((6+(-6+d)d)Q^4+4(-4+d)(-1+d)Q^2 s+4(-3+d)(-1+d)s^2) +$$
$$((124+d(-110-3(-9+d)d))Q^4-(-204+d(238+d(-69+8d)))Q^2 s-2(-1+d)(54+d(-19+2d))s^2) M_0^2 +$$
$$((-12+d(-46+d(-5+4d)))Q^2+2(8-13(-2+d)d)s) M_0^4 + 2(-74+d(41+4(-5+d)d))M_0^6) +$$
$$m_\Delta^6 (-8 (Q^2+s) ((32+d(-25+3d))Q^4+2(33+d(-34+5d))Q^2 s+8(-5+d)(-1+d)s^2) +$$
$$((528+d(392+d(-100+(10-3d)d)))Q^4-(-352+d(48+d(-36+d(-10+3d))))Q^2 s+16(26+d(-71+17d))s^2) M_0^2 +$$
$$((-16+d(1112+d(-480+(118-15d)d)))Q^2+4 (240+d (18+d (47-39d+6d^2))) s) M_0^4 -$$
$$4(200+d(-230+d(101+3(-9+d)d)))M_0^6) + m_\pi^2 (-8(-2+d)^3 Q^6 M_0^2 (Q^2+s+M_0^2) -$$
$$8(-2+d)Q^4 m_\Delta^2 (-(Q^2+s) ((3+(-5+d)d)Q^2+2(-3+d)(-1+d)s) + ((21+(-5+d)d)Q^2+14s-4ds) M_0^2 + 2(12+(-5+d)d)M_0^4) -$$
$$4Q^2 m_\Delta^4 ((Q^2+s) ((16+d(-56+9d))Q^2+2(-1+d)(-38+7d)s) +$$
$$((-140+d(10+d(-35+8d)))Q^2+2(-8+d(44+d(-33+4d)))s) M_0^2 + 2 (-78+d (33-22d+4d^2)) M_0^4) +$$
$$m_\Delta^6 ((Q^2+s) ((768+d(-576+d(340+d(-74+3d))))Q^2-32(-3+d)(1+2d)s) + ((1312+d(-1824+d(828+d(-158+9d))))$$
$$Q^2-2(576+d(-304+3d(68+5(-6+d)d)))s) M_0^2 + 2(272+d(-624+d(244+3(-14+d)d)))M_0^4)))$$
$$(9+5\tau[3])\mathrm{II}[2+d] [\{\{0,m_\pi\},1\},\{\{p+q,m_\Delta\},2\},\{\{p,m_\Delta\},2\}]) - \frac{1}{12(-1+d)^3 F^2 m_\Delta^4 M_0} (ie^2 \mathrm{g}\pi\mathrm{N}\Delta^2 \pi^2 ((Q^2+s)^2 + 2 (Q^2-s) M_0^2 + M_0^4) (2(304+d(-176+d(142+d(-62+9d))))m_\Delta^4 +$$
$$m_\Delta^2 (-8(-8+d(-19+7d))Q^2-16(6+(-6+d)d)s+(256+d(-224+d(148+d(-58+9d))))m_\pi^2+(-160+d(128+d(-132+(58-9d)d)))M_0^2) -$$
$$4(-2+d)Q^2 ((2+d)Q^2+(2+3d)(m_\pi-M_0)(m_\pi+M_0))) (9+5\tau[3])\mathrm{II}[4+d][\{\{0,m_\pi\},1\},\{\{p,m_\Delta\},1\},\{\{p+q,m_\Delta\},3\}]) + \frac{1}{12(-1+d)^3 F^2 m_\Delta^4 M_0} (ie^2 \mathrm{g}\pi\mathrm{N}\Delta^2 \pi^2 ((Q^2+s)^2 +$$
$$2 (Q^2-s) M_0^2 + M_0^4) (-(-8+3d)(-56+d(32+d(-14+3d)))m_\Delta^6 + 4 (-4+d^2) Q^4 (m_\pi-M_0)(m_\pi+M_0) +$$
$$4Q^2 m_\Delta^2 ((-2+d) ((2+d)Q^2+2(-1+d)s) + (-16+d(-28+9d))m_\pi^2 + (12+(34-11d)d)M_0^2) - m_\Delta^4$$
$$(64Q^2-96s+4d ((28-9d)Q^2+2(37-9d)s) + (-8+3d)(-56+d(32+d(-14+3d)))m_\pi^2 + (-352+d (128+d (-136+66d-9d^2))) M_0^2))$$
$$(9+5\tau[3])\mathrm{II}[4+d] [\{\{0,m_\pi\},1\},\{\{p,m_\Delta\},2\},\{\{p+q,m_\Delta\},3\}]) + \frac{1}{24(-1+d)^3 F^2 m_\Delta^4 M_0} (ie^2 \mathrm{g}\pi\mathrm{N}\Delta^2 \pi^2 ((Q^2+s)^2 + 2 (Q^2-s) M_0^2 + M_0^4)$$
$$(-2(56+d(-252+d(136+3(-10+d)d)))m_\Delta^4 +$$
$$m_\Delta^2 (16(-1+d)(-7+2d)Q^2+8(-3+d)ds+(16+d(96+d(-68-3(-6+d)d)))m_\pi^2+(-16+3(-2+d)d(12+(-4+d)d))M_0^2) +$$
$$4(-2+d)Q^2 (2(-1+d)Q^2+(-2+3d)(m_\pi-M_0)(m_\pi+M_0))) (9+$$
$$5\tau[3])\mathrm{II}[4+d][\{\{0,m_\pi\},1\},\{\{p+q,m_\Delta\},2\},\{\{p,m_\Delta\},2\}]) + \frac{1}{12(-1+d)^3 F^2 m_\Delta^4 M_0} (ie^2 \mathrm{g}\pi\mathrm{N}\Delta^2 \pi^2 ((Q^2+s)^2 + 2 (Q^2-s) M_0^2 + M_0^4)$$
$$(-(320+(-8+d)d(68+3(-6+d)d))m_\Delta^6 + 8(-2+d)(-1+d)Q^4 (m_\pi-M_0)(m_\pi+M_0) + 4Q^2 m_\Delta^2$$
$$(2(-2+d)(-1+d)(Q^2+2s) + (16+(-16+d)d)m_\pi^2 + (-24+(28-5d)d)M_0^2) -$$
$$m_\Delta^4 (-4(16+(-16+d)d)Q^2-8(24+(-17+d)d)s+(320+(-8+d)d(68+3(-6+d)d))m_\pi^2+(-128-3d(-136+d(68+(-14+d)d)))M_0^2)) (9+5\tau[3])$$
$$\mathrm{II}[4+d][\{\{0,m_\pi\},1\},\{\{p+q,m_\Delta\},2\},\{\{p,m_\Delta\},3\}])$$

$$\mathbf{B}(\nu,Q^2) = \frac{1}{384(-1+d)^4F^2m_\Delta^6M_0}\left(ie^2 g\pi N\Delta^2\left(-\frac{4(-2+d)^3\left(s+m_\pi^2-m_\Delta^2\right)\left(4sm_\pi^2+d\left(m_\pi^4+\left(s-m_\Delta^2\right)^2-2m_\pi^2\left(3s+m_\Delta^2\right)\right)\right)M_0^2}{d(1+d)s^2}+64(-2+d)^2(-1+d)m_\Delta^2M_0\left(m_\Delta+M_0\right)-\right.\right.$$
$$\frac{2(-2+d)^3\left(s+m_\pi^2-m_\Delta^2\right)\left(8sm_\pi^2+2d\left(m_\pi^4+\left(s-m_\Delta^2\right)^2-2m_\pi^2\left(s+m_\Delta^2\right)\right)+d^2\left(m_\pi^4+\left(s-m_\Delta^2\right)^2-2m_\pi^2\left(s+m_\Delta^2\right)\right)\right)M_0^2\left(-Q^2-3s+M_0^2\right)}{d(1+d)s^3}-$$
$$32(-2+d)^2(-1+d)m_\Delta^2\left(m_\pi^2-m_\Delta^2+M_0^2\right)-\frac{32(-2+d)(-1+d)M_0\left(2dm_\Delta M_0\left(-m_\Delta+(-2+d)M_0\right)+(-2+d)m_\pi^2\left((2-5d)m_\Delta+(-2+d)M_0\right)\right)}{d}+$$
$$\frac{(-2+d)^3(2+d)\left(s+m_\pi^2-m_\Delta^2\right)\left(16sm_\pi^2+d^2\left(s+m_\pi^2-m_\Delta^2\right)^2+4d\left(s^2-3sm_\pi^2+m_\pi^4-2\left(s+m_\pi^2\right)m_\Delta^2+m_\Delta^4\right)\right)M_0^2\left(-s^2+\left(Q^2+M_0^2\right)^2\right)}{d(1+d)s^4}+$$
$$\frac{16(-2+d)\left(m_\pi^2-m_\Delta^2+M_0^2\right)\left(\left(-6+d+d^2\right)m_\Delta^3+\left(-10+d^2\right)m_\Delta^2M_0+\left(-6+d+d^2\right)m_\Delta\left(-m_\pi^2+M_0^2\right)+(-2+d)^2M_0\left(-m_\pi^2+M_0^2\right)\right)}{M_0}-$$
$$\frac{8(-2+d)^3(-1+d)\left(-m_\pi^2+m_\Delta^2+M_0^2\right)\left(-\frac{4m_\pi^2M_0^2}{d}+\frac{\left(m_\pi^2-m_\Delta^2+M_0^2\right)^2}{-1+d}\right)}{M_0^2}+\frac{1}{ds^2}\left(4\left(4sm_\pi^2+d\left(m_\pi^4+\left(s-m_\Delta^2\right)^2-2m_\pi^2\left(s+m_\Delta^2\right)\right)\right)M_0\left(-4(10+d(-14+5d))m_\Delta^3-\right.\right.$$
$$(-2+d)(40+d(-28+5d))m_\Delta^2M_0-2(-2+d)^2m_\Delta\left((-1+d)Q^2+(-9+d)s+2m_\pi^2+M_0^2\right)+(-2+d)^3M_0\left(-2Q^2-3s-3m_\pi^2+2M_0^2\right)\Big)\Big)-$$
$$\frac{1}{s}\left(8\left(s+m_\pi^2-m_\Delta^2\right)M_0\left(-(36+d(2+d)(-10+3d))m_\Delta^3-(-2+d)(4+d(-4+3d))m_\Delta^2M_0+(-2+d)^3M_0\left(-Q^2-3m_\pi^2+M_0^2\right)+\right.\right.$$
$$(-2+d)^2m_\Delta\left(-dQ^2+6s+(16-3d)m_\pi^2+(-10+d)M_0^2\right)\Big)\Big)-$$
$$\frac{4d\left(m_\pi^2-m_\Delta^2+M_0^2\right)\left((-2+d)^3m_\Delta^4+(-2+d)^3\left(m_\pi^2-M_0^2\right)^2-2(-2+d)m_\Delta^2\left((-2+d)^2m_\pi^2-(2+(-4+d)d)M_0^2\right)\right)}{M_0^2}+$$
$$\frac{1}{M_0}\left(8(-2+d)(-1+d)\left(\left(-4+d^2\right)m_\Delta^5+\left(-16+d^2\right)m_\Delta^4M_0+\left(-4+d^2\right)m_\Delta\left(m_\pi^2-M_0^2\right)^2+(-2+d)^2M_0\left(m_\pi^2-M_0^2\right)^2+\right.\right.$$
$$2m_\Delta^2M_0\left((6-(-2+d)d)m_\pi^2+(-4+d)(2+d)M_0^2\right)+2m_\Delta^3\left(-\left(-4+d^2\right)m_\pi^2+\left(-14+d^2\right)M_0^2\right)\Big)\Big)-$$
$$\frac{1}{ds^3}\left(2\left(8sm_\pi^2+d^2\left(s+m_\pi^2-m_\Delta^2\right)^2+2d\left(m_\pi^4+\left(s-m_\Delta^2\right)^2-2m_\pi^2\left(s+m_\Delta^2\right)\right)\right)M_0\left((-2+d)^3M_0\left(Q^2-s+M_0^2\right)\left(3Q^2+2s+3m_\pi^2+M_0^2\right)+\right.\right.$$
$$(-2+d)m_\Delta^2M_0\left(-(32+d(-24+5d))s+(-4+d)^2\left(Q^2+M_0^2\right)\right)+(-2+d)^2m_\Delta\left(Q^2-s+M_0^2\right)\left((-8+d)s+2m_\pi^2+(-2+d)\left(Q^2+M_0^2\right)\right)-$$
$$2m_\Delta^3\left((32+d(-36+11d))s+(-6+d)(-2+d)\left(Q^2+M_0^2\right)\right)\Big)\Big)-$$
$$\frac{1}{s}\left(8(-1+d)M_0\left((56+d(24+d(-24+7d)))m_\Delta^5+(-112+d(116+d(-46+7d)))m_\Delta^4M_0-(-2+d)m_\Delta^2M_0\right.\right.$$
$$\left(-36Q^2+20s-d\left(7(-4+d)Q^2+ds\right)+4(7+(-3+d)d)m_\pi^2+(-2+d)^2M_0^2\right)-m_\Delta^3\left(-(16+d(20+d(-26+7d)))Q^2+44s-d(48+(-20+d)d)s+\right.$$
$$4(14+d(-2+(-2+d)d))m_\pi^2+\left(-4+(-2+d)^2d\right)M_0^2\Big)+(-2+d)^3M_0\left(Q^2\left(Q^2-s\right)+m_\pi^4+\left(-2Q^2+s\right)M_0^2+m_\pi^2\left(5Q^2-3s+M_0^2\right)\right)+$$
$$(-2+d)^2m_\Delta\left(\left(dQ^2-4s\right)\left(Q^2-s\right)+(-4+d)m_\pi^4+\left(-2(-1+d)Q^2+(-8+d)s\right)M_0^2+m_\pi^2\left(-14Q^2+5dQ^2+16s-3ds+(-8+d)M_0^2\right)\right)\Big)\Big)+$$
$$\frac{1}{s^2}\left(4d\left(s+m_\pi^2-m_\Delta^2\right)M_0\left(2(-2+d(-12+5d))m_\Delta^5+(-104+d(104+7(-6+d)d))m_\Delta^4M_0-4(-2+d)m_\Delta^2M_0\right.\right.$$
$$\left(-2(7+(-5+d)d)Q^2+(14+(-6+d)d)s+(-2+(-2+d)d)m_\pi^2+M_0^2\right)+(-2+d)^3M_0\left(\left(Q^2-s\right)\left(3Q^2+s\right)+m_\pi^4+2sM_0^2+m_\pi^2\left(8Q^2-6s+4M_0^2\right)\right)+$$
$$(-2+d)^2m_\Delta\left(-(-12+d)s^2+2m_\pi^4+2(-2+d)Q^2\left(Q^2+M_0^2\right)+s\left(-(12+d)Q^2+(-10+d)M_0^2\right)+m_\pi^2\left(-4Q^2+3dQ^2+6s-3ds+(-10+3d)M_0^2\right)\right)+$$
$$m_\Delta^3\left(-(100+d(-72+5d(2+d)))s-4(13+3(-4+d)d)m_\pi^2+(-2+d)\left((20+(-6+d)d)Q^2+\left(-8+d^2\right)M_0^2\right)\right)\Big)\Big)\Big)$$
$$(9+5\tau[3])\Pi[d]\left[\{\{0,m_\pi\},1\}\right])+\frac{1}{96(-1+d)^4F^2m_\Delta^6M_0^3}\left(i(-2+d)e^2 g\pi N\Delta^2\left(m_\pi^2-\left(m_\Delta+M_0\right)^2\right)\right.$$
$$\left(-(-2+d)^3m_\Delta^6-6(-4+d)(-2+d)m_\Delta^5M_0+(-2+d)^2\left(m_\pi-M_0\right)^2\left(m_\pi+M_0\right)^2\left((-2+d)m_\pi^2+dM_0^2\right)+\right.$$
$$2(-2+d)m_\Delta M_0\left(-m_\pi^2+M_0^2\right)\left(3(-4+d)m_\pi^2+(8+d)M_0^2\right)-4m_\Delta^3M_0\left(-3(-4+d)(-2+d)m_\pi^2+\left(-10+d^2\right)M_0^2\right)+$$
$$m_\Delta^4\left(3(-2+d)^3m_\pi^2-\left(16-16d+d^3\right)M_0^2\right)-m_\Delta^2\left(3(-2+d)^3m_\pi^4-2d(2+(-4+d)d)m_\pi^2M_0^2-(48+d(-56+d(6+d)))M_0^4\right)\Big)$$
$$(9+5\tau[3])\Pi[d]\left[\{\{0,m_\pi\},1\},\{\{p,m_\Delta\},1\}\right])+\frac{1}{384(-1+d)^4F^2m_\Delta^6M_0}\left(ie^2 g\pi N\Delta^2M_0\left(\frac{4(-2+d)^3\left(m_\pi^4+\left(s-m_\Delta^2\right)^2-2m_\pi^2\left(s+m_\Delta^2\right)\right)^2M_0}{(1+d)s^2}+\right.\right.$$

$$\frac{2(-2+d)^3\left(m_\pi^4+\left(s-m_\Delta^2\right)^2-2m_\pi^2\left(s+m_\Delta^2\right)\right)\left(\left(s+m_\pi^2-m_\Delta^2\right)^2+\frac{m_\pi^4+\left(s-m_\Delta^2\right)^2-2m_\pi^2\left(s+m_\Delta^2\right)}{1+d}\right)M_0\left(-Q^2-3s+M_0^2\right)}{s^3}-$$

$$\frac{(-2+d)^3(-1+d)\left(\frac{3\left(m_\pi^4+\left(s-m_\Delta^2\right)^2-2m_\pi^2\left(s+m_\Delta^2\right)\right)^2}{-1+d^2}+\left(s+m_\pi^2-m_\Delta^2\right)^2\left(\left(s+m_\pi^2-m_\Delta^2\right)^2+\frac{6\left(m_\pi^4+\left(s-m_\Delta^2\right)^2-2m_\pi^2\left(s+m_\Delta^2\right)\right)}{-1+d}\right)\right)M_0\left(-s^2+\left(Q^2+M_0^2\right)^2\right)}{s^4}+$$

$$\frac{1}{s^2}\left(4\left(s+m_\pi^2-m_\Delta^2\right)\left(m_\pi^4+\left(s-m_\Delta^2\right)^2-2m_\pi^2\left(s+m_\Delta^2\right)\right)(4(10+d(-14+5d))m_\Delta^3+(-2+d)(40+d(-28+5d))m_\Delta^2M_0+\right.$$

$$(-2+d)^3M_0\left(2Q^2+3s+3m_\pi^2-2M_0^2\right)+2(-2+d)^2m_\Delta\left((-1+d)Q^2+(-9+d)s+2m_\pi^2+M_0^2\right)))+$$

$$\frac{1}{s}\left(8\left(m_\pi^4+\left(s-m_\Delta^2\right)^2-2m_\pi^2\left(s+m_\Delta^2\right)\right)\left(-(36+d(2+d)(-10+3d))m_\Delta^3-(-2+d)(4+d(-4+3d))m_\Delta^2M_0+(-2+d)^3M_0\left(-Q^2-3m_\pi^2+M_0^2\right)+\right.\right.$$

$$(-2+d)^2m_\Delta\left(-dQ^2+6s+(16-3d)m_\pi^2+(-10+d)M_0^2\right)))+\frac{1}{s^3}\left(2\left(s+m_\pi^2-m_\Delta^2\right)\left((2+d)m_\pi^4+(2+d)\left(s-m_\Delta^2\right)^2+2m_\pi^2\left((-4+d)s-(2+d)m_\Delta^2\right)\right)\right.$$

$$\left((-2+d)^3M_0\left(Q^2-s+M_0^2\right)\left(3Q^2+2s+3m_\pi^2+M_0^2\right)+(-2+d)m_\Delta^2M_0\left(-(32+d(-24+5d))s+(-4+d)^2\left(Q^2+M_0^2\right)\right)+\right.$$

$$(-2+d)^2m_\Delta\left(Q^2-s+M_0^2\right)\left((-8+d)s+2m_\pi^2+(-2+d)\left(Q^2+M_0^2\right)\right)-2m_\Delta^3\left((32+d(-36+11d))s+(-6+d)(-2+d)\left(Q^2+M_0^2\right)\right)))+$$

$$32(-1+d)m_\Delta\left((-10+(-2+d)d)m_\Delta^4+4(-6+d)m_\Delta^3M_0+2(-2+d)m_\Delta M_0\left(Q^2+6m_\pi^2-2M_0^2\right)+\right.$$

$$m_\Delta^2\left((-2+d)d\left(Q^2+s\right)+6(5+(-3+d)d)m_\pi^2+(-10+(8-3d)d)M_0^2\right)+(-2+d)^2\left(m_\pi^4+\left(-Q^2+s\right)M_0^2+m_\pi^2\left(3Q^2-3s+M_0^2\right)\right))+$$

$$\frac{1}{s}\left(8(-1+d)\left(s+m_\pi^2-m_\Delta^2\right)\left((56+d(24+d(-24+7d)))m_\Delta^5+(-112+d(116+d(-46+7d)))m_\Delta^4M_0-(-2+d)m_\Delta^2M_0\right.\right.$$

$$\left(-36Q^2+20s-d\left(7(-4+d)Q^2+ds\right)+4(7+(-3+d)d)m_\pi^2+(-2+d)^2M_0^2\right)-m_\Delta^3\left(-(16+d(20+d(-26+7d)))Q^2+44s-d(48+(-20+d)d)s+\right.$$

$$4(14+d(-2+(-2+d)d))m_\pi^2+\left(-4+(-2+d)^2d\right)M_0^2\right)+(-2+d)^3M_0\left(Q^2\left(Q^2-s\right)+m_\pi^4+\left(-2Q^2+s\right)M_0^2+m_\pi^2\left(5Q^2-3s+M_0^2\right)\right)+$$

$$(-2+d)^2m_\Delta\left(\left(dQ^2-4s\right)\left(Q^2-s\right)+(-4+d)m_\pi^4+\left(-2(-1+d)Q^2+(-8+d)s\right)M_0^2+m_\pi^2\left(-14Q^2+5dQ^2+16s-3ds+(-8+d)M_0^2\right)\right)))-$$

$$\frac{1}{s^2}\left(4\left(-4sm_\pi^2+d\left(s+m_\pi^2-m_\Delta^2\right)^2\right)\left(2(-2+d(-12+5d))m_\Delta^5+(-104+d(104+7(-6+d)d))m_\Delta^4M_0-4(-2+d)m_\Delta^2M_0\right.\right.$$

$$\left(-2(7+(-5+d)d)Q^2+(14+(-6+d)d)s+(-2+(-2+d)d)m_\pi^2+M_0^2\right)+(-2+d)^3M_0\left(\left(Q^2-s\right)\left(3Q^2+s\right)+m_\pi^4+2sM_0^2+m_\pi^2\left(8Q^2-6s+4M_0^2\right)\right)+$$

$$(-2+d)^2m_\Delta\left(-(-12+d)s^2+2m_\pi^4+2(-2+d)Q^2\left(Q^2+M_0^2\right)+s\left(-(12+d)Q^2+(-10+d)M_0^2\right)+m_\pi^2\left(-4Q^2+3dQ^2+6s-3ds+(-10+3d)M_0^2\right)\right)+$$

$$m_\Delta^3\left(-(100+d(-72+5d(2+d)))s-4(13+3(-4+d)d)m_\pi^2+(-2+d)\left((20+(-6+d)d)Q^2+\left(-8+d^2\right)M_0^2\right)\right))))$$

$$(9+5\tau[3])\mathrm{II}[d]\left[\left\{\{0,m_\pi\},1\right\},\left\{\{p+q,m_\Delta\},1\right\}\right])+\frac{1}{12(-1+d)^3F^2m_\Delta^5M_0}(ie^2\mathrm{g}\pi\mathrm{N}\Delta^2M_0\left(m_\pi^4\left((-2+d)^2Q^2+2m_\Delta\left(m_\Delta+dm_\Delta+2(-2+d)M_0\right)\right)+\left(-m_\Delta+M_0\right)\left(m_\Delta+M_0\right)\right.$$

$$\left(-2(-15+d)m_\Delta^4-4(-10+d)m_\Delta^3M_0+(-2+d)^2Q^2M_0^2+4(-2+d)m_\Delta M_0\left(-Q^2+M_0^2\right)+m_\Delta^2\left(-(-2+d)\left((2+d)Q^2+2(-1+d)s\right)+2(3+(-2+d)d)M_0^2\right)\right)-$$

$$2m_\pi^2\left(2(-7+d)m_\Delta^4+12m_\Delta^3M_0+(-2+d)^2Q^2M_0^2+2(-2+d)m_\Delta M_0\left(-Q^2+2M_0^2\right)+m_\Delta^2\left(-(-2+d)\left((-2+d)Q^2+(-1+d)s\right)+(4+(-1+d)d)M_0^2\right)\right))$$

$$(9+5\tau[3])\mathrm{II}[d]\left[\left\{\{0,m_\pi\},1\right\},\left\{\{p+q,m_\Delta\},1\right\},\left\{\{p,m_\Delta\},1\right\}\right])+\frac{1}{6(-1+d)^3F^2m_\Delta^4M_0}(i$$

$$e^2\mathrm{g}\pi\mathrm{N}\Delta^2\left((-2+d)Q^2-8m_\Delta^2\right)\left(m_\pi-m_\Delta-M_0\right)\left(m_\pi+m_\Delta-M_0\right)M_0\left(m_\pi-m_\Delta+M_0\right)$$

$$\left(m_\Delta+M_0\right)\left(m_\pi+m_\Delta+M_0\right)(9+5\tau[3])\mathrm{II}[d]\left[\left\{\{0,m_\pi\},1\right\},\left\{\{p+q,m_\Delta\},1\right\},\left\{\{p,m_\Delta\},2\right\}\right])$$

$$-\frac{1}{6(-1+d)^3F^2m_\Delta^6M_0}(2ie^2\mathrm{g}\pi\mathrm{N}\Delta^2\pi M_0\left((-2+d)Q^2\left((2+(-1+d)d)Q^2+2(-1+d)ds\right)m_\Delta^3+\left((-22+(-2+d)d(-4+3d))Q^2+2(-9+d(14+3(-4+d)d))s\right)m_\Delta^5+\right.$$

$$2\left(5+(-4+d)d^2\right)m_\Delta^7+m_\Delta^2\left((-2+d)Q^2\left((8+(-4+d)d)Q^2+2(-2+d)(-1+d)s\right)+\right.$$

$$\left((-84+d(44+d(-16+3d)))Q^2+2(-2+d)(14+3(-3+d)d)s\right)m_\Delta^2+2(-16+(-4+d)d(1+d))m_\Delta^4\right)M_0-$$

$$m_\Delta\left((-2+d)^2(1+d)Q^4+2\left((3+d(16+d(-13+3d)))Q^2+s+(-4+d)(-2+d)ds\right)m_\Delta^2+2(15+d(20+d(-14+3d)))m_\Delta^4\right)M_0^2+$$

$$\left(-(-2+d)^3Q^4-2(-2+d)\left((16+3(-4+d)d)Q^2+(-2+d)(-1+d)s\right)m_\Delta^2-2(-2+3d)(8+(-5+d)d)m_\Delta^4\right)M_0^3+$$

$$m_\Delta\left((-2+d)^3Q^2-4dm_\Delta^2\right)M_0^4+(-2+d)\left((-2+d)^2Q^2-4m_\Delta^2\right)M_0^5+$$

$$m_\pi^4\left((-2+d)^3Q^2m_\Delta-2(1+d(10+(-6+d)d))m_\Delta^3+(-2+d)^3Q^2M_0-2(-2+d)(4+(-3+d)d)m_\Delta^2M_0\right)+$$

$$m_\pi^2\left(-4(2+(-5+d)d)m_\Delta^5-4(4+(-7+d)d)m_\Delta^4M_0-(-2+d)^3Q^2M_0\left(-Q^2+2M_0^2\right)+(-2+d)^2Q^2m_\Delta\left(Q^2+dQ^2-2(-2+d)M_0^2\right)+\right.$$

$$2(-2+d)m_\Delta^2 M_0 ((12+d(-9+2d))Q^2+(-2+d)(-1+d)s+(6+(-3+d)d)M_0^2)+$$
$$2m_\Delta^3 (7Q^2+s+(-2+d)d(2(-3+d)Q^2+(-4+d)s)+(1+d(12+(-6+d)d))M_0^2)))$$
$$(9+5\tau[3])\mathrm{II}[2+d][\{\{0,m_\pi\},1\},\{\{p,m_\Delta\},1\},\{\{p+q,m_\Delta\},2\}])+\frac{1}{6(-1+d)^3F^2m_\Delta^5M_0}(2ie^2\mathrm{g}\pi\mathrm{N}\Delta^2\pi M_0 (-(20+3(-4+d)d)m_\Delta^4-2(18+d(-11+3d))m_\Delta^3M_0-2(-3+d)(-2+d)$$
$$m_\Delta M_0 (Q^2+m_\pi^2-M_0^2)+(-2+d)^2(m_\pi-M_0)(m_\pi+M_0)(2Q^2+m_\pi^2-M_0^2)+2m_\Delta^2((-2+d)(Q^2+(-1+d)s)+(8+(-4+d)d)m_\pi^2+(-6+(5-2d)d)M_0^2))$$
$$(9+5\tau[3])\mathrm{II}[2+d][\{\{0,m_\pi\},1\},\{\{p+q,m_\Delta\},1\},\{\{p,m_\Delta\},1\}])+$$
$$\frac{1}{6(-1+d)^3F^2m_\Delta^5M_0}(2ie^2\mathrm{g}\pi\mathrm{N}\Delta^2\pi M_0(-4(-3+d)m_\Delta^6-4(-9+2d)m_\Delta^5M_0+(-2+d)^2Q^2(m_\pi^2-M_0^2)^2+4(-2+d)m_\Delta(m_\pi-M_0)M_0(m_\pi+M_0)(Q^2+m_\pi^2-M_0^2)+$$
$$4m_\Delta^3M_0(-(-2+d)Q^2+(-7+d)m_\pi^2+(13-3d)M_0^2)+m_\Delta^4(-(-2+d)(dQ^2+2(-1+d)s)+8(-3+d)m_\pi^2+2(30+(-7+d)d)M_0^2)-$$
$$2m_\Delta^2(2(-3+d)m_\pi^4+m_\pi^2(-(-2+d)(Q^2+(-1+d)s)+(14+(-7+d)d)M_0^2)+M_0^2((-2+d)(3Q^2+(-1+d)s)-(8+(-5+d)d)M_0^2)))$$
$$(9+5\tau[3])\mathrm{II}[2+d][\{\{0,m_\pi\},1\},\{\{p+q,m_\Delta\},1\},\{\{p,m_\Delta\},2\}])+\frac{1}{6(-1+d)^3F^2m_\Delta^4M_0}(8ie^2\mathrm{g}\pi\mathrm{N}\Delta^2\pi((-2+d)Q^2-8m_\Delta^2)(m_\pi-m_\Delta-M_0)(m_\pi+m_\Delta-M_0)$$
$$M_0^2(m_\pi-m_\Delta+M_0)(m_\pi+m_\Delta+M_0)(9+5\tau[3])\mathrm{II}[2+d][\{\{0,m_\pi\},1\},\{\{p+q,m_\Delta\},1\},\{\{p,m_\Delta\},3\}]-\frac{1}{6(-1+d)^3F^2m_\Delta^6M_0}$$
$$(ie^2\mathrm{g}\pi\mathrm{N}\Delta^2\pi M_0(m_\pi^4((-2+d)^2dQ^4m_\Delta-4(-3+d)dQ^2m_\Delta^3+4(2+d-4d^2+d^3)m_\Delta^5+((-2+d)^3Q^4-4(-2+d)^2Q^2m_\Delta^2+4(10+d-4d^2+d^3)m_\Delta^4)M_0)+$$
$$(m_\Delta+M_0)(m_\Delta^4((-2+d)((8+(-2+d)d)Q^4+4(3+(-2+d)d)Q^2s+4(-1+d)^2s^2)+4((-10+(-5+d)d)Q^2+2(-5+d)(1+d)s)m_\Delta^2+$$
$$4(2+d-4d^2+d^3)m_\Delta^4)-2(-2+d)Q^2((-2+d)Q^2+2(-3+d)s)m_\Delta^3M_0+4((-4+d)Q^2+4(-5+d)s)m_\Delta^5M_0+32m_\Delta^7M_0-$$
$$2m_\Delta^2((-2+d)Q^2((6+(-3+d)d)Q^2+2(-2+d)(-1+d)s)+2((-20+d(6+(-4+d)d))Q^2+2(-1+d)(7+(-4+d)d)s)m_\Delta^2+4(1+d)$$
$$(1+(-4+d)d)m_\Delta^4)M_0^2+2m_\Delta(Q^2+2m_\Delta^2)((-2+d)^2Q^2-4(-3+d)m_\Delta^2)M_0^3+((-2+d)^3Q^4+4(-2+d)^3Q^2m_\Delta^2+8(-1+d(9+(-5+d)d))m_\Delta^4)M_0^4)-$$
$$2m_\pi^2(4(2+d-4d^2+d^3)m_\Delta^7+4(10+d-4d^2+d^3)m_\Delta^6M_0+(-2+d)^2dQ^4m_\Delta M_0^2+(-2+d)^3Q^4M_0^3+(-2+d)Q^2m_\Delta^2M_0(-(6+(-4+d)d)Q^2-$$
$$2(-2+d)(-1+d)s+2(-3+d)(-2+d)M_0^2)-Q^2m_\Delta^3(d(2+(-4+d)d)Q^2+2(-3+d)(-1+d)ds+4(Q^2+s)-2(2+(-3+d)^2d)M_0^2)+$$
$$4m_\Delta^5(-(5+d)Q^2+(-5+d)(1+d)s+(7+d(5+(-5+d)d))M_0^2)+4m_\Delta^4M_0((11-4d)Q^2+(-5+d)(-1+d)s+(5+d(7+(-5+d)d))M_0^2)))$$
$$(9+5\tau[3])\mathrm{II}[2+d][\{\{0,m_\pi\},1\},\{\{p+q,m_\Delta\},2\},\{\{p,m_\Delta\},2\}])-\frac{1}{6(-1+d)^3F^2m_\Delta^6M_0}(16ie^2\mathrm{g}\pi\mathrm{N}\Delta^2\pi^2M_0(-2(-6+d)(-1+d)m_\Delta^7+2(4+d(18+(-7+d)d))m_\Delta^6M_0+$$
$$(-2+d)m_\pi^4(3(-2+d)Q^2m_\Delta-2(-1+d)m_\Delta^3+(-2+d)((-2+d)Q^2-2(-1+d)m_\Delta^2)M_0)+$$
$$(-2+d)^3Q^2M_0^3(-Q^2+M_0^2)-m_\Delta^5((86+d(-44+7d))Q^2+2(29+d(-15+2d))s+2(-17+d)M_0^2)-$$
$$m_\Delta^4M_0(-(-2+d)(32+3(-6+d)d)Q^2-2(-1+d)(26+d(-14+3d))s+(-68+6d(10+(-5+d)d))M_0^2)-$$
$$(-2+d)m_\Delta^2M_0(-Q^2((6+(-6+d)d)Q^2+2(-3+d)ds)+2((22+d(-14+3d))Q^2+(-2+d)(-1+d)s)M_0^2)+(-2+d)^2Q^2m_\Delta(-Q^2(Q^2+s)+2sM_0^2+M_0^4)-$$
$$m_\Delta^3((36+d(-30+7d))Q^4+2(14+3(-4+d)d)Q^2s+2(-2+d)(-1+d)s^2-2((13+(-10+d)d)Q^2+(-1+d)s)M_0^2+2(-1+d)M_0^4)+$$
$$m_\pi^2(4(-4+d)(-1+d)m_\Delta^5+4(-8+(-1+d)d)m_\Delta^4M_0+(-2+d)^2Q^2m_\Delta(Q^2-2s-4M_0^2)-(-2+d)^3Q^2M_0(-Q^2+2M_0^2)+2m_\Delta^3$$
$$((13+2(-4+d)d)Q^2+(-1+d)(-5+2d)s+(-1+d)M_0^2)+2(-2+d)m_\Delta^2M_0((20+d(-11+2d))Q^2+(-2+d)(-1+d)s+(-2+d)(-1+d)M_0^2)))$$
$$(9+5\tau[3])\mathrm{II}[4+d][\{\{0,m_\pi\},1\},\{\{p,m_\Delta\},1\},\{\{p+q,m_\Delta\},3\}])-\frac{1}{6(-1+d)^3F^2m_\Delta^6M_0}$$
$$(8ie^2\mathrm{g}\pi\mathrm{N}\Delta^2\pi^2M_0(4(2+d-4d^2+d^3)m_\Delta^8M_0+(-2+d)^3Q^4M_0(m_\pi^2-M_0^2)^2+2(-2+d)^2Q^4m_\Delta(m_\pi-M_0)(m_\pi+M_0)(-Q^2-s+2m_\pi^2-M_0^2)+$$
$$8m_\Delta^7(Q^2-5s-d((-4+d)Q^2+(-6+d)s)+(-5+d)(-1+d)M_0^2)-4m_\Delta^6M_0(3(-8+(-3+d)d)Q^2+2(-5+d)(-1+d)s+2(2+d-4d^2+d^3)m_\pi^2+2(-3+d)(-1+d)^2M_0^2)-$$
$$4m_\Delta^5((2+d(-9+2d))Q^4+(16+5(-5+d)d)Q^2s+2(-5+d)(-1+d)s^2+(-(62+3(-7+d)d)Q^2-2(-5+d)(-1+d)s)M_0^2+$$
$$2m_\pi^2(7Q^2-5s-d((-2+d)Q^2+(-6+d)s)+(-5+d)(-1+d)M_0^2))+m_\Delta^4M_0((40+(-8+d)(-2+d)d)Q^4+4(-2+d(16+(-7+d)d))Q^2s+$$
$$4(-2+d)(-1+d)^2s^2+4(2+d-4d^2+d^3)m_\pi^4+4(-(-44+d(22+(-7+d)d))Q^2-2(-1+d)(7+(-4+d)d)s)M_0^2+$$
$$8(-1+d)(5+(-4+d)d)M_0^4-8m_\pi^2((19-5d)Q^2+5s+(-6+d)ds+(-3+d)(-1+d)^2M_0^2))-$$
$$2(-2+d)Q^2m_\Delta^2M_0(2Q^2(Q^2+s)+2(-3+d)m_\pi^4+((12+(-6+d)d)Q^2+2(-2+d)(-1+d)s)M_0^2-2(5+(-4+d)d)M_0^4+$$

$m_\pi^2 (-(12+(-6+d)d)Q^2 - 2(-2+d)(-1+d)s + 2(8+(-5+d)d)M_0^2)) - 2Q^2 m_\Delta^3 ((-2+d)(Q^2+s)(dQ^2+2(-1+d)s) + 8(-3+d)m_\pi^4 +$
$(-(44+5(-6+d)d)Q^2 - 2(8+(-5+d)d)s) M_0^2 + 4(-3+d)M_0^4 + 2m_\pi^2 (-(-4+(-1+d)d)s + (-4+d)((-3+d)Q^2+(-5+d)M_0^2))))$
$(9+5\tau[3])\mathrm{II}[4+d][\{\{0,m_\pi\},1\},\{\{p,m_\Delta\},2\},\{\{p+q,m_\Delta\},3\}]) - \frac{1}{6(-1+d)^3F^2m_\Delta^6M_0}$
$(8ie^2 g\pi N\Delta^2\pi^2 M_0^2 (2(18+d(2+(-5+d)d))m_\Delta^6 + (-2+d)^3Q^2(m_\pi - M_0)(m_\pi + M_0)(Q^2+m_\pi^2-M_0^2) +$
$m_\Delta^4 ((12+d(6+d(-10+3d)))Q^2 + 2(-1+d)(16+3(-4+d)d)s + 4(-11+3d)m_\pi^2 + (60-2d(26+3(-5+d)d))M_0^2) +$
$(-2+d)m_\Delta^2 (-2(-2+d)(-1+d)m_\pi^4 + 2(-2+d)(-1+d)s(Q^2-M_0^2) +$
$2m_\pi^2 ((11+2(-4+d)d)Q^2 + (-2+d)(-1+d)s + (-2+d)(-1+d)M_0^2) + Q^2((-2+d)dQ^2 + (-30+22d-6d^2)M_0^2)))$
$(9+5\tau[3])\mathrm{II}[4+d][\{\{0,m_\pi\},1\},\{\{p+q,m_\Delta\},2\},\{\{p,m_\Delta\},2\}]) - \frac{1}{6(-1+d)^3F^2m_\Delta^6M_0}(8ie^2g\pi N\Delta^2\pi^2M_0^2(4(2+d-4d^2+d^3)m_\Delta^8 + (-2+d)^3Q^4(m_\pi^2-M_0^2)^2 -$
$4m_\Delta^6(-(4+(-7+d)d)Q^2 - 2(-5+d)(-1+d)s + 2(2+d-4d^2+d^3)m_\pi^2 + 2(-1+d)(-7+(-2+d)d)M_0^2) +$
$m_\Delta^4((-2+d)(dQ^2+2(-1+d)s)^2 + 4(2+d-4d^2+d^3)m_\pi^4 + 4((2-(-3+d)d^2)Q^2 - 2(-1+d)(7+(-4+d)d)s)M_0^2 +$
$8(-1+d)(5+(-4+d)d)M_0^4 - 8m_\pi^2(-(1+d)Q^2 + (-5+d)(-1+d)s + (-3+d)(-1+d)^2M_0^2)) - 2(-2+d)Q^2m_\Delta^2$
$(2(-3+d)m_\pi^4 + m_\pi^2(-(-2+d)(dQ^2+2(-1+d)s) + 2(8+(-5+d)d)M_0^2) + M_0^2((2+(-2+d)d)Q^2 + 2(-2+d)(-1+d)s - 2(5+(-4+d)d)M_0^2)))$
$(9+5\tau[3])\mathrm{II}[4+d][\{\{0,m_\pi\},1\},\{\{p+q,m_\Delta\},2\},\{\{p,m_\Delta\},3\}])$

Diagram k)+l)

$\mathbf{A}(\nu,Q^2) = -\frac{1}{96(-1+d)^2F^2m_\Delta^2M_0^2}$
$(e^2 g\pi N\Delta^2(1+\tau[3])(\frac{1}{dsM_0^2}(2(-24sm_\pi^2M_0^2 + d^2((Q^2-s)(m_\pi^2-m_\Delta^2)^2 - (-Q^2s-2s^2+m_\pi^4+2sm_\Delta^2+m_\Delta^4+2m_\pi^2(9s-m_\Delta^2))M_0^2+2sM_0^4) - 2d((Q^2-s)(m_\pi^2-m_\Delta^2)^2 +$
$(Q^2-s)m_\Delta(-m_\pi^2+m_\Delta^2)M_0 - (-Q^2s-2s^2+m_\pi^4+2sm_\Delta^2+m_\Delta^4+m_\pi^2(24s-2m_\Delta^2))M_0^2 + (-m_\pi^2m_\Delta+m_\Delta^3)M_0^3+2sM_0^4))\mathrm{II}[d][\{\{0,m_\pi\},1\}]) -$
$\frac{1}{-sM_0^2+M_0^4}(2(Q^2-s+M_0^2)(-(-2+d)(m_\pi^2-m_\Delta^2) - 2m_\Delta M_0 + (-2+d)M_0^2)((m_\pi^2-m_\Delta^2)^2 - 2(m_\pi^2+m_\Delta^2)M_0^2+M_0^4)\mathrm{II}[d][\{\{0,m_\pi\},1\},\{\{p,m_\Delta\},1\}]) +$
$(-2+d)(Q^2+4m_\pi^2)(s-6m_\pi^2-2m_\Delta^2+M_0^2)\mathrm{II}[d][\{\{0,m_\pi\},1\},\{\{q,m_\pi\},1\}] + \frac{1}{s(-s+M_0^2)}(2(m_\pi^4+(s-m_\Delta^2)^2-2m_\pi^2(s+m_\Delta^2))$
$(-(-2+d)(-s+m_\pi^2)(Q^2+s-M_0^2) + (-2+d)m_\Delta^2(Q^2+s-M_0^2) - 2m_\Delta M_0(Q^2-s+M_0^2))\mathrm{II}[d][\{\{0,m_\pi\},1\},\{\{p+q,m_\Delta\},1\}]) +$
$\frac{1}{-s+M_0^2}(16(-1+d)\pi(4Q^2m_\Delta^3M_0 + (-2+d)m_\Delta^4(-Q^2-s+M_0^2) + (-2+d)(-s+m_\pi^2)(m_\pi-M_0)(m_\pi+M_0)(-Q^2-s+M_0^2) + 2m_\Delta M_0$
$(s(Q^2+s) - 2Q^2m_\pi^2 + (Q^2-2s)M_0^2 + M_0^4) - m_\Delta^2(-(Q^2+s)(2(-1+d)Q^2+ds+2(-2+d)m_\pi^2) + ((-6+d)Q^2+4s+2(-2+d)m_\pi^2)M_0^2 + (-4+d)M_0^4))$
$\mathrm{II}[2+d][\{\{0,m_\pi\},1\},\{\{q,m_\pi\},1\},\{\{p+q,m_\Delta\},1\}]) - \frac{1}{-s+M_0^2}(16(-1+d)\pi Q^2(-(-2+d)m_\pi^4 + m_\Delta^2(2(-1+d)Q^2 - (-2+d)m_\Delta^2) + 4m_\Delta^3M_0 +$
$2(2+d)m_\Delta^2M_0^2 + 4m_\Delta M_0^3 - (-2+d)M_0^4 + 2m_\pi^2((-2+d)m_\Delta^2 - 2m_\Delta M_0 + (-2+d)M_0^2))\mathrm{II}[2+d][\{\{0,m_\pi\},1\},\{\{q,m_\pi\},1\},\{\{\mathrm{pr}+q,m_\Delta\},1\}])))$

$\mathbf{B}(\nu,Q^2) = \frac{1}{24(-1+d)^2F^2m_\Delta^2M_0^3}(ie^2g\pi N\Delta^2(1+\tau[3])$
$(\frac{1}{Q^2s^2}((2sM_0(Q^2m_\Delta(m_\pi^2-m_\Delta^2) + 3Q^2(m_\pi^2-m_\Delta^2)M_0 + 4sm_\Delta M_0^2 + 4sM_0^3) + d(2Q^2s(m_\pi^2-m_\Delta^2)^2 + 6Q^2sm_\Delta(-m_\pi^2+m_\Delta^2)M_0 + Q^2(-9s+2m_\pi^2-2m_\Delta^2)(m_\pi^2-m_\Delta^2)M_0^2 - 8s^2$

$$m_\Delta M_0^3 - 8s^2 M_0^4\right) + d^2 \left(-Q^2 s \left(m_\pi^2 - m_\Delta^2\right)^2 + 2Q^2 s m_\Delta \left(m_\pi^2 - m_\Delta^2\right) M_0 - Q^2 \left(m_\pi^2 - m_\Delta^2\right) \left(-3s + m_\pi^2 - m_\Delta^2\right) M_0^2 + 2s^2 m_\Delta M_0^3 + 2s^2 M_0^4\right)\right) \mathrm{II}[d]\left[\{\{0, m_\pi\}, 1\}\right]\right) +$$

$$\frac{1}{-s+M_0^2} \left(\left(-m_\pi^2 + m_\Delta^2 + 2m_\Delta M_0 + M_0^2\right) \left((-2+d)d\left(m_\pi^2 - m_\Delta^2\right)^2 + 2(-1+d)m_\Delta\left(m_\pi^2 - m_\Delta^2\right) M_0 + \left(\left(2+d-d^2\right) m_\pi^2 + \left(2-d+d^2\right) m_\Delta^2\right) M_0^2 - 2(-1+d)m_\Delta M_0^3 + (-2+d)M_0^4\right)\right.$$

$$\mathrm{II}[d]\left[\{\{0, m_\pi\}, 1\}, \{\{p, m_\Delta\}, 1\}\right][\{\}]) - \frac{(-2+d)\left(Q^2+4m_\pi^2\right) M_0^3 (m_\Delta + M_0) \mathrm{II}[d][\{\{0,m_\pi\},1\},\{\{q,m_\pi\},1\}]}{Q^2} -$$

$$\frac{1}{s^2\left(-s+M_0^2\right)} \left(M_0^3 \left(2s\left(-s+m_\pi^2\right)\left(s + (1+(-3+d)d)m_\pi^2\right) m_\Delta + 2s\left((4+(-3+d)d)s - 2(1+(-3+d)d)m_\pi^2\right) m_\Delta^3 +\right.\right.$$

$$2(1+(-3+d)d)s m_\Delta^5 - (-2+d)\left(-s+m_\pi^2\right)^2 \left(-s + dm_\pi^2\right) M_0 + (-2+d)\left(-s+m_\pi^2\right)\left(2s - ds + 3dm_\pi^2\right) m_\Delta^2 M_0 +$$

$$\left.(-2+d)\left(-3s + 2ds - 3dm_\pi^2\right) m_\Delta^4 M_0 + (-2+d)dm_\Delta^6 M_0\right) \mathrm{II}[d]\left[\{\{0, m_\pi\}, 1\}, \{\{p+q, m_\Delta\}, 1\}\right]) +$$

$$\frac{1}{-s+M_0^2} \left(2(-1+d)M_0^3 \left(m_\pi^4 \left(m_\Delta - dm_\Delta - (-2+d)M_0\right) + m_\pi^2 \left((-1+d)m_\Delta\left(s+2m_\Delta^2\right) + (-2+d)sM_0 + 2(-1+d)m_\Delta^2 M_0 + (-3+d)m_\Delta M_0^2 + (-2+d)M_0^3\right) +\right.\right.$$

$$\left.(m_\Delta + M_0)\left((-1+d)m_\Delta^2\left(2Q^2 + s - m_\Delta^2\right) - m_\Delta\left(s + m_\Delta^2\right) M_0 + (-2+d)\left(-s+m_\pi^2\right) M_0^2 + 2m_\Delta M_0^3\right)\right) \mathrm{II}[d]\left[\{\{0, m_\pi\}, 1\}, \{\{q, m_\pi\}, 1\}, \{\{p+q, m_\Delta\}, 1\}\right]) +$$

$$\frac{1}{-s+M_0^2} \left(2(-1+d)M_0^3 \left(m_\pi^4 \left((-1+d)m_\Delta + (-2+d)M_0\right) - 2m_\pi^2 (m_\Delta + M_0)\left((-1+d)m_\Delta^2 + (-2+d)M_0^2\right) +\right.\right.$$

$$\left.(m_\Delta + M_0)\left((-1+d)m_\Delta^2\left(-2Q^2 + m_\Delta^2\right) + m_\Delta^3 M_0 + (3-2d)m_\Delta^2 M_0^2 - m_\Delta M_0^3 + (-2+d)M_0^4\right)\right) \mathrm{II}[d]\left[\{\{0, m_\pi\}, 1\}, \{\{q, m_\pi\}, 1\}, \{\{\mathrm{pr}+q, m_\Delta\}, 1\}\right]) +$$

$$\frac{1}{-s+M_0^2} \left(8(-1+d)\pi M_0^4 \left(-(-2+d)m_\Delta^4 + 2m_\Delta^3 M_0 + (-2+d)\left(-s+m_\pi^2\right)(-m_\pi + M_0)(m_\pi + M_0) + 2m_\Delta M_0\left(-m_\pi^2 + M_0^2\right) + m_\Delta^2\left(-2Q^2 + 2(-2+d)m_\pi^2 + d\left(2Q^2 + s + M_0^2\right)\right)\right)\right)$$

$$\mathrm{II}[2+d]\left[\{\{0, m_\pi\}, 1\}, \{\{q, m_\pi\}, 1\}, \{\{p+q, m_\Delta\}, 2\}\right]) + \frac{1}{-s+M_0^2}$$

$$\left(8(-1+d)\pi M_0^4 \left((-2+d)m_\pi^4 + m_\Delta^2\left(-2(-1+d)Q^2 + (-2+d)m_\Delta^2\right) - 2m_\Delta^3 M_0 - 2dm_\Delta^2 M_0^2 - 2m_\Delta M_0^3 + (-2+d)M_0^4 + m_\pi^2\left(-2(-2+d)m_\Delta^2 + 2m_\Delta M_0 - 2(-2+d)M_0^2\right)\right)\right)$$

$$\mathrm{II}[2+d]\left[\{\{0, m_\pi\}, 1\}, \{\{q, m_\pi\}, 1\}, \{\{\mathrm{pr}+q, m_\Delta\}, 2\}\right]) +$$

$$\frac{1}{-s+M_0^2} \left(8(-1+d)\pi M_0^3 \left(-2(-1+d)m_\Delta^5 - 2dm_\Delta^4 M_0 + 2(-2+d)\left(-s+m_\pi^2\right) M_0\left(-m_\pi^2 + M_0^2\right) + 2m_\Delta^2 M_0\left((-1+2d)Q^2 + (-3+d)s + 2(-1+d)m_\pi^2 + (1+d)M_0^2\right) +\right.\right.$$

$$m_\Delta^3\left(-3\left(Q^2+s\right) + 2d\left(2Q^2+s\right) + 4(-1+d)m_\pi^2 + (-5+2d)M_0^2\right) + m_\Delta\left(s\left(Q^2+s\right) - \left(Q^2 + s - 2ds\right) m_\pi^2 - 2(-1+d)m_\pi^4 +\right.$$

$$\left.\left.\left(-(1+2d)s + (-7+2d)m_\pi^2\right) M_0^2 + 6M_0^4\right)\right) \mathrm{II}[2+d]\left[\{\{0, m_\pi\}, 1\}, \{\{q, m_\pi\}, 2\}, \{\{-p, m_\Delta\}, 1\}\right]) +$$

$$\frac{1}{-s+M_0^2} \left(8(-1+d)\pi M_0^3 \left(2m_\pi^4 \left((-1+d)m_\Delta + (-2+d)M_0\right) + m_\pi^2\left(m_\Delta\left(Q^2 - 4(-1+d)m_\Delta^2\right) - 4(-1+d)m_\Delta^2 M_0 - 4(-2+d)m_\Delta M_0^2 - 4(-2+d)M_0^3\right) +\right.\right.$$

$$\left.(m_\Delta + M_0)\left(m_\Delta^2\left((3-4d)Q^2 + 2(-1+d)m_\Delta^2\right) + \left(-Q^2 m_\Delta + 2m_\Delta^3\right) M_0 + 2(3-2d)m_\Delta^2 M_0^2 - 2m_\Delta M_0^3 + 2(-2+d)M_0^4\right)\right)$$

$$\mathrm{II}[2+d]\left[\{\{0, m_\pi\}, 1\}, \{\{q, m_\pi\}, 2\}, \{\{-\mathrm{pr}, m_\Delta\}, 1\}\right]) + \frac{1}{-s+M_0^2}$$

$$\left(64(-1+d)\pi^2 M_0^4 \left(-(-2+d)m_\Delta^4 + 2m_\Delta^3 M_0 + (-2+d)\left(-s+m_\pi^2\right)(-m_\pi + M_0)(m_\pi + M_0) + 2m_\Delta M_0\left(-m_\pi^2 + M_0^2\right) + m_\Delta^2\left(-2Q^2 + 2(-2+d)m_\pi^2 + d\left(2Q^2 + s + M_0^2\right)\right)\right)\right)$$

$$\mathrm{II}[4+d]\left[\{\{0, m_\pi\}, 1\}, \{\{q, m_\pi\}, 2\}, \{\{-p, m_\Delta\}, 2\}\right]) + \frac{1}{-s+M_0^2}$$

$$\left(64(-1+d)\pi^2 M_0^4 \left((-2+d)m_\pi^4 + m_\Delta^2\left(-2(-1+d)Q^2 + (-2+d)m_\Delta^2\right) - 2m_\Delta^3 M_0 - 2dm_\Delta^2 M_0^2 - 2m_\Delta M_0^3 + (-2+d)M_0^4 + m_\pi^2\left(-2(-2+d)m_\Delta^2 + 2m_\Delta M_0 - 2(-2+d)M_0^2\right)\right)\right)$$

$$\mathrm{II}[4+d]\left[\{\{0, m_\pi\}, 1\}, \{\{q, m_\pi\}, 2\}, \{\{-\mathrm{pr}, m_\Delta\}, 2\}\right]) +$$

$$\frac{1}{-s+M_0^2} \left(128(-1+d)\pi^2 m_\Delta M_0^3 \left(Q^2 s + s^2 - 3sM_0^2 + 2M_0^4 - m_\pi^2\left(Q^2 - s + M_0^2\right) + m_\Delta^2\left(Q^2 - s + M_0^2\right) + 2m_\Delta M_0\left(Q^2 - s + M_0^2\right)\right)\right)$$

$$\mathrm{II}[4+d]\left[\{\{0, m_\pi\}, 1\}, \{\{q, m_\pi\}, 3\}, \{\{-p, m_\Delta\}, 1\}\right]) - \frac{128(-1+d)\pi^2 Q^2 m_\Delta M_0^3\left(-m_\pi^2 + (m_\Delta + M_0)^2\right) \mathrm{II}[4+d][\{\{0,m_\pi\},1\},\{\{q,m_\pi\},3\},\{\{-\mathrm{pr},m_\Delta\},1\}]}{-s+M_0^2}\Bigg)\Bigg)$$

Diagram m)+n)

$$\mathbf{A}(\nu, Q^2) = \frac{1}{48(-1+d)^3 dF^2 s^2 m_\Delta^4 M_0^5} \left(ie^2 \mathrm{g}\pi\mathrm{N}\Delta^2 \left(16d^5 s^2 m_\pi^4 M_0^4 -\right.\right.$$

$$32sm_\pi^2 M_0^2 \left(\left(Q^4 - 2Q^2 s - s^2\right) m_\Delta^2 + \left(Q^4 - 2Q^2 s - s^2\right) m_\Delta M_0 + 2s\left(-4Q^2 - 3s + m_\Delta^2\right) M_0^2 + 2Q^2 m_\Delta M_0^3 - \left(10s + m_\Delta^2\right) M_0^4 + m_\Delta M_0^5 + m_\pi^2\left(-Q^4 + 2Q^2 s + s^2 + 10sM_0^2 + M_0^4\right)\right) +$$

$$d^4M_0^2\left(Q^2sm_\Delta^4\left(3Q^2-4s+2m_\Delta^2\right)+8Q^2\left(Q^2-s\right)sm_\Delta^3M_0+\left(Q^2sm_\Delta^2\left(-11s+m_\Delta^2\right)+Q^4\left(2s^2+sm_\Delta^2+2m_\Delta^4\right)+2s\left(-s^3-3s^2m_\Delta^2+4sm_\Delta^4+m_\Delta^6\right)\right)M_0^2+\right.$$
$$8Q^2sm_\Delta\left(-2s+m_\Delta^2\right)M_0^3-\left(2sm_\Delta^2\left(s+m_\Delta^2\right)+Q^2\left(7s^2+sm_\Delta^2-4m_\Delta^4\right)\right)M_0^4+2\left(s^2-sm_\Delta^2+m_\Delta^4\right)M_0^6+$$
$$m_\pi^4\left(Q^2s\left(3Q^2-4s+2m_\Delta^2\right)+\left(2Q^4-5Q^2s+2s\left(-85s+m_\Delta^2\right)\right)M_0^2+4\left(Q^2-2s\right)M_0^4+2M_0^6\right)-m_\pi^2\left(2Q^2sm_\Delta^2\left(5Q^2-4s+2m_\Delta^2\right)+8Q^2\left(Q^2-s\right)sm_\Delta M_0+\right.$$
$$\left.\left(Q^2s\left(-23s+4m_\Delta^2\right)+Q^4\left(7s+4m_\Delta^2\right)-2s\left(10s^2+19sm_\Delta^2-2m_\Delta^4\right)\right)M_0^2+8Q^2sm_\Delta M_0^3+\left(11Q^2s+8Q^2m_\Delta^2-6sm_\Delta^2\right)M_0^4+4\left(s+m_\Delta^2\right)M_0^6\right)\right)-$$
$$4d\left(-2s\left(-Q^4+2Q^2s+s^2\right)m_\Delta^6-2s\left(-Q^4+2Q^2s+s^2\right)m_\Delta^5M_0+m_\Delta^4\left(6s^2\left(s-m_\Delta^2\right)+2Q^4\left(s+m_\Delta^2\right)+Q^2s\left(4s+17m_\Delta^2\right)\right)M_0^2+\right.$$
$$m_\Delta^3\left(2s^2\left(2s-11m_\Delta^2\right)+Q^4\left(11s+2m_\Delta^2\right)+Q^2s\left(-4s+49m_\Delta^2\right)\right)M_0^3+s\left(-2s^3+19s^2m_\Delta^2-15sm_\Delta^4+3m_\Delta^6+2Q^4\left(3s+m_\Delta^2\right)+Q^2\left(4s^2+16sm_\Delta^2+15m_\Delta^4\right)\right)M_0^4+$$
$$m_\Delta\left(-Q^4s+Q^2\left(-2s^2+sm_\Delta^2+4m_\Delta^4\right)+s\left(-8s^2+11sm_\Delta^2+39m_\Delta^4\right)\right)M_0^5-\left(8s^3-5s^2m_\Delta^2-29sm_\Delta^4+2m_\Delta^6+Q^2s\left(7s+m_\Delta^2\right)\right)M_0^6+$$
$$\left(-Q^2sm_\Delta+2\left(4s^2m_\Delta-5sm_\Delta^3+m_\Delta^5\right)\right)M_0^7+s\left(2s-3m_\Delta^2\right)M_0^8+2m_\pi^6\left(s+M_0^2\right)\left(-Q^4+2Q^2s+s^2-4sM_0^2+M_0^4\right)+$$
$$m_\pi^4\left(-6s\left(-Q^4+2Q^2s+s^2\right)m_\Delta^2-2s\left(-Q^4+2Q^2s+s^2\right)m_\Delta M_0+\left(-Q^2s\left(36s+m_\Delta^2\right)+3Q^4\left(7s+2m_\Delta^2\right)+2s^2\left(-7s+3m_\Delta^2\right)\right)M_0^2+\right.$$
$$\left.2\left(Q^4+6Q^2s-s^2\right)m_\Delta M_0^3+s\left(-13Q^2-248s+5m_\Delta^2\right)M_0^4+2\left(2Q^2+s\right)m_\Delta M_0^5-6\left(3s+m_\Delta^2\right)M_0^6+2m_\Delta M_0^7\right)-$$
$$m_\pi^2\left(-6s\left(-Q^4+2Q^2s+s^2\right)m_\Delta^4-4s\left(-Q^4+2Q^2s+s^2\right)m_\Delta^3M_0+2m_\Delta^2\left(-s^2\left(4s+3m_\Delta^2\right)+Q^4\left(13s+3m_\Delta^2\right)+Q^2s\left(-16s+9m_\Delta^2\right)\right)M_0^2+\right.$$
$$m_\Delta\left(-8s^2\left(s+3m_\Delta^2\right)+Q^4\left(17s+4m_\Delta^2\right)+Q^2s\left(-28s+27m_\Delta^2\right)\right)M_0^3+s\left(7Q^4+Q^2\left(-151s+m_\Delta^2\right)-s\left(120s+19m_\Delta^2\right)\right)M_0^4+$$
$$\left.\left.\left(25Q^2sm_\Delta+8Q^2m_\Delta^3+7sm_\Delta^3\right)M_0^5+\left(11Q^2s-164s^2+7sm_\Delta^2-6m_\Delta^4\right)M_0^6+4\left(2sm_\Delta+m_\Delta^3\right)M_0^7+4sM_0^8\right)\right)+$$
$$2d^2\left(-4s\left(-Q^4+2Q^2s+s^2\right)m_\Delta^6-2s\left(-Q^4+2Q^2s+s^2\right)m_\Delta^5M_0+m_\Delta^4\left(8s^2\left(2s-m_\Delta^2\right)+Q^4\left(7s+4m_\Delta^2\right)+2Q^2s\left(4s+19m_\Delta^2\right)\right)M_0^2+\right.$$
$$m_\Delta^3\left(2s^2\left(2s-17m_\Delta^2\right)+Q^4\left(41s+2m_\Delta^2\right)+4Q^2s\left(-7s+22m_\Delta^2\right)\right)M_0^3+$$
$$\left(Q^4\left(16s^2+7sm_\Delta^2+4m_\Delta^4\right)+Q^2s\left(8s^2-11sm_\Delta^2+31m_\Delta^4\right)+s\left(-8s^3+7s^2m_\Delta^2+4sm_\Delta^4+18m_\Delta^6\right)\right)M_0^4+$$
$$m_\Delta\left(-3Q^4s+Q^2\left(-38s^2+23sm_\Delta^2+4m_\Delta^4\right)+2s\left(-8s^2+17sm_\Delta^2+39m_\Delta^4\right)\right)M_0^5-2\left(8s^3-3s^2m_\Delta^2-28sm_\Delta^4+2m_\Delta^6+2Q^2\left(7s^2+sm_\Delta^2-2m_\Delta^4\right)\right)M_0^6+$$
$$\left(-3Q^2sm_\Delta+2\left(8s^2m_\Delta-9sm_\Delta^3+m_\Delta^5\right)\right)M_0^7+\left(8s^2-11sm_\Delta^2+4m_\Delta^4\right)M_0^8+4m_\pi^6\left(s+M_0^2\right)\left(-Q^4+2Q^2s+s^2-4sM_0^2+M_0^4\right)+$$
$$2m_\pi^4\left(-6s\left(-Q^4+2Q^2s+s^2\right)m_\Delta^2+s\left(Q^4-2Q^2s-s^2\right)m_\Delta M_0+\left(8s^2\left(-s+m_\Delta^2\right)+6Q^4\left(3s+m_\Delta^2\right)-Q^2s\left(28s+m_\Delta^2\right)\right)M_0^2+\right.$$
$$\left.\left(Q^4+10Q^2s-5s^2\right)m_\Delta M_0^3+\left(2Q^4-18Q^2s+3s\left(-98s+3m_\Delta^2\right)\right)M_0^4+\left(2Q^2+5s\right)m_\Delta M_0^5+\left(4Q^2-20s-6m_\Delta^2\right)M_0^6+m_\Delta M_0^7+2M_0^8\right)-$$
$$m_\pi^2\left(-12s\left(-Q^4+2Q^2s+s^2\right)m_\Delta^4-4s\left(-Q^4+2Q^2s+s^2\right)m_\Delta^3M_0+m_\Delta^2\left(-4s^2m_\Delta^2+16Q^2s\left(-3s+2m_\Delta^2\right)+Q^4\left(55s+12m_\Delta^2\right)\right)M_0^2+\right.$$
$$m_\Delta\left(Q^4\left(31s+4m_\Delta^2\right)+4Q^2s\left(-11s+5m_\Delta^2\right)-4s^2\left(s+11m_\Delta^2\right)\right)M_0^3+\left(4Q^4\left(7s+2m_\Delta^2\right)+Q^2s\left(-252s+5m_\Delta^2\right)-2s\left(104s^2+57sm_\Delta^2-6m_\Delta^4\right)\right)M_0^4+$$
$$\left.\left.Q^2m_\Delta\left(35s+8m_\Delta^2\right)M_0^5+2\left(22Q^2s-104s^2+8Q^2m_\Delta^2+7sm_\Delta^2-6m_\Delta^4\right)M_0^6+4m_\Delta\left(s+m_\Delta^2\right)M_0^7+8\left(2s+m_\Delta^2\right)M_0^8\right)\right)-$$
$$d^3\left(-2s\left(-Q^4+2Q^2s+s^2\right)m_\Delta^6+m_\Delta^4\left(2s^2\left(5s-m_\Delta^2\right)+Q^4\left(11s+2m_\Delta^2\right)+Q^2\left(-4s^2+22sm_\Delta^2\right)\right)M_0^2+2sm_\Delta^3\left(23Q^4-20Q^2s+14Q^2m_\Delta^2-4sm_\Delta^2\right)M_0^3+\right.$$
$$\left(Q^4\left(14s^2+7sm_\Delta^2+8m_\Delta^4\right)+Q^2s\left(4s^2-49sm_\Delta^2+11m_\Delta^4\right)+2s\left(-5s^3-12s^2m_\Delta^2+15sm_\Delta^4+8m_\Delta^6\right)\right)M_0^4-$$
$$2sm_\Delta\left(Q^4+34Q^2s+4s^2-21Q^2m_\Delta^2-12sm_\Delta^2-14m_\Delta^4\right)M_0^5+\left(-35Q^2s^2-8s^3-5Q^2sm_\Delta^2+2s^2m_\Delta^2+16Q^2m_\Delta^4+16sm_\Delta^4-2m_\Delta^6\right)M_0^6-$$
$$2sm_\Delta\left(Q^2-4s+2m_\Delta^2\right)M_0^7+2\left(5s^2-6sm_\Delta^2+4m_\Delta^4\right)M_0^8+2m_\pi^6\left(s+M_0^2\right)\left(-Q^4+2Q^2s+s^2-4sM_0^2+M_0^4\right)+$$
$$m_\pi^4\left(-6s\left(-Q^4+2Q^2s+s^2\right)m_\Delta^2+\left(6Q^2s\left(-6s+m_\Delta^2\right)+2s^2\left(-3s+5m_\Delta^2\right)+Q^4\left(25s+6m_\Delta^2\right)\right)M_0^2+\right.$$
$$\left.8\left(Q^2-s\right)sm_\Delta M_0^3+\left(8Q^4-33Q^2s+4s\left(-164s+5m_\Delta^2\right)\right)M_0^4+8sm_\Delta M_0^5+2\left(8Q^2-3\left(7s+m_\Delta^2\right)\right)M_0^6+8M_0^8\right)-$$
$$m_\pi^2\left(-6s\left(-Q^4+2Q^2s+s^2\right)m_\Delta^4+2m_\Delta^2\left(s^2\left(2s+m_\Delta^2\right)+4Q^2s\left(-5s+2m_\Delta^2\right)+Q^4\left(26s+3m_\Delta^2\right)\right)M_0^2-2sm_\Delta\left(-17Q^4+8sm_\Delta^2+2Q^2\left(10s+m_\Delta^2\right)\right)M_0^3+\right.$$
$$\left(Q^2s\left(-179s+10m_\Delta^2\right)+Q^4\left(35s+16m_\Delta^2\right)-2s\left(76s^2+85sm_\Delta^2-7m_\Delta^4\right)\right)M_0^4+\left(34Q^2sm_\Delta-4sm_\Delta^3\right)M_0^5+$$
$$\left.\left.\left.\left(55Q^2s-84s^2+32Q^2m_\Delta^2-10sm_\Delta^2-6m_\Delta^4\right)M_0^6+4\left(5s+4m_\Delta^2\right)M_0^8\right)\right)\right)(1+\tau[3])\Pi[d]\left[\{\{0,m_\pi\},1\}\right]\right)-$$
$$\frac{1}{24(-1+d)^3F^2m_\Delta^4M_0^5}\left(ie^2\mathrm{g}\pi\mathrm{N}\Delta^2\left(-(-2+d)^2\left(2Q^2+s\right)m_\Delta^8+2(-2+d)\left(2Q^2+s\right)m_\Delta^7M_0+(-2+d)m_\Delta^6\left(2\left(-4+2d+d^2\right)Q^2+(-8+6d)s-(-6+d)m_\Delta^2\right)M_0^2+\right.\right.$$
$$2m_\Delta^5\left(2d^3Q^2+6s-22m_\Delta^2-2d^2\left(5Q^2+m_\Delta^2\right)+d\left(12Q^2-3s+17m_\Delta^2\right)\right)M_0^3+2m_\Delta^4\left(\left(4-12d+13d^2-4d^3\right)Q^2-2\left(2-d-2d^2+d^3\right)s+\left(10-6d-7d^2+2d^3\right)m_\Delta^2\right)M_0^4+$$
$$2m_\Delta^3\left(\left(28-62d+40d^2-8d^3\right)Q^2+3(-2+d)s+\left(78-77d+12d^2\right)m_\Delta^2\right)M_0^5+2m_\Delta^2\left(-(-2+d)^2(-3+4d)Q^2+34m_\Delta^2+d^2\left(s+4m_\Delta^2\right)-2d\left(s+14m_\Delta^2\right)\right)M_0^6-$$
$$2m_\Delta\left(2d^3Q^2+d\left(4Q^2+s-17m_\Delta^2\right)-2\left(s-7m_\Delta^2\right)+d^2\left(-6Q^2+8m_\Delta^2\right)\right)M_0^7+\left(-2(-2+d)^2(-1+d)Q^2+(-2+d)^2s-2\left(12-6d+d^2\right)m_\Delta^2\right)M_0^8-$$
$$2\left(6-7d+2d^2\right)m_\Delta M_0^9+(-2+d)^2M_0^{10}+(-2+d)^2m_\pi^8\left(-2Q^2-s+3M_0^2\right)-$$

$$2(-2+d)m_\pi^6\left(-2(-2+d)\left(2Q^2+s\right)m_\Delta^2+\left(2Q^2+s\right)m_\Delta M_0+\left(\left(8-6d+d^2\right)Q^2-(-2+d)s+2(-3+2d)m_\Delta^2\right)M_0^2+\left(m_\Delta-2dm_\Delta\right)M_0^3+5(-2+d)M_0^4\right)+$$
$$2m_\pi^4\left(-3(-2+d)^2\left(2Q^2+s\right)m_\Delta^4+3(-2+d)\left(2Q^2+s\right)m_\Delta^3M_0+(-2+d)m_\Delta^2\left(\left(12-10d+3d^2\right)Q^2+d\left(s+3m_\Delta^2\right)\right)M_0^2+\right.$$
$$m_\Delta\left(2\left(-8+10d-5d^2+d^3\right)Q^2+(-2+d)s+\left(-26+27d-6d^2\right)m_\Delta^2\right)M_0^3+$$
$$\left.(-2+d)\left(\left(10-9d+2d^2\right)Q^2+\left(4+5d-2d^2\right)m_\Delta^2\right)M_0^4+\left(-10+17d-6d^2\right)m_\Delta M_0^5+6(-2+d)^2M_0^6\right)-$$
$$2m_\pi^2\left(-2(-2+d)^2\left(2Q^2+s\right)m_\Delta^6+3(-2+d)\left(2Q^2+s\right)m_\Delta^5M_0+(-2+d)m_\Delta^4\left(-2dQ^2+3d^2Q^2-6s+5ds+6m_\Delta^2\right)M_0^2+\right.$$
$$m_\Delta^3\left(4(-2+d)^2(-1+d)Q^2-2(-2+d)s+\left(-46+39d-6d^2\right)m_\Delta^2\right)M_0^3+$$
$$m_\Delta^2\left(-2\left(-8+16d-8d^2+d^3\right)Q^2-2\left(-6+11d-6d^2+d^3\right)s+\left(-18+8d-3d^2\right)m_\Delta^2\right)M_0^4+m_\Delta\left(2(-2+d)Q^2-(-2+d)s+6\left(-2-2d+d^2\right)m_\Delta^2\right)M_0^5+$$
$$\left.\left(-(-2+d)^2Q^2+(-2+d)^2s-4(-3+d)^2m_\Delta^2\right)M_0^6+\left(-14+19d-6d^2\right)m_\Delta M_0^7+3(-2+d)^2M_0^8\right)\Big)$$
$$(1+\tau[3])\mathrm{II}[d]\left[\{\{0,m_\pi\},1\},\{\{p,m_\Delta\},1\}\right]\Big)-\frac{1}{48(-1+d)^3F^2m_\Delta^4M_0^5\left(-s+M_0^2\right)}\left(ie^2\mathrm{g}\pi\mathrm{N}\Delta^2\left(2(-2+d)^2Q^2m_\pi^8\left(-Q^2+2M_0^2\right)+\right.\right.$$
$$m_\pi^6\left(8(-2+d)^2Q^4m_\Delta^2-4(-2+d)Q^4m_\Delta M_0+Q^2\left(-(-2+d)^2(-11+3d)Q^2+2\left(-10+10d+d^2-d^3\right)m_\Delta^2\right)M_0^2+\right.$$
$$\left.8\left(6-5d+d^2\right)Q^2m_\Delta M_0^3+2\left((-2+d)^2(-13+5d)Q^2-2\left(2+2d-5d^2+d^3\right)m_\Delta^2\right)M_0^4\right)+$$
$$m_\pi^4\left(-12(-2+d)^2Q^4m_\Delta^4+12(-2+d)Q^4m_\Delta^3M_0+Q^2m_\Delta^2\left(\left(-100+142d-71d^2+13d^3\right)Q^2+2\left(-34+30d-11d^2+3d^3\right)m_\Delta^2\right)M_0^2+2Q^2m_\Delta\right.$$
$$\left(\left(-22+29d-17d^2+4d^3\right)Q^2-2\left(39-20d+d^2\right)m_\Delta^2\right)M_0^3+\left((-2+d)^2(-23+11d)Q^4-4\left(29-10d-4d^2+d^3\right)Q^2m_\Delta^2+4\left(-10+18d-11d^2+3d^3\right)m_\Delta^4\right)$$
$$\left.M_0^4-4m_\Delta\left(\left(14+11d-17d^2+4d^3\right)Q^2-2\left(17-22d+5d^2\right)m_\Delta^2\right)M_0^5+\left(-2(-2+d)^2(-29+17d)Q^2+4\left(10+6d-21d^2+5d^3\right)m_\Delta^2\right)M_0^6\right)+$$
$$\left(-m_\Delta^2+M_0^2\right)\left(2(-2+d)^2Q^4m_\Delta^6-4(-2+d)Q^4m_\Delta^5M_0+Q^2m_\Delta^4\left(\left(8-14d+11d^2-3d^3\right)Q^2-2\left(-34+38d-11d^2+d^3\right)m_\Delta^2\right)M_0^2+\right.$$
$$2Q^2m_\Delta^3\left(\left(22-41d+23d^2-4d^3\right)Q^2+2\left(49-44d+7d^2\right)m_\Delta^2\right)M_0^3-$$
$$2m_\Delta^2\left(\left(-10+19d-9d^2+d^3\right)Q^4+\left(-76+62d-9d^2+d^3\right)Q^2m_\Delta^2+2\left(-18+26d-9d^2+d^3\right)m_\Delta^4\right)M_0^4+$$
$$2m_\Delta\left(\left(-18+31d-19d^2+4d^3\right)Q^4-2\left(3+21d-20d^2+4d^3\right)Q^2m_\Delta^2+4\left(17-22d+5d^2\right)m_\Delta^4\right)M_0^5+$$
$$\left((-2+d)^2(-7+5d)Q^4-2\left(70-20d-27d^2+7d^3\right)Q^2m_\Delta^2+8\left(-2+2d+d^2-d^3\right)m_\Delta^4\right)M_0^6-$$
$$\left.4m_\Delta\left(\left(-6+25d-19d^2+4d^3\right)Q^2+6\left(-1+d^2\right)m_\Delta^2\right)M_0^7+2\left(-(-2+d)^2(-9+7d)Q^2+2\left(6+2d-11d^2+3d^3\right)m_\Delta^2\right)M_0^8\right)+$$
$$m_\pi^2\left(8(-2+d)^2Q^4m_\Delta^6-12(-2+d)Q^4m_\Delta^5M_0+Q^2m_\Delta^4\left(\left(56-92d+57d^2-13d^3\right)Q^2+2\left(70-70d+19d^2-3d^3\right)m_\Delta^2\right)M_0^2-\right.$$
$$8Q^2m_\Delta^3\left(\left(-10+17d-10d^2+2d^3\right)Q^2+\left(-38+27d-3d^2\right)m_\Delta^2\right)M_0^3-$$
$$2m_\Delta^2\left(\left(-10+12d-9d^2+3d^3\right)Q^4+\left(-188+160d-31d^2+3d^3\right)Q^2m_\Delta^2+6\left(-10+14d-5d^2+d^3\right)m_\Delta^4\right)M_0^4-4Q^2m_\Delta$$
$$\left(\left(-18+29d-18d^2+4d^3\right)Q^2-2\left(18-7d+d^2\right)m_\Delta^2\right)M_0^5+\left(-(-2+d)^2(-21+13d)Q^4+2\left(158-98d+d^2+3d^3\right)Q^2m_\Delta^2+8\left(2-10d+11d^2-3d^3\right)m_\Delta^4\right)$$
$$\left.\left.M_0^6+8m_\Delta\left(\left(-2+23d-19d^2+4d^3\right)Q^2-2\left(10-11d+d^2\right)m_\Delta^2\right)M_0^7+2\left((-2+d)^2(-27+19d)Q^2-2\left(14+6d-27d^2+7d^3\right)m_\Delta^2\right)M_0^8\right)\right)$$
$$(1+\tau[3])\mathrm{II}[d]\left[\{\{0,m_\pi\},1\},\{\{p,m_\Delta\},1\}\right]\Big)+\frac{1}{48(-1+d)^3F^2s^2m_\Delta^4M_0\left(-s+M_0^2\right)}\left(ie^2\mathrm{g}\pi\mathrm{N}\Delta^2\left(\left(Q^2+s\right)\left(2(-2+d)^2s^4\left((-3+d)Q^2+s-ds\right)+(-2+d)s^3+\right.\right.\right.$$
$$\left((-22+(23-3d)d)Q^2+54s+4(-4+d)ds\right)m_\Delta^2 2s^2\left((-2+(9-4d)d)Q^2+(92+d(-43+d(-19+7d)))s\right)m_\Delta^4-(-2+d)s\left(-d(1+d)Q^2+2(-6+(-6+d)d)s\right)m_\Delta^6-$$
$$2\left((-2+d)^2Q^2+30s-d(34+(-10+d)d)s\right)m_\Delta^8\Big)+2m_\Delta\left((-2+d)Q^4\left(s-m_\Delta^2\right)\left(2(4+d(-5+2d))s^2-(-1+d)(-11+4d)sm_\Delta^2-2m_\Delta^4\right)+\right.$$
$$2s^2\left((-2+d)(-3+2d)s^3-(21+2d(-19+6d))s^2m_\Delta^2+(-30+d(12+5d))sm_\Delta^4+(5+(5-3d)d)m_\Delta^6\right)+$$
$$\left.Q^2s\left(-2(2+(-3+d)d(-3+2d))s^3+(-52+d(103+d(-71+16d)))s^2m_\Delta^2+(98+d(-65+d(-5+4d)))sm_\Delta^4-2(45+7(-6+d)d)m_\Delta^6\right)\right)$$
$$M_0+\left((-2+d)^2s^3\left((-1+3d)Q^4+(11-7d)Q^2s+2(2+d)s^2\right)-\right.$$
$$s^2\left(2(-2+d)(1+2(-2+d)d)Q^4-(-156+d(186+d(-57+7d)))Q^2s+2(-68+d(53+d(-13+3d)))s^2\right)m_\Delta^2-$$
$$s\left((-2+d)\left(-4+d+d^2\right)Q^4-(108+d(-72+d(-11+3d)))Q^2s+2(232+d(-165+d(-31+19d)))s^2\right)m_\Delta^4+$$
$$\left.\left(2(-2+d)^2dQ^4-(60+d(-46+d(-5+3d)))Q^2s+2(118+d(-114+d(15+4d)))s^2\right)m_\Delta^6+2(-5+d)(2+(-4+d)d)sm_\Delta^8\right)M_0^2+$$
$$2m_\Delta\left(-(-2+d)s^2\left((-1+d)Q^4+(-1+d)(-9+4d)Q^2s+4(-2+d)s^2\right)+s\left((-2+d)(-1+d)Q^4-4(-8+d(23+2d(-9+2d)))Q^2s+\right.\right.$$
$$\left.2(39+d(-48+7d))s^2\right)m_\Delta^2+s\left((6+d(19+d(-21+4d)))Q^2+4(23+d(-23+5d))s\right)m_\Delta^4+2\left(2(-2+d)Q^2+(-37+(38-7d)d)s\right)m_\Delta^6\Big)M_0^3+$$

$$\left((-2+d)^2s^3\left((11-7d)Q^2+2(-5+d)s\right)+2s^2\left(-2(-2+d)(1+2(-2+d)d)Q^2+(12+d(-9+d(-11+5d)))s\right)m_\Delta^2+\right.$$
$$s\left(-(-2+d)^2(-1+5d)Q^2+2(114+d(-103+4d(1+d)))s\right)m_\Delta^4+2\left(2(-2+d)^2dQ^2-(62+d(-56+d(5+2d)))s\right)m_\Delta^6+2(-2+d)^2m_\Delta^8\Big)M_0^4-$$
$$2m_\Delta\left(s-m_\Delta^2\right)\left((-2+d)s\left((-1+d)Q^2+2s\right)+2(10+(-9+d)d)sm_\Delta^2+2(-2+d)m_\Delta^4\right)M_0^5-$$
$$2(-2+d)\left(s-m_\Delta^2\right)\left((-2+d)^2s^2-(-3+d)(-1+d)sm_\Delta^2+(-2+d)dm_\Delta^4\right)M_0^6+$$
$$2(-2+d)^2m_\pi^8\left(Q^2-3s+M_0^2\right)\left(-Q^2-s+M_0^2\right)+$$
$$m_\pi^4\left(\left(Q^2+s\right)\left(2(-2+d)^2s^2\left(3(-4+d)Q^2+(16-7d)s\right)-s\left((100+d(-118+(47-7d)d))Q^2+2(32+d(8+5(-4+d)d))s\right)m_\Delta^2-\right.\right.$$
$$2\left(6(-2+d)^2Q^2-(-5+3d)\left(2+d^2\right)s\right)m_\Delta^4\Big)+2(-2+d)s\left((11+d(-9+4d))Q^4+(1+3(9-4d)d)Q^2s+2(-5+6d)s^2\right)m_\Delta M_0+$$
$$4\left(3(-2+d)Q^4-(27+(-14+d)d)Q^2s+(43+d(-49+11d))s^2\right)m_\Delta^3M_0+\left((-2+d)^2s\left((1+5d)Q^4+(45-33d)Q^2s+6(-6+5d)s^2\right)+\right.$$
$$\left(6(-2+d)^2dQ^4-(-48+d(110+d(-71+13d)))Q^2s+2(74+d(26+d(-85+22d)))s^2\right)m_\Delta^2+2(-1+d)(34+d(-20+3d))sm_\Delta^4\Big)M_0^2+$$
$$2m_\Delta\left((-2+d)s\left((13+d(-9+4d))Q^2+4(2-3d)s\right)-2\left(3s+(-2+d)\left(-6Q^2+ds\right)\right)m_\Delta^2\right)M_0^3+$$
$$\left((-2+d)^2s\left((5+7d)Q^2-18ds\right)+2\left(6(-2+d)^2dQ^2-(22+d(18+5d(-7+2d)))s\right)m_\Delta^2+12(-2+d)^2m_\Delta^4\right)M_0^4+$$
$$4(-2+d)m_\Delta\left(s+3m_\Delta^2\right)M_0^5+2(-2+d)^2\left((2+d)s+3dm_\Delta^2\right)M_0^6\Big)+$$
$$m_\pi^2\left(-\left(Q^2+s\right)\left((-2+d)^2s^3\left(7(-3+d)Q^2+2(8-5d)s\right)+2s^2\left(\left(-8+d+d^3\right)Q^2-(-3+d)(-44+d(17+2d))s\right)m_\Delta^2-\right.\right.$$
$$s\left((56+d(-68+(33-7d)d))Q^2+2(128+d(-90+d(4+3d)))s\right)m_\Delta^4-2\left(4(-2+d)^2Q^2+54s-3d(18+(-5+d)d)s\right)m_\Delta^6\Big)+$$
$$2m_\Delta\left((-2+d)s^2\left((-19+(21-8d)d)Q^4+(11+d(-47+16d))Q^2s+2(7-6d)s^2\right)-\right.$$
$$2s\left(2(-2+d)(5+2(-3+d)d)Q^4-(26+d(11+d(-17+4d)))Q^2s+(66+d(-53+5d))s^2\right)m_\Delta^2-$$
$$2\left(3(-2+d)Q^4-2(32+3(-8+d)d)Q^2s+\left(46-39d+6d^2\right)s^2\right)m_\Delta^4\Big)M_0+\left(-(-2+d)^2s^2\left(d\left(2Q^2-9s\right)\left(3Q^2-2s\right)+\left(43Q^2-8s\right)s\right)-\right.$$
$$2s\left((-2+d)(1+d(-5+2d))Q^4-(30+d(-10+d(-11+3d)))Q^2s+(248+d(-195+d(9+8d)))s^2\right)m_\Delta^2-$$
$$\left(6(-2+d)^2dQ^4-(64+d(-24+d(-27+7d)))Q^2s+2(220+d(-136+d(-34+19d)))s^2\right)m_\Delta^4-2(-22+d(46+d(-19+3d)))sm_\Delta^6\Big)M_0^2+$$
$$2m_\Delta\left((-2+d)s\left(-(-1+d)Q^4+4(-2+d)Q^2s+4(-4+3d)s^2\right)+2s\left(-(-1+d)(10+d(-15+4d))Q^2+2(-6+d)(-3+d)s\right)m_\Delta^2+\right.$$
$$4\left(-3(-2+d)Q^2+(20+3(-6+d)d)s\right)m_\Delta^4\Big)M_0^3+2\left((-2+d)^2(8+3d)s^3-\left(-102+d\left(131-60d+9d^2\right)\right)s^2m_\Delta^2+\right.$$
$$(100+d(-74+d(-6+7d)))sm_\Delta^4-4(-2+d)^2m_\Delta^6+(-2+d)^2Q^2\left(2(-4+d)s^2-(3+d)sm_\Delta^2-6dm_\Delta^4\right)\Big)M_0^4+2m_\Delta\left(-(-2+d)\left((-1+d)Q^2-2s\right)s-\right.$$
$$2(10+(-9+d)d)sm_\Delta^2-6(-2+d)m_\Delta^4\Big)M_0^5+2(-2+d)\left((-4+d)(-2+d)s^2+(7+(-6+d)d)sm_\Delta^2-3(-2+d)dm_\Delta^4\right)M_0^6\Big)-$$
$$m_\pi^6\left(4s\left(-\left(11Q^2-24s\right)\left(Q^2+s\right)+\left(13Q^2-32s\right)M_0^2+8M_0^4\right)+4m_\Delta^2\left(-\left(8Q^2-13s\right)\left(Q^2+s\right)-19sM_0^2+8M_0^4\right)+\right.$$
$$d^2\left(-\left(Q^2+s\right)\left(-6s\left(8s+m_\Delta^2\right)+Q^2\left(15s+8m_\Delta^2\right)\right)-8\left(Q^2-s\right)sm_\Delta M_0-\left(8Q^4-49Q^2s+88s^2+34sm_\Delta^2\right)M_0^2-8sm_\Delta M_0^3+8\left(-2Q^2+6s+m_\Delta^2\right)M_0^4-8M_0^6\right)+$$
$$d^3\left(s\left(Q^2+s\right)\left(Q^2-6s+2m_\Delta^2\right)+\left(2Q^4-9Q^2s+2s\left(7s+m_\Delta^2\right)\right)M_0^2+2\left(2Q^2-5s\right)M_0^4+2M_0^6\right)+$$
$$4d\left(\left(Q^2+s\right)\left(6\left(2Q^2-5s\right)s+\left(8Q^2-13s\right)m_\Delta^2\right)+\left(Q^4+8Q^2s-5s^2\right)m_\Delta M_0+\left(2Q^4-22Q^2s+23s\left(2s+m_\Delta^2\right)\right)M_0^2+\right.$$
$$2\left(Q^2+2s\right)m_\Delta M_0^3+2\left(2Q^2-9s-4m_\Delta^2\right)M_0^4+m_\Delta M_0^5+2M_0^6\Big)-8m_\Delta M_0\left(4Q^2s-s^2+\left(Q^2+M_0^2\right)^2\right)\Big)\Big)\Big)$$
$$(1+\tau[3])\mathrm{II}[d]\left[\left\{\{0,m_\pi\},1\right\},\left\{\{p+q,m_\Delta\},1\right\}\right]\Big)-\frac{1}{12(-1+d)^2F^2m_\Delta^4M_0\left(-s+M_0^2\right)}\left(ie^2\mathrm{g}\pi\mathrm{N}\Delta^2\left(m_\pi^6\left((-2+d)^2Q^2-2(-2+(-2+d)d)m_\Delta^2\right)M_0^2+\right.\right.$$
$$2(-2+d)Q^2\left((-3+2d)Q^2-s\right)sm_\Delta M_0^3+(-2+d)^2Q^2\left(2Q^2-s\right)sM_0^4-$$
$$2m_\Delta^7M_0\left((2+d(-13+4d))Q^2+(-10+3d)s+(-4+5d)M_0^2\right)+$$
$$2m_\Delta^8\left(-(1+2(-3+d)d)\left(Q^2+s\right)+(13+(-8+d)d)M_0^2\right)+$$
$$2m_\Delta^5M_0\left((-2+d)(-3+2d)Q^4+2(2+d(-7+2d))Q^2s+(-30+7d)s^2+\left((2+d(-13+4d))Q^2+4(6+d)s\right)M_0^2+(-22+5d)M_0^4\right)-$$
$$m_\Delta^6\left(-\left(Q^2+s\right)\left((-2+d)(-3+2d)Q^2+2(-1+d)(-5+2d)s\right)+\left((-5+d)dQ^2-4\left(-9+d+d^2\right)s\right)M_0^2+2(13+(-7+d)d)M_0^4\right)-2m_\Delta^3M_0$$
$$\left((-2+d)Q^2s\left((-3+2d)Q^2+(-7+4d)s\right)+\left((-2+d)(-3+2d)Q^4-2(10+d(-7+2d))Q^2s+(-12+7d)s^2\right)M_0^2+(-2+d)\left((-6+4d)Q^2+s\right)M_0^4\right)-$$
$$m_\Delta^2M_0^2\left((-2+d)s\left(-2Q^4+(-4+d)Q^2s-2(-1+d)s^2\right)+\left((-2+d)(-3+2d)Q^4-(42+d(-21+4d))Q^2s+2(6+(-4+d)d)s^2\right)M_0^2+\right.$$

$$(-2+d)\left((-7+4d)Q^2-2(-1+d)s\right)M_0^4\right)+m_\Delta^4\left(-2(-2+d)(-1+d)s\left(Q^2+s\right)\left(Q^2+2s\right)+\right.$$
$$2s\left((-5+d(-7+3d))Q^2+(-9+2(-2+d)d)s\right)M_0^2+\left((-8+(-4+d)d)Q^2+86s-8d^2s\right)M_0^4+2\left(-14+d^2\right)M_0^6\right)+$$
$$m_\pi^4\left(2(-2+d)Q^2\left((-3+2d)Q^2-s\right)m_\Delta M_0-(-2+d)^2Q^2M_0^2\left(-2Q^2+2s+M_0^2\right)-2m_\Delta^3M_0\left((2+d(-13+4d))Q^2+(-10+3d)s+(-4+5d)M_0^2\right)+\right.$$
$$2m_\Delta^4\left(-(-17+2(-3+d)d)\left(Q^2+s\right)+3(-3+(-4+d)d)M_0^2\right)+$$
$$m_\Delta^2\left((8+d(-7+2d))Q^2\left(Q^2+s\right)+\left((4+(7-3d)d)Q^2+4(-2+(-2+d)d)s\right)M_0^2+2(-2+(-2+d)d)M_0^4\right)\right)+$$
$$m_\pi^2\left(-4m_\Delta^5M_0\left((18+(11-4d)d)Q^2-5(-6+d)s+(-44+13d)M_0^2\right)-2m_\Delta^6\left(-4(-3+d)d\left(Q^2+s\right)+(-18+d(-2+3d))M_0^2\right)-\right.$$
$$(-2+d)^2Q^2M_0^2\left(\left(2Q^2-s\right)s+2\left(Q^2-s\right)M_0^2\right)-2(-2+d)Q^2m_\Delta M_0\left(\left((-5+2d)Q^2-s\right)s+\left((-1+2d)Q^2-s\right)M_0^2\right)+$$
$$2m_\Delta^3M_0\left(2(2+d(-7+2d))Q^4+(-30+d(-3+4d))Q^2s+(-10+3d)s^2+\left((38+d(-25+4d))Q^2+4(-4+3d)s\right)M_0^2+(-2+d)M_0^4\right)-$$
$$m_\Delta^4\left(-2\left(Q^2+s\right)\left((-3+d)(-1+2d)Q^2+(-15+d(-3+2d))s\right)+\left((20+(40-11d)d)Q^2+4(18-7d)s\right)M_0^2+2(-27+7d)M_0^4\right)-$$
$$2m_\Delta^2\left((-3+d)(-2+d)Q^2s\left(Q^2+s\right)+\left(-\left(3-6d+d^2\right)Q^4+3(5-2d)Q^2s+d(-5+2d)s^2\right)M_0^2+\right.$$
$$\left.\left.\left(-(-5+d)(-1+2d)Q^2+2(-4+d)s\right)M_0^4+(-2+d)(-1+d)M_0^6\right)\right)\right)$$
$$(1+\tau[3])\mathrm{II}[d]\left[\left\{\left\{0,m_\pi\right\},1\right\},\left\{\left\{p+q,m_\Delta\right\},1\right\},\left\{\left\{p,m_\Delta\right\},1\right\}\right]+\frac{1}{12(-1+d)^2F^2m_\Delta^4M_0\left(-s+M_0^2\right)}\left(ie^2\mathrm{g}\pi\mathrm{N}\Delta^2\left(m_\pi^6\left((-2+d)^2Q^2-2\left(-2-2d+d^2\right)m_\Delta^2\right)M_0^2+\right.\right.$$
$$m_\pi^4\left(Q^2m_\Delta^2\left(\left(8-7d+2d^2\right)Q^2+2\left(17+6d-2d^2\right)m_\Delta^2\right)+2Q^2m_\Delta\left(\left(6-7d+2d^2\right)Q^2+\left(-2+13d-4d^2\right)m_\Delta^2\right)M_0+\right.$$
$$\left.\left(2(-2+d)^2Q^4-\left(-12+d^2\right)Q^2m_\Delta^2+2\left(8-6d+d^2\right)m_\Delta^4\right)M_0^2-2m_\Delta\left((-2+d)Q^2+2(-7+4d)m_\Delta^2\right)M_0^3-3\left((-2+d)^2Q^2-2\left(-2-2d+d^2\right)m_\Delta^2\right)M_0^4\right)+$$
$$\left(-m_\Delta^2+M_0^2\right)^2\left(Q^2m_\Delta^2\left(\left(6-7d+2d^2\right)Q^2-2\left(1-6d+2d^2\right)m_\Delta^2\right)+2Q^2m_\Delta\left(\left(6-7d+2d^2\right)Q^2+\left(-2+13d-4d^2\right)m_\Delta^2\right)M_0+\right.$$
$$\left.\left(2(-2+d)^2Q^4+\left(12+8d-3d^2\right)Q^2m_\Delta^2-2\left(-12+2d+d^2\right)m_\Delta^4\right)M_0^2-2m_\Delta\left((-2+d)Q^2+2(-7+4d)m_\Delta^2\right)M_0^3-\left((-2+d)^2Q^2-2\left(-2-2d+d^2\right)m_\Delta^2\right)M_0^4\right)+$$
$$m_\pi^2\left(m_\Delta+M_0\right)\left(2(-3+d)Q^2m_\Delta^3\left((-1+2d)Q^2+4dm_\Delta^2\right)+2Q^2m_\Delta^2\left(\left(1-7d+2d^2\right)Q^2+2\left(-18-5d+2d^2\right)m_\Delta^2\right)M_0+\right.$$
$$m_\Delta\left(-4\left(2-3d+d^2\right)Q^4+\left(28-40d+11d^2\right)Q^2m_\Delta^2+2\left(18-10d+d^2\right)m_\Delta^4\right)M_0^2-\left(4(-2+d)^2Q^4+\left(12+16d-5d^2\right)Q^2m_\Delta^2+2\left(-10+6d+d^2\right)m_\Delta^4\right)M_0^3+$$
$$\left.\left.m_\Delta\left(\left(-20+16d-3d^2\right)Q^2+2\left(-34+10d+3d^2\right)m_\Delta^2\right)M_0^4+3\left((-2+d)^2Q^2-2\left(-2-2d+d^2\right)m_\Delta^2\right)M_0^5\right)\right)$$
$$(1+\tau[3])\mathrm{II}[d]\left[\left\{\left\{0,m_\pi\right\},1\right\},\left\{\left\{\mathrm{pr}+q,m_\Delta\right\},1\right\},\left\{\left\{\mathrm{pr},m_\Delta\right\},1\right\}\right]-\frac{1}{6(-1+d)^2F^2m_\Delta^4M_0\left(-s+M_0^2\right)}\left(ie^2\mathrm{g}\pi\mathrm{N}\Delta^2\pi\right.$$
$$\left(4M_0^2\left(Q^2+\frac{\left(Q^2+s-M_0^2\right)^2}{4M_0^2}\right)\left(2(18+(-8+d)d)m_\Delta^6-4(-17+5d)m_\Delta^5M_0-(-2+d)^2Q^2\left(-s+m_\pi^2\right)\left(m_\pi-M_0\right)\left(m_\pi+M_0\right)+2(-2+d)Q^2m_\Delta M_0\left(2s-3m_\pi^2+M_0^2\right)+\right.\right.$$
$$2m_\Delta^3M_0\left(6Q^2-3dQ^2-22s+8ds+(34-10d)m_\pi^2+2(-6+d)M_0^2\right)-m_\Delta^4\left(44s+2(-10+d)ds+4(6+(-2+d)d)m_\pi^2-12M_0^2+(-2+d)d\left(Q^2+2M_0^2\right)\right)+$$
$$m_\Delta^2\left((-2+d)s\left((2+d)Q^2+2(-1+d)s\right)+2(-2+(-4+d)d)m_\pi^4+\left((-4+d)(-2+d)Q^2-2(6+(-2+d)d)s\right)M_0^2+\right.$$
$$\left.\left.2(-2+d)(-1+d)M_0^4-2m_\pi^2\left((4+(-3+d)d)Q^2+(-2+(-4+d)d)s+(-2+(-4+d)d)M_0^2\right)\right)\right)-$$
$$\left(Q^2+s+M_0^2\right)\left((-2+d)s\left((-2+d)Q^4+(-8+d)Q^2s-2(-1+d)s^2\right)m_\Delta^2-\left((-2+d)^2Q^4+2(6+(-9+d)d)Q^2s+2(-32+5d)s^2\right)m_\Delta^4+\right.$$
$$\left((20+d(-20+3d))Q^2+4(-3+2d)s\right)m_\Delta^6-2(-12+d(2+d))m_\Delta^8+m_\pi^6\left(-(-2+d)^2Q^2+2(-2+(-2+d)d)m_\Delta^2\right)+$$
$$2m_\Delta^3\left(-(-2+d)Q^4+(-2+4d)s^2+4(13-2d)sm_\Delta^2+2(7-4d)m_\Delta^4+Q^2\left((8+d)s+(16-9d)m_\Delta^2\right)\right)M_0+$$
$$\left((-2+d)^2Q^2s\left(-Q^2+s\right)+\left((-2+d)^2Q^4-2(12+(-5+d)d)Q^2s+2(6+(-4+d)d)s^2\right)m_\Delta^2+\left((-4+(10-3d)d)Q^2+4\left(-17+d+d^2\right)s\right)m_\Delta^4+\right.$$
$$\left.2(4+(-6+d)d)m_\Delta^6\right)M_0^2+2m_\Delta\left((-2+d)Q^2\left(Q^2+s\right)+\left((-22+7d)Q^2+4(-3+d)s\right)m_\Delta^2+4(-13+2d)m_\Delta^4\right)M_0^3+$$
$$2m_\Delta^2\left((-2+d)\left(d\left(Q^2-s\right)+s\right)-(10+(-5+d)d)m_\Delta^2\right)M_0^4+m_\pi^4\left(-2(4+d(-10+3d))m_\Delta^4+2(-2+d)Q^2m_\Delta M_0+4(-7+4d)m_\Delta^3M_0-\right.$$
$$\left.(-2+d)^2Q^2\left(Q^2-2s-M_0^2\right)+m_\Delta^2\left(8s+d\left((-4+d)Q^2-4(-2+d)s\right)+(4-2(-2+d)d)M_0^2\right)\right)+$$
$$m_\pi^2\left(6(-2+(-2+d)d)m_\Delta^6-2(-2+d)Q^2m_\Delta M_0\left(Q^2+s+M_0^2\right)-4m_\Delta^3M_0\left(-7Q^2+4dQ^2-8s+6ds+2(-3+d)M_0^2\right)+\right.$$
$$m_\Delta^4\left((24+(4-3d)d)Q^2+4(3+(-8+d)d)s+4(5+2d)M_0^2\right)+(-2+d)^2Q^2\left(\left(Q^2-s\right)s+\left(Q^2-2s\right)M_0^2\right)+$$
$$\left.\left.\left.2m_\Delta^2\left(-(-2+d)^2Q^4-3(-2+d)Q^2s+d(-5+2d)s^2+\left(-(2+(-7+d)d)Q^2+2(-4+d)s\right)M_0^2+(-2+d)(-1+d)M_0^4\right)\right)\right)\right)$$
$$(1+\tau[3])\mathrm{II}[2+d]\left[\left\{\left\{0,m_\pi\right\},1\right\},\left\{\left\{p,m_\Delta\right\},1\right\},\left\{\left\{p+q,m_\Delta\right\},2\right\}\right]-4i\pi$$

$$\Big(-\frac{1}{12(-1+d)^2F^2m_\Delta^4M_0(-s+M_0^2)}\left(e^2 g\pi N\Delta^2M_0^2\left(-m_\pi^6\left((-2+d)^2Q^2-2\left(-2-2d+d^2\right)m_\Delta^2\right)-\right.\right.$$
$$m_\pi^4\left((-2+d)^2Q^4-(-4+d)dQ^2m_\Delta^2+2\left(4-10d+3d^2\right)m_\Delta^4-2m_\Delta\left((-2+d)Q^2+2(-7+4d)m_\Delta^2\right)M_0-3\left((-2+d)^2Q^2-2\left(-2-2d+d^2\right)m_\Delta^2\right)M_0^2\right)-$$
$$\left(-m_\Delta^2+M_0^2\right)\left(-m_\Delta^2\left((-2+d)^2Q^4+\left(-20+20d-3d^2\right)Q^2m_\Delta^2+2\left(-12+2d+d^2\right)m_\Delta^4\right)-2m_\Delta\left(Q^2+m_\Delta^2\right)\left((-2+d)Q^2+2(-7+4d)m_\Delta^2\right)M_0+\right.$$
$$\left((-2+d)^2Q^4-2\left(-2-4d+d^2\right)Q^2m_\Delta^2+4(5-2d)m_\Delta^4\right)M_0^2-2m_\Delta\left((-2+d)Q^2+2(-7+4d)m_\Delta^2\right)M_0^3-\left((-2+d)^2Q^2-2\left(-2-2d+d^2\right)m_\Delta^2\right)M_0^4\Big)-$$
$$m_\pi^2\left(m_\Delta^2\left(2(-2+d)^2Q^4+\left(-24-4d+3d^2\right)Q^2m_\Delta^2-6\left(-2-2d+d^2\right)m_\Delta^4\right)+2Q^2m_\Delta\left((-2+d)Q^2+2(-7+4d)m_\Delta^2\right)M_0-2\left((-2+d)^2Q^4+\right.\right.$$
$$\left.\left(4+4d-d^2\right)Q^2m_\Delta^2+2\left(8-6d+d^2\right)m_\Delta^4\right)M_0^2+4m_\Delta\left((-2+d)Q^2+2(-7+4d)m_\Delta^2\right)M_0^3+3\left((-2+d)^2Q^2-2\left(-2-2d+d^2\right)m_\Delta^2\right)M_0^4\Big)\Big)(1+\tau[3])\Big)+$$
$$\frac{1}{24(-1+d)^2F^2m_\Delta^4M_0(-s+M_0^2)}\left(e^2 g\pi N\Delta^2\left(-m_\pi^6\left((-2+d)^2Q^2-2\left(-2-2d+d^2\right)m_\Delta^2\right)\left(Q^2+2M_0^2\right)+m_\pi^4\left(Q^2m_\Delta^2\left(\left(4+4d-d^2\right)Q^2-2\left(4-10d+3d^2\right)m_\Delta^2\right)+\right.\right.\right.$$
$$2Q^2m_\Delta\left((-2+d)Q^2+2(-7+4d)m_\Delta^2\right)M_0+\left(5(-2+d)^2Q^4-4\left(-7-9d+3d^2\right)Q^2m_\Delta^2-4\left(4-10d+3d^2\right)m_\Delta^4\right)M_0^2+$$
$$4m_\Delta\left((-2+d)Q^2+2(-7+4d)m_\Delta^2\right)M_0^3+6\left((-2+d)^2Q^2-2\left(-2-2d+d^2\right)m_\Delta^2\right)M_0^4\Big)+$$
$$m_\pi^2\left(Q^2m_\Delta^2\left(48Q^2m_\Delta^2-12m_\Delta^4+d^2m_\Delta^2\left(Q^2+6m_\Delta^2\right)+2d\left(Q^4-2Q^2m_\Delta^2-6m_\Delta^4\right)\right)+4Q^4m_\Delta\left((-2+d)Q^2+(-10+d)m_\Delta^2\right)M_0+\right.$$
$$2m_\Delta^2\left(\left(8-8d+3d^2\right)Q^4+\left(88-24d+7d^2\right)Q^2m_\Delta^2+6\left(-2-2d+d^2\right)m_\Delta^4\right)M_0^2+16Q^2m_\Delta\left((-2+d)Q^2+(-10+d)m_\Delta^2\right)M_0^3+\left(-7(-2+d)^2Q^4+2\right.$$
$$\left.\left(-14-30d+9d^2\right)Q^2m_\Delta^2+8\left(8-6d+d^2\right)m_\Delta^4\right)M_0^4-8m_\Delta\left((-2+d)Q^2+2(-7+4d)m_\Delta^2\right)M_0^5-6\left((-2+d)^2Q^2-2\left(-2-2d+d^2\right)m_\Delta^2\right)M_0^6\Big)+$$
$$\left(-m_\Delta^2+M_0^2\right)\left(Q^2m_\Delta^2\left(-2(-2+d)Q^4+\left(16+4d-d^2\right)Q^2m_\Delta^2+2\left(-12+2d+d^2\right)m_\Delta^4\right)-2Q^2m_\Delta\left(2(-2+d)Q^4+(-18+d)Q^2m_\Delta^2+2(7-4d)m_\Delta^4\right)M_0+\right.$$
$$2m_\Delta^2\left(-\left(-4+d^2\right)Q^4+\left(42-8d+d^2\right)Q^2m_\Delta^2+2\left(-12+2d+d^2\right)m_\Delta^4\right)M_0^2-2m_\Delta\left(9(-2+d)Q^4+2(-45+7d)Q^2m_\Delta^2+4(7-4d)m_\Delta^4\right)M_0^3+\left(3(-2+d)^2Q^4+\right.$$
$$\left.2\left(6+10d-3d^2\right)Q^2m_\Delta^2+8(-5+2d)m_\Delta^4\right)M_0^4+4m_\Delta\left((-2+d)Q^2+2(-7+4d)m_\Delta^2\right)M_0^5+2\left((-2+d)^2Q^2-2\left(-2-2d+d^2\right)m_\Delta^2\right)M_0^6\Big)\Big)(1+\tau[3])\Big)\Big)$$
$$\mathrm{II}[2+d]\left[\left\{\{0,m_\pi\},1\right\},\left\{\{\mathrm{pr},m_\Delta\},1\right\},\left\{\{\mathrm{pr}+q,m_\Delta\},2\right\}\right]-\frac{1}{6(-1+d)^2F^2m_\Delta^4M_0(-s+M_0^2)}\Big(2ie^2 g\pi N\Delta^2\pi\left((-2+d)^2Q^2sM_0^2\left((-3+d)\left(Q^2+s\right)+2M_0^2\right)-\right.$$
$$2m_\Delta^5M_0\left(-(-11+d)d\left(Q^2+s\right)-2\left(Q^2+5s\right)+3(-1+d)(-8+3d)M_0^2\right)-2(-1+d)m_\Delta^6\left((7+(-5+d)d)\left(Q^2+s\right)+(-19+7d)M_0^2\right)+$$
$$2(-2+d)Q^2m_\Delta M_0\left(2s\left(Q^2+s\right)+\left(-(-3+d)Q^2+(3+d)s\right)M_0^2+2M_0^4\right)+$$
$$2m_\Delta^3M_0\left((2+(-7+d)d)Q^4+2\left(-13+d^2\right)Q^2s+(-20+d(3+d))s^2+\left(-2(21+d(-11+2d))Q^2+(52+d(-27+7d))s\right)M_0^2+2(-9+d)M_0^4\right)+$$
$$m_\Delta^4\left(\left(Q^2+s\right)\left((-2+(-5+d)(-1+d)d)Q^2+2(-9+d(5+(-5+d)d))s\right)+\left((-66+d(31+d(-13+2d)))Q^2+2(37+d(-25+d(3+d)))s\right)M_0^2+4(-7+3d)M_0^4\right)-$$
$$m_\Delta^2\left((-2+d)s\left(Q^2+s\right)\left((2+(-5+d)d)Q^2+2(-3+d)(-1+d)s\right)+\left(\left(6+(-3+d)^2d\right)Q^4-(-74+d(37+(-4+d)d))Q^2s-\right.\right.$$
$$\left.2(-2+d)(13+(-6+d)d)s^2\right)M_0^2+\left(\left(26+d-7d^2+2d^3\right)Q^2+2(-34+d(24+(-7+d)d))s\right)M_0^4+4(8+(-5+d)d)M_0^6\Big)+$$
$$m_\pi^4\left(16(-2+d)Q^2m_\Delta M_0+(-2+d)^2Q^2\left((-3+d)\left(Q^2+s\right)+2M_0^2\right)-2(-1+d)m_\Delta^2\left((2+(-5+d)d)\left(Q^2+s\right)+(-4+3d)M_0^2\right)\right)+$$
$$m_\pi^2\left(-(-2+d)^2Q^2\left(s+M_0^2\right)\left((-3+d)\left(Q^2+s\right)+2M_0^2\right)+2(-2+d)Q^2m_\Delta M_0\left((-5+d)Q^2+(-9+d)s-2(3+d)M_0^2\right)+\right.$$
$$2(-1+d)m_\Delta^4\left((17+2(-5+d)d)\left(Q^2+s\right)+(-11+2d)M_0^2\right)-2m_\Delta^3M_0\left(-(-17+d)(-2+d)Q^2-(-10+d)(-1+d)s+3(-1+d)(-8+3d)M_0^2\right)+$$
$$m_\Delta^2\left(\left(Q^2+s\right)\left((-4+d(19+d(-13+2d)))Q^2+2(-5+d)(-2+d)(1+d)s\right)+\right.$$
$$\left.\left.\left((3+d)(24+d(-15+2d))Q^2+2(-22+d(8+(-3+d)d))s\right)M_0^2+\left(32-22d+6d^2\right)M_0^4\right)\right)\Big)$$
$$(1+\tau[3])\mathrm{II}[2+d]\left[\left\{\{0,m_\pi\},1\right\},\left\{\{p+q,m_\Delta\},1\right\},\left\{\{p,m_\Delta\},1\right\}\right]\Big)+\frac{1}{12(-1+d)^2F^2m_\Delta^4M_0(-s+M_0^2)}\left(e^2 g\pi N\Delta^2M_0^2\left((-2+d)s\left((-2+d)Q^4+(-8+d)Q^2s-2(-1+d)s^2\right)m_\Delta^2-\right.\right.$$
$$\left((-2+d)^2Q^4+2(6+(-9+d)d)Q^2s+2(-32+5d)s^2\right)m_\Delta^4+\left((20+d(-20+3d))Q^2+4(-3+2d)s\right)m_\Delta^6-2(-12+d(2+d))m_\Delta^8+m_\pi^6\left(-(-2+d)^2Q^2+2(-2+(-2+d)d)m_\Delta^2\right)+$$
$$2m_\Delta^3\left(-(-2+d)Q^4+(-2+4d)s^2+4(13-2d)sm_\Delta^2+2(7-4d)m_\Delta^4+Q^2\left((8+d)s+(16-9d)m_\Delta^2\right)\right)M_0+$$
$$\left((-2+d)^2Q^2s\left(-Q^2+s\right)+\left((-2+d)^2Q^4-2(12+(-5+d)d)Q^2s+2(6+(-4+d)d)s^2\right)m_\Delta^2+\left((-4+(10-3d)d)Q^2+4\left(-17+d+d^2\right)s\right)m_\Delta^4+2(4+(-6+d)d)m_\Delta^6\right)M_0^2+$$
$$2m_\Delta\left((-2+d)Q^2\left(Q^2+s\right)+\left((-22+7d)Q^2+4(-3+d)s\right)m_\Delta^2+4(-13+2d)m_\Delta^4\right)M_0^3+2m_\Delta^2\left((-2+d)\left(d\left(Q^2-s\right)+s\right)-(10+(-5+d)d)m_\Delta^2\right)M_0^4+$$
$$m_\pi^4\left(-2(4+d(-10+3d))m_\Delta^4+2(-2+d)Q^2m_\Delta M_0+4(-7+4d)m_\Delta^3M_0-(-2+d)^2Q^2\left(Q^2-2s-M_0^2\right)+m_\Delta^2\left(8s+d\left((-4+d)Q^2-4(-2+d)s\right)+\left(4-2(-2+d)d\right)M_0^2\right)\right)+$$
$$m_\pi^2\left(6(-2+(-2+d)d)m_\Delta^6-2(-2+d)Q^2m_\Delta M_0\left(Q^2+s+M_0^2\right)-4m_\Delta^3M_0\left(-7Q^2+4dQ^2-8s+6ds+2(-3+d)M_0^2\right)+\right.$$
$$m_\Delta^4\left((24+(4-3d)d)Q^2+4(3+(-8+d)d)s+4(5+2d)M_0^2\right)+(-2+d)^2Q^2\left(\left(Q^2-s\right)s+\left(Q^2-2s\right)M_0^2\right)+$$

$$2m_\Delta^2\left(-(-2+d)^2Q^4-3(-2+d)Q^2s+d(-5+2d)s^2+\left(-(2+(-7+d)d)Q^2+2(-4+d)s\right)M_0^2+(-2+d)(-1+d)M_0^4\right)\Big)\Big)\left(1+\tau[3]\right)$$
$$\left(-4i\pi\mathrm{II}[2+d]\left[\{\{0,m_\pi\},1\},\{\{p,m_\Delta\},1\},\{\{p+q,m_\Delta\},2\}\right]+4i\left(\pi\mathrm{II}[2+d]\left[\{\{0,m_\pi\},1\},\{\{p,m_\Delta\},1\},\{\{p+q,m_\Delta\},2\}\right]+\right.\right.$$
$$\left.\left.\pi\mathrm{II}[2+d]\left[\{\{0,m_\pi\},1\},\{\{p+q,m_\Delta\},1\},\{\{p,m_\Delta\},2\}\right]\right)\right)\Big)+\frac{1}{6(-1+d)^2F^2m_\Delta^4M_0\left(-s+M_0^2\right)}$$
$$\Big(2ie^2\mathrm{g}\pi\mathrm{N}\Delta^2\pi\Big(m_\pi^4\left(Q^2\left((-3+d)(-2+d)^2Q^2-2\left(-2+7d-6d^2+d^3\right)m_\Delta^2\right)+16(-2+d)Q^2m_\Delta M_0+(-1+d)\left((-2+d)^2Q^2-2\left(-2-2d+d^2\right)m_\Delta^2\right)M_0^2\right)+$$
$$m_\pi^2\Big(Q^2m_\Delta^2\left(\left(-4+19d-13d^2+2d^3\right)Q^2+2\left(-17+27d-12d^2+2d^3\right)m_\Delta^2\right)+2(-2+d)Q^2m_\Delta\left((-5+d)Q^2+(-17+d)m_\Delta^2\right)M_0+$$
$$\left(-2(-3+d)(-2+d)^2Q^4+2\left(44+2d-17d^2+3d^3\right)Q^2m_\Delta^2+4(-3+d)(-1+d)^2m_\Delta^4\right)M_0^2-2m_\Delta\left(\left(-30+13d+d^2\right)Q^2+2\left(7-11d+4d^2\right)m_\Delta^2\right)$$
$$M_0^3-2(-1+d)\left((-2+d)^2Q^2-2\left(-2-2d+d^2\right)m_\Delta^2\right)M_0^4\Big)-$$
$$\left(m_\Delta+M_0\right)\Big(Q^2m_\Delta^3\left(-\left(-2+5d-6d^2+d^3\right)Q^2+2\left(-7+12d-6d^2+d^3\right)m_\Delta^2\right)+Q^2m_\Delta^2\left(\left(-6+19d-8d^2+d^3\right)Q^2-2\left(-5+d-5d^2+d^3\right)m_\Delta^2\right)M_0+$$
$$m_\Delta\left(\left(8+2d-5d^2+d^3\right)Q^4+\left(76-44d+19d^2-3d^3\right)Q^2m_\Delta^2+2\left(12-14d+d^2+d^3\right)m_\Delta^4\right)M_0^2+\left(-(-3+d)(-2+d)^2Q^4+3\left(20-5d^2+d^3\right)Q^2m_\Delta^2-2\left(-2+8d-7d^2+d^3\right)m_\Delta^4\right)M_0^3+$$

$$(-2+d)m_\Delta\left(\left(-12-5d+d^2\right)Q^2-2\left(-8+7d+d^2\right)m_\Delta^2\right)M_0^4-(-1+d)\left((-2+d)^2Q^2-2\left(-2-2d+d^2\right)m_\Delta^2\right)M_0^5\Big)\Big)$$
$$(1+\tau[3])\mathrm{II}[2+d]\left[\{\{0,m_\pi\},1\},\{\{\mathrm{pr}+q,m_\Delta\},1\},\{\{\mathrm{pr},m_\Delta\},1\}\right]\Big)-\frac{1}{6(-1+d)^2F^2m_\Delta^4M_0\left(-s+M_0^2\right)}\Big(2ie^2\mathrm{g}\pi\mathrm{N}\Delta^2M_0^2\Big(-m_\pi^6\left((-2+d)^2Q^2-2\left(-2-2d+d^2\right)m_\Delta^2\right)-$$
$$m_\pi^4\left((-2+d)^2Q^4-(-4+d)dQ^2m_\Delta^2+2\left(4-10d+3d^2\right)m_\Delta^4-2m_\Delta\left((-2+d)Q^2+2(-7+4d)m_\Delta^2\right)M_0-3\left((-2+d)^2Q^2-2\left(-2-2d+d^2\right)m_\Delta^2\right)M_0^2\right)-$$
$$\left(-m_\Delta^2+M_0^2\right)\Big(-m_\Delta^2\left((-2+d)^2Q^4+\left(-20+20d-3d^2\right)Q^2m_\Delta^2+2\left(-12+2d+d^2\right)m_\Delta^4\right)-2m_\Delta\left(Q^2+m_\Delta^2\right)\left((-2+d)Q^2+2(-7+4d)m_\Delta^2\right)M_0+$$
$$\left((-2+d)^2Q^4-2\left(-2-4d+d^2\right)Q^2m_\Delta^2+4(5-2d)m_\Delta^4\right)M_0^2-2m_\Delta\left((-2+d)Q^2+2(-7+4d)m_\Delta^2\right)M_0^3-\left((-2+d)^2Q^2-2\left(-2-2d+d^2\right)m_\Delta^2\right)M_0^4\Big)-$$
$$m_\pi^2\Big(m_\Delta^2\left(2(-2+d)^2Q^4+\left(-24-4d+3d^2\right)Q^2m_\Delta^2-6\left(-2-2d+d^2\right)m_\Delta^4\right)+2Q^2m_\Delta\left((-2+d)Q^2+2(-7+4d)m_\Delta^2\right)M_0-$$
$$2\left((-2+d)^2Q^4+\left(4+4d-d^2\right)Q^2m_\Delta^2+2\left(8-6d+d^2\right)m_\Delta^4\right)M_0^2+4m_\Delta\left((-2+d)Q^2+2(-7+4d)m_\Delta^2\right)M_0^3+3\left((-2+d)^2Q^2-2\left(-2-2d+d^2\right)m_\Delta^2\right)M_0^4\Big)\Big)$$
$$(1+\tau[3])\left(\pi\mathrm{II}[2+d]\left[\{\{0,m_\pi\},1\},\{\{\mathrm{pr},m_\Delta\},1\},\{\{\mathrm{pr}+q,m_\Delta\},2\}\right]+\pi\mathrm{II}[2+d]\left[\{\{0,m_\pi\},1\},\{\{\mathrm{pr}+q,m_\Delta\},1\},\{\{\mathrm{pr},m_\Delta\},2\}\right]\right)\Big)-$$
$$\frac{1}{6(-1+d)^2F^2m_\Delta^4M_0\left(-s+M_0^2\right)}\left(8ie^2\mathrm{g}\pi\mathrm{N}\Delta^2\pi^2\left(\left(Q^2+s\right)^2+2\left(Q^2-s\right)M_0^2+M_0^4\right)\Big(2(18+(-8+d)d)m_\Delta^6-4(-17+5d)m_\Delta^5M_0-(-2+d)^2Q^2\left(-s+m_\pi^2\right)\left(m_\pi-M_0\right)\left(m_\pi+M_0\right)+\right.$$
$$2(-2+d)Q^2m_\Delta M_0\left(2s-3m_\pi^2+M_0^2\right)+2m_\Delta^3M_0\left(6Q^2-3dQ^2-22s+8ds+(34-10d)m_\pi^2+2(-6+d)M_0^2\right)-$$
$$m_\Delta^4\left(44s+2(-10+d)ds+4(6+(-2+d)d)m_\pi^2-12M_0^2+(-2+d)d\left(Q^2+2M_0^2\right)\right)+m_\Delta^2\Big((-2+d)s\left((2+d)Q^2+2(-1+d)s\right)+$$
$$+2(-2+(-4+d)d)m_\pi^4+\left((-4+d)(-2+d)Q^2-2(6+(-2+d)d)s\right)M_0^2 2(-2+d)(-1+d)M_0^4-2m_\pi^2\left((4+(-3+d)d)Q^2+(-2+(-4+d)d)s+(-2+(-4+d)d)M_0^2\right)\Big)\Big)$$
$$(1+\tau[3])\mathrm{II}[4+d]\left[\{\{0,m_\pi\},1\},\{\{p,m_\Delta\},1\},\{\{p+q,m_\Delta\},3\}\right]\Big)+32i\pi^2\left(-\frac{1}{24(-1+d)^2F^2m_\Delta^4M_0\left(-s+M_0^2\right)}\left(e^2\mathrm{g}\pi\mathrm{N}\Delta^2Q^2\left(Q^2+4M_0^2\right)\Big(m_\pi^4\left((-2+d)^2Q^2-2\left(-2-4d+d^2\right)m_\Delta^2\right)-\right.\right.$$
$$2m_\pi^2\left(-m_\Delta^2\left(\left(4-3d+d^2\right)Q^2+2\left(6-2d+d^2\right)m_\Delta^2\right)+\left(-3(-2+d)Q^2m_\Delta+2(17-5d)m_\Delta^3\right)M_0+\left((-2+d)^2Q^2-2\left(-2-4d+d^2\right)m_\Delta^2\right)M_0^2\right)+\left(-m_\Delta^2+M_0^2\right)$$
$$\left(m_\Delta^2\left(36m_\Delta^2+2d\left(Q^2-8m_\Delta^2\right)-d^2\left(Q^2-2m_\Delta^2\right)\right)+\left(-6(-2+d)Q^2m_\Delta+4(17-5d)m_\Delta^3\right)M_0+\left((-2+d)^2Q^2-2\left(-2-4d+d^2\right)m_\Delta^2\right)M_0^2\right)\Big)(1+\tau[3])\Big)+$$
$$\frac{1}{12(-1+d)^2F^2m_\Delta^4M_0\left(-s+M_0^2\right)}\Big(e^2\mathrm{g}\pi\mathrm{N}\Delta^2Q^2\left(Q^2+4M_0^2\right)\Big(m_\pi^4\left((-2+d)^2Q^2-2\left(-2-3d+d^2\right)m_\Delta^2\right)+$$
$$m_\pi^2\left(m_\Delta^2\left(\left(4-7d+2d^2\right)Q^2+2\left(9-6d+2d^2\right)m_\Delta^2\right)+2m_\Delta\left((-2+d)Q^2+(-10+d)m_\Delta^2\right)M_0-2\left((-2+d)^2Q^2-2\left(-2-3d+d^2\right)m_\Delta^2\right)M_0^2\right)+$$
$$\left(-m_\Delta^2+M_0^2\right)\left(m_\Delta^2\left(-\left(2-3d+d^2\right)Q^2+2\left(3-3d+d^2\right)m_\Delta^2\right)-2m_\Delta\left((-2+d)Q^2+(-10+d)m_\Delta^2\right)M_0+\left((-2+d)^2Q^2-2\left(-2-3d+d^2\right)m_\Delta^2\right)M_0^2\right)\Big)(1+\tau[3])\Big)\Big)$$
$$\mathrm{II}[4+d]\left[\{\{0,m_\pi\},1\},\{\{\mathrm{pr},m_\Delta\},1\},\{\{\mathrm{pr}+q,m_\Delta\},3\}\right]+\frac{1}{12(-1+d)^2F^2m_\Delta^4M_0\left(-s+M_0^2\right)}\left(e^2\mathrm{g}\pi\mathrm{N}\Delta^2\left(\left(Q^2+s\right)^2+2\left(Q^2-s\right)M_0^2+M_0^4\right)\right.$$
$$\Big(2(3+(-3+d)d)m_\Delta^6-2(-10+d)m_\Delta^5M_0-(-2+d)^2Q^2\left(-s+m_\pi^2\right)\left(m_\pi-M_0\right)\left(m_\pi+M_0\right)+2(-2+d)Q^2m_\Delta M_0\left(-m_\pi^2+M_0^2\right)-2m_\Delta^3M_0\Big((-2+d)Q^2+ds+(-10+d)m_\pi^2-$$
$$-2(-5+d)M_0^2\Big)m_\Delta^4\left((-2+d)(-1+d)Q^2+2(-3+d)(1+d)s+2(9+2(-3+d)d)m_\pi^2+2(-2+d)^2M_0^2\right)+$$
$$m_\Delta^2\Big((-2+d)s\left((-2+d)Q^2+2(-1+d)s\right)+2(-2+(-3+d)d)m_\pi^4+\left((-2+d)(-1+d)Q^2-2(6+(-3+d)d)s\right)M_0^2+$$
$$2(-2+d)(-1+d)M_0^4+m_\pi^2\left((-4+(7-2d)d)Q^2+4s-2(-3+d)ds+(4-2(-3+d)d)M_0^2\right)\Big)\Big)$$
$$(1+\tau[3])\left(32i\pi^2\mathrm{II}[4+d]\left[\{\{0,m_\pi\},1\},\{\{p,m_\Delta\},1\},\{\{p+q,m_\Delta\},3\}\right]-\right.$$

$$16i\left(2\pi^2\mathrm{II}[4+d]\left[\{\{0,m_\pi\},1\},\{\{p,m_\Delta\},1\},\{\{p+q,m_\Delta\},3\}\right]+\pi^2\mathrm{II}[4+d]\left[\{\{0,m_\pi\},1\},\{\{p+q,m_\Delta\},2\},\{\{p,m_\Delta\},2\}\right]\right)\Big)\Big) -$$

$$\frac{1}{6(-1+d)^2F^2m_\Delta^4M_0\left(-s+M_0^2\right)}\left(8ie^2\mathrm{g}\pi\mathrm{N}\Delta^2Q^2\left(Q^2+4M_0^2\right)\left(m_\pi^4\left((-2+d)^2Q^2-2\left(-2-3d+d^2\right)m_\Delta^2\right)+\right.\right.$$

$$m_\pi^2\left(m_\Delta^2\left(\left(4-7d+2d^2\right)Q^2+2\left(9-6d+2d^2\right)m_\Delta^2\right)+2m_\Delta\left((-2+d)Q^2+(-10+d)m_\Delta^2\right)M_0-2\left((-2+d)^2Q^2-2\left(-2-3d+d^2\right)m_\Delta^2\right)M_0^2\right)+$$

$$\left(-m_\Delta^2+M_0^2\right)\left(m_\Delta^2\left(-\left(2-3d+d^2\right)Q^2+2\left(3-3d+d^2\right)m_\Delta^2\right)-2m_\Delta\left((-2+d)Q^2+(-10+d)m_\Delta^2\right)M_0+\left((-2+d)^2Q^2-2\left(-2-3d+d^2\right)m_\Delta^2\right)M_0^2\right)\Big)(1+\tau[3])$$

$$\left(2\pi^2\mathrm{II}[4+d]\left[\{\{0,m_\pi\},1\},\{\{\mathrm{pr},m_\Delta\},1\},\{\{\mathrm{pr}+q,m_\Delta\},3\}\right]+\pi^2\mathrm{II}[4+d]\left[\{\{0,m_\pi\},1\},\{\{\mathrm{pr}+q,m_\Delta\},2\},\{\{\mathrm{pr},m_\Delta\},2\}\right]\right)\Big)$$

$$\mathbf{B}(\nu,Q^2)=\frac{1}{24(-1+d)^3F^2m_\Delta^4M_0^5}\left(ie^2\mathrm{g}\pi\mathrm{N}\Delta^2(1+\tau[3])\right.$$

$$\Big(\frac{1}{ds^3}\Big(\Big(-32sm_\pi^2M_0^2\left(Q^2s\left(m_\pi^2-m_\Delta^2\right)-Q^2sm_\Delta M_0+\left(\left(Q^2+s\right)m_\pi^2-sm_\Delta^2-Q^2\left(s+m_\Delta^2\right)\right)M_0^2+\left(-s+m_\pi^2-m_\Delta^2\right)M_0^4\right)+d^4\Big(-Q^2s^2\left(m_\pi^2-m_\Delta^2\right)^3+2Q^2s^2m_\Delta\left(m_\pi^2-m_\Delta^2\right)^2M_0+$$

$$s\left(m_\pi^2-m_\Delta^2\right)\left(-\left(Q^2-2s\right)m_\pi^4-2sm_\Delta^4+Q^2m_\Delta^2\left(s-m_\Delta^2\right)+Q^2m_\pi^2\left(3s+2m_\Delta^2\right)\right)M_0^2+2sm_\Delta\left(\left(Q^2-2s\right)m_\pi^4+2sm_\Delta^4-2Q^2m_\pi^2\left(s+m_\Delta^2\right)+Q^2\left(-2sm_\Delta^2+m_\Delta^4\right)\right)M_0^3-$$

$$\left(\left(Q^2-2s\right)m_\pi^6-2sm_\Delta^4\left(s+m_\Delta^2\right)+Q^2m_\Delta^2\left(s^2+sm_\Delta^2-m_\Delta^4\right)+m_\pi^4\left(-3Q^2\left(s+m_\Delta^2\right)+2s\left(3s+m_\Delta^2\right)\right)+m_\pi^2\left(2sm_\Delta^2\left(-2s+m_\Delta^2\right)+Q^2\left(3s^2+2sm_\Delta^2+3m_\Delta^4\right)\right)\right)M_0^4+$$

$$2sm_\Delta\left(s^2+m_\pi^4-4sm_\Delta^2-2m_\pi^2m_\Delta^2+m_\Delta^4\right)M_0^5+\left(s^3-m_\pi^6-5s^2m_\Delta^2-3sm_\Delta^4+m_\Delta^6+m_\pi^4\left(s+3m_\Delta^2\right)+m_\pi^2\left(s^2+2sm_\Delta^2-3m_\Delta^4\right)\right)M_0^6\Big)+$$

$$2d^3\Big(Q^2s^2\left(m_\pi^2-m_\Delta^2\right)^3-3Q^2s^2m_\Delta\left(m_\pi^2-m_\Delta^2\right)^2M_0+s\left(m_\pi^2-m_\Delta^2\right)\left(\left(Q^2-5s\right)m_\pi^4-Q^2sm_\Delta^2+Q^2m_\Delta^4+5sm_\Delta^4-Q^2m_\pi^2\left(3s+2m_\Delta^2\right)\right)M_0^2+$$

$$sm_\Delta\left(\left(-3Q^2+16s\right)m_\pi^4+6Q^2sm_\Delta^2-3Q^2m_\Delta^4-8sm_\Delta^4+m_\pi^2\left(-8sm_\Delta^2+6Q^2\left(s+m_\Delta^2\right)\right)\right)M_0^3+$$

$$\left(\left(Q^2-5s\right)m_\pi^6+3Q^2s^2m_\Delta^2+Q^2sm_\Delta^4+7s^2m_\Delta^4-Q^2m_\Delta^6-5sm_\Delta^6+m_\pi^4\left(-3Q^2\left(s+m_\Delta^2\right)+s\left(19s+5m_\Delta^2\right)\right)+m_\pi^2\left(5sm_\Delta^2\left(-2s+m_\Delta^2\right)+Q^2\left(5s^2+2sm_\Delta^2+3m_\Delta^4\right)\right)\right)$$

$$M_0^4+sm_\Delta\left(-3s^2-5m_\pi^4+24sm_\Delta^2-5m_\Delta^4+2m_\pi^2\left(2s+5m_\Delta^2\right)\right)M_0^5+\left(-2s^3+m_\pi^6+13s^2m_\Delta^2+5sm_\Delta^4-m_\Delta^6+m_\pi^4\left(s-3m_\Delta^2\right)+m_\pi^2\left(-5s^2-6sm_\Delta^2+3m_\Delta^4\right)\right)M_0^6\Big)-$$

$$8d\Big(Q^2s^2\left(m_\pi^2-m_\Delta^2\right)^3-Q^2s^2m_\Delta\left(m_\pi^2-m_\Delta^2\right)^2M_0+s\left(m_\pi^2-m_\Delta^2\right)\left(\left(Q^2+s\right)m_\pi^4-Q^2sm_\Delta^2+Q^2m_\Delta^4+sm_\Delta^4-m_\pi^2\left(7Q^2s+2Q^2m_\Delta^2+2sm_\Delta^2\right)\right)M_0^2-$$

$$sm_\Delta\left(\left(Q^2+7s\right)m_\pi^4+\left(Q^2+6s\right)m_\Delta^4-m_\pi^2\left(5Q^2s+2Q^2m_\Delta^2+13sm_\Delta^2\right)\right)M_0^3+$$

$$\left(\left(Q^2+s\right)m_\pi^6-m_\pi^4\left(7Q^2s+11s^2+3Q^2m_\Delta^2+3sm_\Delta^2\right)-m_\Delta^2\left(sm_\Delta^2\left(11s+m_\Delta^2\right)+Q^2\left(2s^2-sm_\Delta^2+m_\Delta^4\right)\right)+m_\pi^2\left(sm_\Delta^2\left(14s+3m_\Delta^2\right)+Q^2\left(5s^2+6sm_\Delta^2+3m_\Delta^4\right)\right)\right)$$

$$M_0^4-s^2m_\Delta\left(s+2m_\pi^2+5m_\Delta^2\right)M_0^5+\left(m_\pi^6-4s^2m_\Delta^2+sm_\Delta^4-m_\Delta^6-m_\pi^4\left(7s+3m_\Delta^2\right)+m_\pi^2\left(7s^2+6sm_\Delta^2+3m_\Delta^4\right)\right)M_0^6\Big)+$$

$$4d^2\Big(Q^2s^2\left(m_\pi^2-m_\Delta^2\right)^3+s\left(m_\pi^2-m_\Delta^2\right)\left(\left(Q^2+4s\right)m_\pi^4-2sm_\Delta^4-m_\pi^2\left(5Q^2s+2Q^2m_\Delta^2+2sm_\Delta^2\right)+Q^2\left(-sm_\Delta^2+m_\Delta^4\right)\right)M_0^2+$$

$$s^2m_\Delta\left(-19m_\pi^4-3m_\Delta^2\left(Q^2+m_\Delta^2\right)+m_\pi^2\left(Q^2+22m_\Delta^2\right)\right)M_0^3+\left(\left(Q^2+4s\right)m_\pi^6+2sm_\Delta^4\left(-11s+m_\Delta^2\right)-m_\pi^4\left(5Q^2s+22s^2+3Q^2m_\Delta^2+6sm_\Delta^2\right)-\right.$$

$$Q^2\left(5s^2m_\Delta^2-sm_\Delta^4+m_\Delta^6\right)+m_\pi^2\left(20s^2m_\Delta^2+Q^2\left(s^2+4sm_\Delta^2+3m_\Delta^4\right)\right)\Big)M_0^4+sm_\Delta\left(3m_\pi^4-20sm_\Delta^2+3m_\Delta^4-6m_\pi^2\left(s+m_\Delta^2\right)\right)M_0^5+$$

$$\left(s^3+m_\pi^6-13s^2m_\Delta^2-sm_\Delta^4-m_\Delta^6-m_\pi^4\left(7s+3m_\Delta^2\right)+m_\pi^2\left(9s^2+8sm_\Delta^2+3m_\Delta^4\right)\right)M_0^6\Big)\Big)\mathrm{II}[d]\left[\{\{0,m_\pi\},1\}\right]\Big)+$$

$$\frac{1}{s-M_0^2}\Big(M_0^5\Big(\frac{1}{M_0^5}\left(\left(m_\pi^2-m_\Delta^2+M_0^2\right)\left((-2+d)^2Q^2\left(m_\pi^2-m_\Delta^2\right)+2m_\Delta\left((-2+d)Q^2-2(-2+d)m_\pi^2-2m_\Delta^2\right)M_0-\left((-2+d)^2Q^2+4\left(5-4d+d^2\right)m_\Delta^2\right)\right.\right.$$

$$M_0^2+4(-2+d)m_\Delta M_0^3\Big)\Big((2+d)\left(m_\pi^2-m_\Delta^2\right)^2+2\left((-4+d)m_\pi^2-(2+d)m_\Delta^2\right)M_0^2+(2+d)M_0^4\Big)\Big)+$$

$$\frac{2\left(m_\pi^2-m_\Delta^2+M_0^2\right)\left(\left(m_\pi^2-m_\Delta^2\right)^2-2\left(m_\pi^2+m_\Delta^2\right)M_0^2+M_0^4\right)\left(4m_\Delta^3+(16+3(-4+d)d)m_\Delta^2M_0+(-2+d)^2M_0\left(m_\pi^2-M_0^2\right)-2(-2+d)m_\Delta\left(-2m_\pi^2+M_0^2\right)\right)}{M_0^4}+$$

$$8(-1+d)m_\Delta\Big(3(-2+d)m_\pi^4-\left(-m_\Delta+M_0\right)\left(m_\Delta+M_0\right)^2\left(dm_\Delta+(-2+d)M_0\right)+m_\pi^2\left(2(-1+2d)m_\Delta^2+2(3+d)m_\Delta M_0-2(-2+d)M_0^2\right)\Big)-$$

$$\frac{4\left(\left(m_\pi^2-m_\Delta^2\right)^2-2\left(m_\pi^2+m_\Delta^2\right)M_0^2+M_0^4\right)\left((-2+d)m_\pi^2\left((2+d)m_\Delta+(-2+d)M_0\right)-\left(m_\Delta+M_0\right)\left(-\left(4-2d+d^2\right)m_\Delta^2+2(-2+d)m_\Delta M_0+(-2+d)^2M_0^2\right)\right)}{M_0^2}-$$

$$\frac{1}{M_0^4}\Big(2\Big(-4m_\pi^2M_0^2+d\left(m_\pi^2-m_\Delta^2+M_0^2\right)^2\Big)\left(2m_\Delta^5+(2+(-2+d)d)m_\Delta^4M_0+(-2+d)^2M_0\left(m_\pi^2-M_0^2\right)\left(2Q^2+m_\pi^2-M_0^2\right)+m_\Delta^3\left(-(-2+d)dQ^2+2(-3+d)m_\pi^2-\right.\right.$$

$$2\left(13-7d+2d^2\right)M_0^2\right)-2m_\Delta^2 M_0\left(\left(3-3d+d^2\right)m_\pi^2+\left(25-15d+3d^2\right)M_0^2\right)+(-2+d)m_\Delta\left(dQ^2m_\pi^2-2m_\pi^4-\left((-4+d)Q^2+8m_\pi^2\right)M_0^2+10M_0^4\right)\right)\right)+$$
$$\frac{1}{M_0^2}\left(4(-1+d)\left(m_\pi^2-m_\Delta^2+M_0^2\right)\left((-2+d)m_\pi^4\left((-8+d)m_\Delta+(-2+d)M_0\right)-m_\pi^2\left(m_\Delta\left(\left(2+d-d^2\right)Q^2+2\left(15-7d+d^2\right)m_\Delta^2\right)+\right.\right.\right.$$
$$\left(-(-2+d)^2Q^2+2\left(10-4d+d^2\right)m_\Delta^2\right)M_0+2(-2+d)^2m_\Delta M_0^2+2(-2+d)^2M_0^3\right)+(m_\Delta+M_0)$$
$$\left(m_\Delta^2\left(\left(4-3d+d^2\right)Q^2+\left(14-4d+d^2\right)m_\Delta^2\right)+\left(-(-2+d)Q^2m_\Delta+2(9-4d)m_\Delta^3\right)M_0-\left((-2+d)^2Q^2+2\left(6-3d+d^2\right)m_\Delta^2\right)M_0^2+\right.$$
$$\left.6(-2+d)m_\Delta M_0^3+(-2+d)^2M_0^4\right)\right)\right)\right)\mathrm{II}[d]\left[\{\{0,m_\pi\},1\},\{\{p,m_\Delta\},1\}\right]\right)-$$
$$\frac{1}{-s+M_0^2}\left(M_0^5\left(\frac{4\left(m_\pi^4+\left(s-m_\Delta^2\right)^2-2m_\pi^2\left(s+m_\Delta^2\right)\right)\left((-2+d)\left(-ds+(2+d)m_\pi^2\right)m_\Delta+(4+(-2+d)d)m_\Delta^3+(-2+d)^2\left(-s+m_\pi^2\right)M_0+(8+(-4+d)d)m_\Delta^2M_0\right)}{s}-\right.\right.$$
$$\frac{2\left(s+m_\pi^2-m_\Delta^2\right)\left(m_\pi^4+\left(s-m_\Delta^2\right)^2-2m_\pi^2\left(s+m_\Delta^2\right)\right)\left(4m_\Delta^3+(-2+d)^2\left(-s+m_\pi^2\right)M_0+(16+3(-4+d)d)m_\Delta^2M_0-2(-2+d)m_\Delta\left(-2m_\pi^2+M_0^2\right)\right)}{s^2}+$$
$$\frac{1}{s^3}\left(\left(s+m_\pi^2-m_\Delta^2\right)\left((2+d)m_\pi^4+(2+d)\left(s-m_\Delta^2\right)^2+2m_\pi^2\left((-4+d)s-(2+d)m_\Delta^2\right)\right)\right.$$
$$\left(4sm_\Delta^3+(-2+d)^2\left(s-m_\pi^2\right)M_0\left(Q^2-s+M_0^2\right)-2(-2+d)sm_\Delta\left(Q^2+s-2m_\pi^2+M_0^2\right)+m_\Delta^2M_0\left((16+3(-4+d)d)s+(-2+d)^2\left(Q^2+M_0^2\right)\right)\right)\right)-$$
$$8(-1+d)m_\Delta\left(3(-2+d)m_\pi^4-(m_\Delta+M_0)^2\left((-2+d)s+m_\Delta\left(-dm_\Delta+2M_0\right)\right)+m_\pi^2\left(2(-1+2d)m_\Delta^2+2(3+d)m_\Delta M_0+(-2+d)\left(-3s+M_0^2\right)\right)\right)-$$
$$\frac{1}{s}\left(4(-1+d)\left(s+m_\pi^2-m_\Delta^2\right)\left((14+(-4+d)d)m_\Delta^5+(-2+d)^2\left(-s+m_\pi^2\right)\left(Q^2-s+m_\pi^2\right)M_0+(-8+d)(-4+d)m_\Delta^4M_0+m_\Delta^3\right.\right.$$
$$\left((4+(-3+d)d)Q^2-4(-1+d)^2s-2(15+(-7+d)d)m_\pi^2+2(5+(-5+d)d)M_0^2\right)+m_\Delta^2M_0\left(6Q^2-34s-4(-5+d)ds-2(10+(-4+d)d)m_\pi^2+\right.$$
$$\left.10M_0^2+(-4+d)d\left(Q^2+2M_0^2\right)\right)+(-2+d)m_\Delta\left((-8+d)m_\pi^4+m_\pi^2\left(Q^2+dQ^2+6s-2ds-2M_0^2\right)+s\left(Q^2-dQ^2+(2+d)s+2M_0^2\right)\right)\right)\right)+$$
$$\frac{1}{s^2}\left(2\left(-4sm_\pi^2+d\left(s+m_\pi^2-m_\Delta^2\right)^2\right)\left(2m_\Delta^5+(2+(-2+d)d)m_\Delta^4M_0+(-2+d)^2\left(-s+m_\pi^2\right)M_0\left(2Q^2-2s+m_\pi^2+M_0^2\right)-m_\Delta^3\right.\right.$$
$$\left((-2+d)dQ^2+(22+d(-10+3d))s-2(-3+d)m_\pi^2+(-2+d)^2M_0^2\right)+m_\Delta^2M_0\left((-54+(34-7d)d)s-2(3+(-3+d)d)m_\pi^2+(-2+d)^2M_0^2\right)+$$
$$\left.\left.(-2+d)m_\Delta\left(-2m_\pi^4+m_\pi^2\left(dQ^2-(6+d)s+(-2+d)M_0^2\right)+s\left((6+d)s-(-4+d)\left(Q^2+M_0^2\right)\right)\right)\right)\right)\right)\right)\mathrm{II}[d]\left[\{\{0,m_\pi\},1\},\{\{p+q,m_\Delta\},1\}\right]\right)+$$
$$\frac{1}{-s+M_0^2}\left(8(-1+d)M_0^5\left(m_\Delta\left(m_\pi^2-(m_\Delta+M_0)^2\right)\left((-2+d)m_\pi^4-(-m_\Delta+M_0)(m_\Delta+M_0)\left((-2+d)s+m_\Delta\left(-dm_\Delta+2M_0\right)\right)+m_\pi^2\left(2m_\Delta^2+6m_\Delta M_0+(-2+d)\left(-s+M_0^2\right)\right)\right)\right.\right.$$
$$\mathrm{II}[d]\left[\{\{0,m_\pi\},1\},\{\{p+q,m_\Delta\},1\},\{\{p,m_\Delta\},1\}\right]-m_\Delta\left(-m_\pi^2+(m_\Delta+M_0)^2\right)$$
$$\left(-(-2+d)m_\pi^4-2m_\pi^2m_\Delta(m_\Delta+3M_0)+(-m_\Delta+M_0)^2(m_\Delta+M_0)\left(dm_\Delta+(-2+d)M_0\right)\right)\mathrm{II}[d]\left[\{\{0,m_\pi\},1\},\{\{\mathrm{pr}+q,m_\Delta\},1\},\{\{\mathrm{pr},m_\Delta\},1\}\right]+$$
$$4\pi\left(2(-2+d)m_\pi^6m_\Delta+m_\pi^4\left(2(7+(-5+d)d)m_\Delta^3-(-2+d)^2Q^2M_0+2(10+(-5+d)d)m_\Delta^2M_0-(-2+d)m_\Delta\left(dQ^2+s-M_0^2\right)\right)+\right.$$
$$m_\pi^2\left(-2(8+d(-7+2d))m_\Delta^5-4(9+(-4+d)d)m_\Delta^4M_0+(-2+d)^2Q^2M_0\left(s+M_0^2\right)-2m_\Delta^3\left((-2+d)dQ^2+(-5+(-1+d)d)s+(19+(-7+d)d)M_0^2\right)-\right.$$
$$2m_\Delta^2M_0\left((4+(-3+d)d)Q^2+(-4+(-1+d)d)s+(6+(-3+d)d)M_0^2\right)+(-2+d)m_\Delta\left(\left((-2+d)Q^2-s\right)s+\left(dQ^2-2s\right)M_0^2-3M_0^4\right)\right)+$$
$$(m_\Delta+M_0)\left(2(3+(-3+d)d)m_\Delta^6+10m_\Delta^5M_0-(-2+d)^2Q^2sM_0^2+(-2+d)sm_\Delta M_0\left(s+3M_0^2\right)-m_\Delta^3M_0\left((4+d)s+(-2+3d)M_0^2\right)-\right.$$
$$m_\Delta^4\left((4+(-2+d)d)Q^2+(16+d(-7+2d))s+(-18+d(-1+2d))M_0^2\right)+m_\Delta^2\left((-2+d)s\left(-s+d\left(Q^2+2s\right)\right)+\left((8+(-4+d)d)Q^2+\right.\right.$$
$$\left.\left.\left.(-22+(11-2d)d)s\right)M_0^2+2(6+(-4+d)d)M_0^4\right)\right)\right)\mathrm{II}[2+d]\left[\{\{0,m_\pi\},1\},\{\{p,m_\Delta\},1\},\{\{p+q,m_\Delta\},2\}\right]-$$
$$4\pi\left(2(-2+d)m_\pi^6m_\Delta+m_\pi^4\left(14m_\Delta^3+2dm_\Delta\left(Q^2-5m_\Delta^2\right)+d^2\left(-Q^2m_\Delta+2m_\Delta^3\right)+\left(-(-2+d)^2Q^2+2\left(10-5d+d^2\right)m_\Delta^2\right)M_0\right)+\right.$$
$$(-m_\Delta+M_0)(m_\Delta+M_0)^2\left(m_\Delta^2\left(\left(4-2d+d^2\right)Q^2-2\left(3-3d+d^2\right)m_\Delta^2\right)-10m_\Delta^3M_0-\left((-2+d)^2Q^2+2\left(4+d-d^2\right)m_\Delta^2\right)M_0^2+4(-2+d)m_\Delta M_0^3\right)-$$
$$2m_\pi^2\left(m_\Delta^3\left(8m_\Delta^2+d^2\left(Q^2+2m_\Delta^2\right)-d\left(2Q^2+7m_\Delta^2\right)\right)+m_\Delta^2\left(\left(4-3d+d^2\right)Q^2+2\left(9-4d+d^2\right)m_\Delta^2\right)M_0+\right.$$
$$\left.\left.m_\Delta\left(-\left(2-3d+d^2\right)Q^2+2\left(7-4d+d^2\right)m_\Delta^2\right)M_0^2+\left(-(-2+d)^2Q^2+2(-1+d)^2m_\Delta^2\right)M_0^3+3(-2+d)m_\Delta M_0^4\right)\right)$$
$$\mathrm{II}[2+d]\left[\{\{0,m_\pi\},1\},\{\{\mathrm{pr},m_\Delta\},1\},\{\{\mathrm{pr}+q,m_\Delta\},2\}\right]-8\pi m_\Delta M_0\left(-m_\pi+m_\Delta+M_0\right)\left(m_\pi+m_\Delta+M_0\right)$$
$$\left(2m_\Delta\left(m_\pi^2-m_\Delta^2\right)+\left(2s-ds+(-2+d)m_\pi^2+dm_\Delta^2\right)M_0\right)\mathrm{II}[2+d]\left[\{\{0,m_\pi\},1\},\{\{p+q,m_\Delta\},1\},\{\{p,m_\Delta\},2\}\right]-8\pi m_\Delta M_0\left(-m_\pi+m_\Delta+M_0\right)$$
$$\left(m_\pi+m_\Delta+M_0\right)\left(-2m_\pi^2m_\Delta+2m_\Delta^3-\left((-2+d)m_\pi^2+dm_\Delta^2\right)M_0+(-2+d)M_0^3\right)\mathrm{II}[2+d]\left[\{\{0,m_\pi\},1\},\{\{\mathrm{pr}+q,m_\Delta\},1\},\{\{\mathrm{pr},m_\Delta\},2\}\right]+$$

$$32\pi^2 \left(2(-2+d)(-1+d)m_\Delta^6 M_0 - 2m_\Delta^5 \left(-Q^2+s+M_0^2\right) + (-2+d)^2 Q^2 \left(-s+m_\pi^2\right) M_0 \left(-m_\pi^2+M_0^2\right) - m_\Delta^4 M_0 \left(\left(-8+d^2\right) Q^2 + 2(10+(-3+d)d)s + 4(-2+d)\right.\right.$$
$$(-1+d)m_\pi^2 + 2(-2+d)(-1+d)M_0^2\big) + (-2+d)m_\Delta \left(2m_\pi^4 \left(-Q^2+s+M_0^2\right) + s\left(Q^2+s+M_0^2\right)\left(-Q^2+2M_0^2\right) - m_\pi^2 \left(-Q^2+s+M_0^2\right)\left(Q^2+2s+2M_0^2\right)\right) -$$
$$m_\Delta^3 \left(2(-4+d)s^2 + \left(-Q^2+M_0^2\right)\left(dQ^2+2(-3+d)m_\pi^2+2M_0^2\right) + s\left(3(-4+d)Q^2+2(-3+d)m_\pi^2+(34-6d)M_0^2\right)\right) +$$
$$m_\Delta^2 M_0 \left(2Q^4 + (-2+d)^2Q^2 s + 2(-1+(-2+d)d)s^2 + 2(-2+d)(-1+d)m_\pi^4 + \left((8+(-4+d)d)Q^2 - 2(14+(-7+d)d)s\right) M_0^2 + 2(5+(-4+d)d)\right.$$
$$M_0^4 - 2m_\pi^2 \left((6+(-4+d)d)Q^2 + \left(-10+d+d^2\right)s + (-2+d)(-1+d)M_0^2\right)\big)\big)\, \mathrm{II}[4+d]\left[\{\{0,m_\pi\},1\},\{\{p,m_\Delta\},1\},\{\{p+q,m_\Delta\},3\}\right] -$$
$$32\pi^2 \left((-2+d)m_\pi^4 \left(-2Q^2 m_\Delta + \left(-(-2+d)Q^2 + 2(-1+d)m_\Delta^2\right) M_0 + 4m_\Delta M_0^2\right) + m_\pi^2 \left(Q^2 m_\Delta \left((-2+d)Q^2 + 2(-3+d)m_\Delta^2\right) -\right.\right.$$
$$2m_\Delta^2 \left(\left(6-4d+d^2\right)Q^2 + 2\left(2-3d+d^2\right)m_\Delta^2\right) M_0 + 2m_\Delta \left((-2+d)Q^2 - 2(-3+d)m_\Delta^2\right) M_0^2 + 2\left((-2+d)^2Q^2 + 2\left(4+d-d^2\right)m_\Delta^2\right) M_0^3 - 8(-2+d)m_\Delta M_0^4\big) +$$
$$(m_\Delta+M_0)\left(Q^2 m_\Delta^2 \left(dQ^2+2m_\Delta^2\right) + \left(-(-2+d)Q^4 m_\Delta - \left(-6+d^2\right)Q^2 m_\Delta^3 + 2\left(2-3d+d^2\right)m_\Delta^5\right) M_0 + m_\Delta^2 \left(\left(8-4d+d^2\right)Q^2 - 2\left(4-3d+d^2\right)m_\Delta^2\right) M_0^2 +\right.$$
$$m_\Delta \left((-2+d)^2Q^2 - 2\left(8-3d+d^2\right)m_\Delta^2\right) M_0^3 - \left((-2+d)^2Q^2 + 2\left(6+d-d^2\right)m_\Delta^2\right) M_0^4 + 4(-2+d)m_\Delta M_0^5\big)\big)$$
$$\mathrm{II}[4+d]\left[\{\{0,m_\pi\},1\},\{\{\mathrm{pr},m_\Delta\},1\},\{\{\mathrm{pr}+q,m_\Delta\},3\}\right] + 16\pi^2 M_0 \left(2(-2+d)(-1+d)m_\Delta^6 - 4m_\Delta^5 M_0 + 2(-2+d)\right.$$
$$\left(-Q^2-2s+2m_\pi^2\right) m_\Delta (m_\pi - M_0) M_0 (m_\pi + M_0) + (-2+d)^2 Q^2 \left(-s+m_\pi^2\right)(-m_\pi+M_0)(m_\pi+M_0) - 2m_\Delta^3 M_0 \left(dQ^2+2s+2(-3+d)m_\pi^2 - 2(-6+d)M_0^2\right) -$$
$$m_\Delta^4 \left(d^2Q^2 + 2(6+(-3+d)d)s + 4(-2+d)(-1+d)m_\pi^2 + 2(6+(-3+d)d)M_0^2\right) + m_\Delta^2 \left((-2+d)s\left(dQ^2+2(-1+d)s\right) + 2(-2+d)(-1+d)m_\pi^4 +\right.$$
$$\left((-2+d)dQ^2 - 2(14+(-7+d)d)s\right) M_0^2 + 2(-2+d)(-1+d)M_0^4 + m_\pi^2 \left(8s - 2d^2\left(Q^2+s\right) + 2d\left(2Q^2+s\right) + (8-2(-1+d)d)M_0^2\right)\big)\big)$$
$$\mathrm{II}[4+d]\left[\{\{0,m_\pi\},1\},\{\{p+q,m_\Delta\},2\},\{\{p,m_\Delta\},2\}\right] - 16\pi^2 M_0 \left(-(-2+d)m_\pi^4 \left((-2+d)Q^2 + 2m_\Delta (m_\Delta - dm_\Delta - 2M_0)\right) -\right.$$
$$2m_\pi^2 \left((-2+d)m_\Delta^2 \left(-2m_\Delta^2 + d\left(Q^2+2m_\Delta^2\right)\right) + m_\Delta \left((-2+d)Q^2 + 2(-3+d)m_\Delta^2\right) M_0 - \left((-2+d)^2Q^2 + 2\left(4+d-d^2\right)m_\Delta^2\right) M_0^2 + 4(-2+d)m_\Delta M_0^3\right) +$$
$$(m_\Delta+M_0)\left(4m_\Delta^5 - 6dm_\Delta^5 + d^2\left(-Q^2m_\Delta^3 + 2m_\Delta^5\right) + m_\Delta^2 \left(-8m_\Delta^2 - 2d\left(Q^2-3m_\Delta^2\right) + d^2\left(Q^2-2m_\Delta^2\right)\right) M_0 + m_\Delta \left(-16m_\Delta^2 - 2d\left(Q^2-3m_\Delta^2\right) + d^2\left(Q^2-2m_\Delta^2\right)\right) M_0^2 -\right.$$
$$\left((-2+d)^2Q^2 + 2\left(6+d-d^2\right)m_\Delta^2\right) M_0^3 + 4(-2+d)m_\Delta M_0^4\big)\big)\, \mathrm{II}[4+d]\left[\{\{0,m_\pi\},1\},\{\{\mathrm{pr}+q,m_\Delta\},2\},\{\{\mathrm{pr},m_\Delta\},2\}\right]\big)\big)\big)$$

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